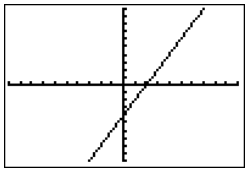
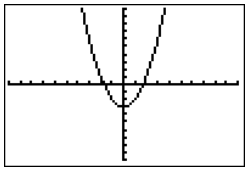
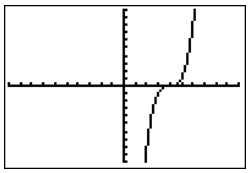
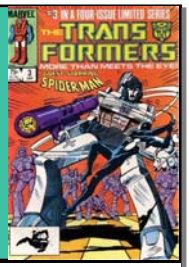
Math 30-1

Unit 1: Transformations

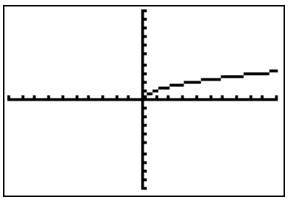
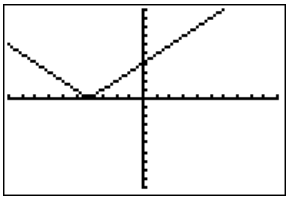
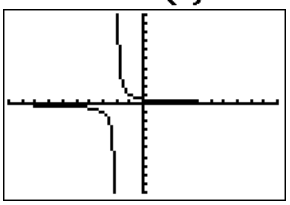
1.1 Translations

Types of Functions:

Linear Functions	Quadratic Functions	Cubic Functions
$f(x) = ax + b$	$f(x) = ax^2 + bx + c$	$f(x) = ax^3 + bx^2 + cx + d$
		



Types of Functions Continued:

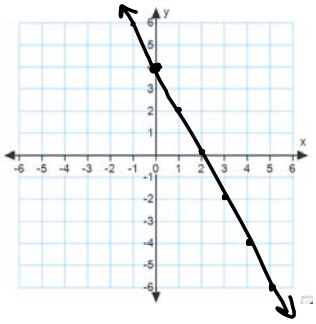
Radical Functions	Absolute Value Functions	Rational Functions/ Reciprocals
$f(x) = \sqrt{x}$	$f(x) = \lvert x \rvert$	$f(x) = \frac{n(x)}{d(x)}$
		

Review: Sketch the following graphs by hand and state the domain and range of the functions in both set notation and interval notation.

Ex.) $y = -2x + 4$ $y = mx + b$

Domain: $\{x \in \mathbb{R}\}$ $(-\infty, \infty)$

Range: $\{y \in \mathbb{R}\}$ $(-\infty, \infty)$



Set-Builder Notation

A *set* is simply a collection of numbers, such as {1, 4, 5}. We use *set-builder notation* to outline the rules governing members of a set.

$\{x \mid x \in R, x \geq -1\}$

State the variable. List conditions on the variable.

In words: "The variable is x, such that x can be any real number with the condition that $x \geq -1$".

As a shortcut, set-builder notation can be reduced to just the most important condition.

$x \geq -1$

While this resource uses the shortcut for brevity, as set-builder notation is covered in previous courses, Math 30-1 students *are* expected to know how to read and write full set-builder notation.

Interval Notation

Math 30-1 students are expected to know that domain and range can be expressed using *interval notation*.

() - **Round Brackets:** Exclude point from interval.
[] - **Square Brackets:** Include point in interval.
Infinity ∞ always gets a round bracket.

Examples: $x \geq -5$ becomes $[-5, \infty)$;
$1 < x \leq 4$ becomes $(1, 4]$;
$x \in R$ becomes $(-\infty, \infty)$;
$-8 \leq x < 2$ or $5 \leq x < 11$ becomes $[-8, 2) \cup [5, 11)$, where U means "or", or *union of sets*;
$x \in R, x \neq 2$ becomes $(-\infty, 2) \cup (2, \infty)$;
$-1 \leq x \leq 3, x \neq 0$ becomes $[-1, 0) \cup (0, 3]$.

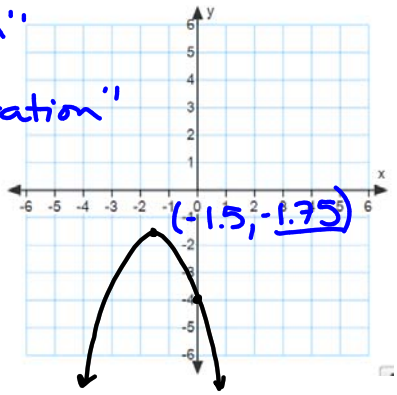
Ex.) $y = -x^2 - 3x - 4$

Domain: $\{x \in \mathbb{R}\}$ "set notation"

$(-\infty, \infty)$ "interval notation"

Range: $\{y \mid y \leq -1.75, y \in \mathbb{R}\}$

$(-\infty, -1.75]$



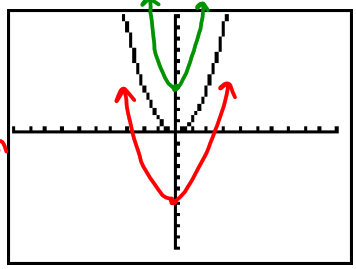
$y = af[b(x-h)]+k$

Vertical Translations

Given the "mother function" y = $f(x) = x^2$. Deduce a meaning for the value of k.

Graph:

- $y = f(x) + 4 = x^2 + 4$ 4up
- $y = f(x) - 6 = x^2 - 6$ 6 down



Thus, $y = f(x)+k$ results in a vertical translation of k units.

Mapping Notation: (x , y) ⟶ (x , y ± k)

x remains the same

$y = af[b(x-h)]+k$

Ex.) Describe the following transformations:

a) $y = f(x) + 12$ VT up 12

b) $y + 4 = f(x) - 3$ VT 7 down

$-4 \quad -4$

$y = f(x) - 7$

c) $(x , y) \longrightarrow (x , y + 3)$ VT 3up

$y = af[b(x-h)]+k$

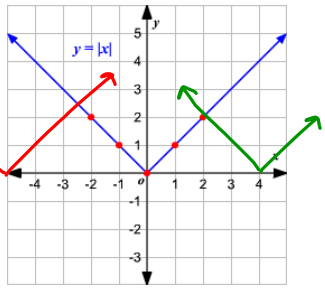
Horizontal Translations

Given the "mother function" y = *f(x)* = |*x*|. Deduce a meaning for the value of *h.*

Graph:

- *y = f(x + 5)* = |x+5|
- *y = f(x - 4)* = |x-4|

|x-5|



Thus, $y = f(x-h)$ results in a horizontal translation of h units.

opposite direction than sign

Mapping Notation: (x , y) ⟶ (x ± h , y)

"What you see is what you get."

do not switch direction/sign

$y = af[b(x-h)]+k$

Ex.) Describe the following transformations:

right +
left −

a) *y = f(x + 1)* f(x-⁻1) HT 1 left

b) *y = f(x - 2)* HT right 2

c) *(x , y)* ⟶ *(x + 3 , y)* HT 3 right

d) *(x , y)* ⟶ *(x - 5 , y)* HT 5 left

$y = af[b(x-h)]+k$

Horizontal and Vertical Translations

Ex.) Given the functions $y = f(x)$, describe the following transformations:

a) $g(x) = f(x + 2) - 7$ HT 2 left VT down 7

b) $(x , y) \longrightarrow (x + 3 , y - 1)$ HT 3 right VT down 1

Ex.) Given point A (1, 7) is on $y = f(x)$, determine the new coordinates $y = f(x + 3) - 5$.

(1,7)

(1-3, 7-5)

(-2,2)

HT 3 left
VT 5 down

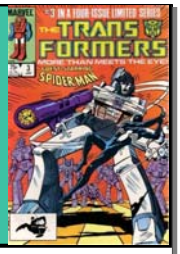
Ex.) Given $y = |x|$, translate the graph 5 units right and 3 units up. Determine the new equation, domain and range.

+

$y = |x-5| + 3$

Domain: $(-\infty, \infty)$

Range: $[3, \infty)$



Ex.) If function f is transformed to a new function $g(x) = f(x - 3) + 2$, then the range of function g will be

A $g(x) \geq -11$

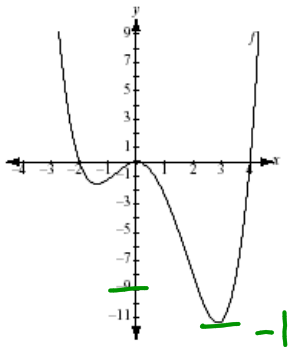
B $g(x) \geq -9$

C $g(x) \geq -8$

D $g(x) \geq 0$

Use the following information to answer the next question.

The partial graph of the function $y = f(x)$ is shown below. The range of function f is $f(x) \geq -11$.



Ex.) As a result of the transformation of the graph of $y = x^3$ into the graph of $y - 4 = (x - 3)^3$, the point (3, 27) becomes point (6, y).

The value of y is 31.

HT 3 right

+4

Round your answer to the nearest whole number.

$y = (x-3)^3 + 4$

HT right 3

VT 4 up

3	1		

Pg. 12 # 2, 4, 5, 10, 11, 18