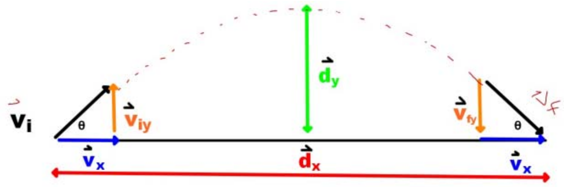
1.12 Projectile Motion (II)

Projectiles Launched at an Angle:



Notice the symmetry:

$\vec{v}_{iy} = -\vec{v}_{fy}$ - equal in magnitude, opposite in direction.

$\vec{v}_x = \vec{v}_x$ - the horizontal velocity does not change.

$\theta = \theta$ - the angle of launch = angle of landing

$t = t$ - the time it takes the projectile to travel up and down is equal to the time it takes the projectile to travel $\vec{d}_x$.

1. How long is the projectile in the air?

If you are given $\vec{v}_{iy}$, you can find t.

Recall:

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

As $\vec{v}_{iy} = -\vec{v}_{fy}$ and $\vec{a} = \vec{g}$,

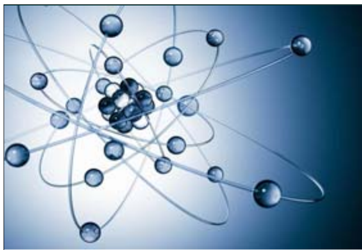
$$\vec{a} = \frac{(-\vec{v}_{iy} - \vec{v}_{iy})}{t}$$

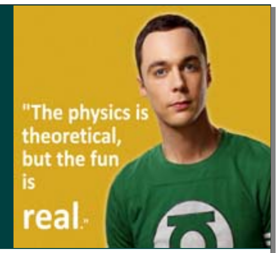
$$\vec{g} = \frac{(-\vec{v}_{iy} - \vec{v}_{iy})}{t}$$

$$\boxed{t = \frac{-2\vec{v}_{iy}}{\vec{g}}}$$

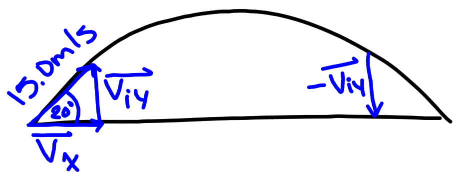
Full-Time Expression

- This is called the full time expression because it gives the amount of time it takes the projectile to travel up and down.





Ex.) An object is thrown at an angle of 20º N of E with a velocity of 15 m/s. How long is the object in the air?



$\sin 20° = \dfrac{\vec{V}_{iy}}{15.0}$

$\vec{V}_{iy} = 5.13\ldots$ m/s

$\vec{a} = \dfrac{\vec{V}_{fy} - \vec{V}_{iy}}{t}$

$\vec{a} = \dfrac{-\vec{V}_{iy} - \vec{V}_{iy}}{t}$

$\vec{a} = \dfrac{-2\vec{V}_{iy}}{t}$

$t = \dfrac{-2\vec{v}_{iy}}{\vec{a}_y}$

$t = \dfrac{-2(5.13\ldots)}{-9.81} = \boxed{1.0 s}$

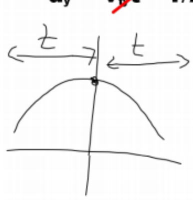
2. How long was the object in the air given d_y?

Recall: at $\vec{d}_y$, the $\vec{v}_y = 0$. This occurs at half the total time of the motion.

$\vec{d}_y = \vec{v}_{fy}t - 1/2\vec{a}t^2$

If we consider the point when half the time has elapsed...

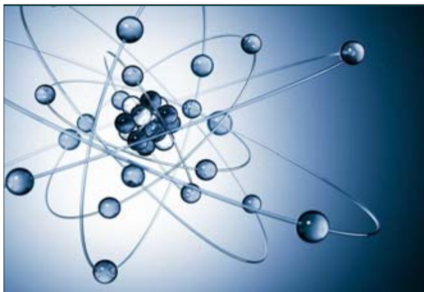
$\vec{d}_y = \cancel{\vec{v}_{fy}t}^{\,0} - 1/2\vec{g}t^2$

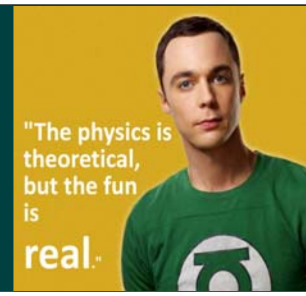


$$\boxed{t = \sqrt{\frac{-2\vec{d}_y}{\vec{g}}}}$$

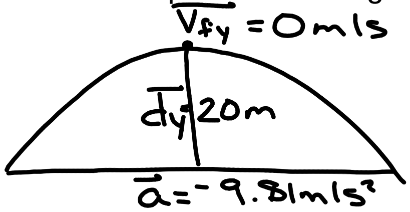
Half-time Expression

This will give half the time the projectile is in the air.





Ex.) Rick shoots a basketball from half-court. The ball reaches a height of 20 m from its release point. How long was the ball in the air?



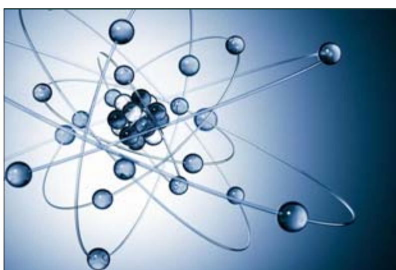
$$\vec{d}_y = \vec{v}_{fy}t - \tfrac{1}{2}\vec{a}_y t^2$$

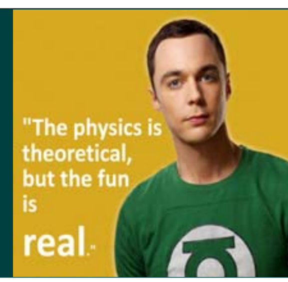
$$20 = \cancel{0t} - (\tfrac{1}{2})(-9.81)t^2$$

$$\sqrt{\frac{20}{4.905}} = \frac{4.905\,t^2}{4.905}$$

$$t = 2.019\ldots$$

$$t_{total} = 2t = \boxed{4.0s}$$





3. How high does it go?

Because we are talking about the y-direction, and there is an acceleration in this direction, we must use the kinematics equations with acceleration:

Use either:

$$\vec{d}_y = \vec{v}_{iy}t + 1/2\vec{g}t^2$$

$$\vec{d}_y = \vec{v}_{fy}t - 1/2\vec{g}t^2$$

$$\vec{v}_{fy}^2 = \vec{v}_{iy}^2 + 2\vec{g}\vec{d}$$

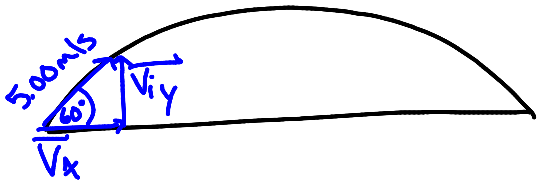
4. How far does it go (range)?

Because the x-direction undergoes uniform motion, we just use:

$$\vec{d}_x = \vec{v}_x t$$

Ex.) A potato gun is fired with a velocity of 5.00 m/s [60°].

a) How long is the potato in the air?



$\sin 60° = \frac{\vec{V_{iy}}}{5.00}$

$\vec{V_{iy}} = 4.3301\ldots \text{ m/s}$

$\vec{a} = \frac{\vec{V_{fy}} - \vec{V_{iy}}}{t}$

$a = \frac{-\vec{V_{iy}} - \vec{V_{iy}}}{t}$

$t = \frac{-2\vec{V_{iy}}}{\vec{a}} = \frac{-2(4.3301\ldots)}{-9.81}$

$\boxed{t = 0.883 s}$

b) How high does the potato go?

$\vec{d_y} = ?$

$\Delta\vec{d_y} = \cancel{\vec{V_{fy}}} t - \frac{1}{2}\vec{a_y} t^2$ (0)

$\Delta\vec{d_y} = -(\frac{1}{2})(-9.81)(\frac{0.883}{2})^2 = \boxed{0.956 \text{ m}}$

c) What is the potato's range?

$\vec{d_x}$

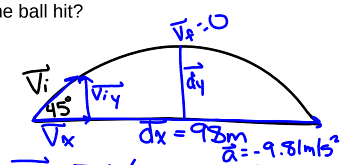
$\vec{d_x} = \vec{V_x} t$

$\cos 60° = \frac{\vec{V_x}}{5.00}$

$\vec{V_x} = 2.5 \text{ m/s}$

$= 2.5(0.883) = \boxed{2.21 \text{ m}}$

Ex.) A baseball player hits a home run into left field. If the player hits the ball at a 45º angle, and the fence is 98 m away from home plate, with what velocity was the ball hit?



$\vec{v_x} = \vec{v_{iy}}$ (because of special Δs)

$\vec{d_x} = \vec{v_x} t$

$\frac{98}{\vec{v_x}} = \frac{\cancel{\vec{v_x}} t}{\cancel{\vec{v_x}}}$ $\quad t = \left(\frac{98}{\vec{v_{iy}}}\right)$

$\vec{a} = \frac{\cancel{\vec{v_f}}^{\,0} - \vec{v_i}}{t}$

$t = \frac{-\vec{v_{iy}}}{a}$

$\frac{98}{\vec{v_{iy}}} = \frac{-\vec{v_{iy}}}{-9.81}$

$-961.38 = -v_{iy}^2$

$\sqrt{961.38} = \vec{v_{iy}}$

$\vec{v_{iy}} = 31.1 m/s$

$\sin 45° = \frac{31.1}{\vec{v_i}}$

$\boxed{\vec{v_i} = 44 m/s}$

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