Unit 1:

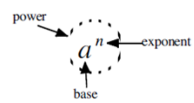
Exponents and Radicals

1.5 Exponents

Exponents can be used to write repeated multiplication.

Ex. 4 x 4 x 4 or 4^3.

A power is a number written in exponential form. It consists of a base and an exponent.



A number that multiplies a variable is called a coefficient. In $4w^2$, 4 is the coefficient.

Ex. Write the following as repeated multiplication while stating the base, exponent and coefficient.

1. $8x^2$ 2. $(-7)^3$ 3. $-2(ab)^4$

1.5(a) The Zero Exponent Law

$(-6)^2 = (-6)(-6) = 36$

Evaluate the following and extend the pattern:

$3^3 = 27$ ÷3
$3^2 = 9$ ÷3
$3^1 = 3$ ÷3
$3^0 = 1$

$-6^3 = -216$ ÷6
$-6^2 = -36$ ÷6
$-6^1 = -6$ ÷6
$-6^0 = -1$

Ex. Evaluate

a) 6^0
$= \boxed{1}$

b) $(-9)^0$
$= \boxed{1}$

c) -9^0 $9^0 = 1$
$= \boxed{-1}$

d) $2(6^2)^0$
$= 2(1)$
$= \boxed{2}$

The Exponent Laws

1. Product Law:
$(a^m)(a^n) = a^{m+n}$

Ex. $a^3 \times a^2 = (axaxa)(axa) = a^5$ +

2. Quotient Law:
$\frac{a^m}{a^n}$ $a^m \div a^n = a^{m-n}$
"top – bottom"

Ex. $a^5 \div a^3 = \frac{\cancel{a}x\cancel{a}x\cancel{a}xaxa}{axaxa} = a^2$

3. Power of a Product Law:
$(ab)^m = a^m b^m$

Ex. $(2a)^3 = (2a)(2a)(2a) = 2^3a^3$ simplify
$= 8a^3$ evaluated

4. Power of a Quotient Law:
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Ex. $\left(\frac{8}{7}\right)^3 = \frac{8\times8\times8}{7\times7\times7} = \frac{8^3}{7^3} = \frac{512}{343}$
exact value

5. Power of a Power Law:
$(a^m)^n = a^{m\times n}$

Ex. $(a^3)^2 = (a^3)(a^3)$
$= (axaxa)(axaxa) = a^6$

"side-by-side, multiply"

Ex 1. Use the exponent laws to simplify and then evaluate.

a) $3^4 \times 3^2$
$= \boxed{3^6}$ simplified
$= \boxed{729}$ evaluated

b) $(2^2)^3$
$= \boxed{2^6}$
$= \boxed{64}$

c) $(-2)^5(-2)^2$
$= \boxed{(-2)^7}$
$= \boxed{-128}$

Ex 2. Use the exponent laws to simplify.

a) $(b)^4(b)^3$
$= \boxed{b^7}$

b) x^6x^1
$= \boxed{x^7}$

c) $b^7 \times b^0 \times b^4$
$= \boxed{b^{11}}$

d) $(a^4)^3$
$= \boxed{a^{12}}$

e) $\frac{h^8}{h^5}$
$= \boxed{h^3}$

f) $y^{10} \div y^4 = \frac{y^{10}}{y^4}$
$= \boxed{y^6}$

g) $(st)^6$
$= \boxed{s^6t^6}$

h) $(-3pq)^4$
$= (-3)^4p^4q^4$
$= \boxed{81p^4q^4}$

i) $\left(\frac{x}{y}\right)^9$
$= \boxed{\frac{x^9}{y^9}}$

j) $\left(\frac{c}{4}\right)^2$
$= \boxed{\frac{c^2}{4^2}} = \boxed{\frac{c^2}{16}}$

#25-31.

1.5(b) Combining Exponent Laws:

When combining exponent laws the rules of BEDMAS still apply. Take one step at a time to ensure you do not make a mistake.

*When brackets are involved think "work from the inside out."

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Ex. 1) $(4x^5)(2x^3)$

$= \boxed{8x^8}$

b) $(3a^4)(a^5)(6a^3)$

$= \boxed{18a^{12}}$

c) $\dfrac{30b^{14}}{45b^{10}}$

$= \boxed{\dfrac{2b^4}{3}}$

$2/3\, b^4$

d) $x^5y^8x^3y^4$

$= \boxed{x^8y^{12}}$

e) $\dfrac{10e^8f^{12}}{4e^4f^2}$

$= \boxed{\dfrac{5e^4f^{10}}{2}}$

f) $(3x^2)^3$

$= 3^3x^6$

$= \boxed{27x^6}$

g) $\dfrac{x^3x^5}{x^2x^1}$

$= \dfrac{x^8}{x^3}$

$= \boxed{x^5}$

h) $(5ab^6)^2(4a^2b)$

$= (5^2a^2b^{12})(4a^2b^1)$

$= \boxed{100a^4b^{13}}$

i) $16(x^3y^5)^2$

$= 16(x^6y^{10})$

$= \boxed{16x^6y^{10}}$

j) $\left(\dfrac{4y^3 \times 3x^6}{6x^5}\right)^4$

$= \left(\dfrac{12x^6y^3}{6x^5}\right)^4$

$= (2x^1y^3)^4$

$= 2^4x^4y^{12}$

$= \boxed{16x^4y^{12}}$

Ax^By^C

7	6	8

ABC

$16 \times 4 \times 12$

13	2	1

A + B + C

1.5(c) Integral Exponents

Evaluate the following and extend the pattern:

$3^3 = 27$ $\searrow \div 3$

$3^2 = 9$ $\searrow \div 3$

$3^1 = 3$ $\searrow \div 3$

$3^0 = 1$ $\searrow \div 3$

$3^{-1} = \frac{1}{3}$ $\searrow \div 3$

$3^{-2} = \frac{1}{9}$ $\searrow \div 3$

Ex.

a) $8^{-2} =$ b) $n^{-3} =$ c) $3b^{-4} =$

The Negative Exponent Law:

A base (not including zero) raised to a negative exponent has the properties:

$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

Ex 1 Simplify and write the answer with positive exponents.

a) $b^{-4} \times b^{-3}$
$= \frac{b^{-7}}{1}$
$= \boxed{\frac{1}{b^7}}$

b) $6x^2 \div 2x^7$
$= 3x^{-5}$
$= \boxed{\frac{3}{x^5}}$

c) $\frac{8a^{-5}}{1}$
$= \boxed{\frac{8}{a^5}}$

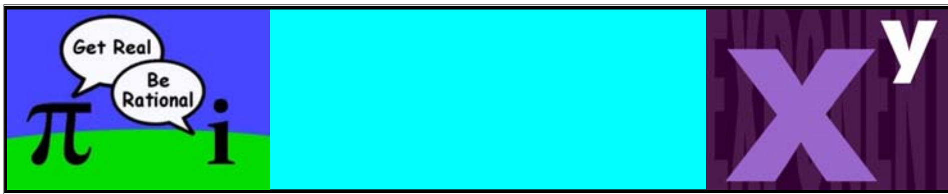
Ex 2) Simplify with positive exponents.

a) $\left(\frac{c}{d}\right)^{-3}$
$= \frac{c^{-3}}{d^{-3}}$
$= \boxed{\frac{d^3}{c^3}}$

b) $\left(\frac{x}{4}\right)^{-3}$
$= \frac{x^{-3}}{4^{-3}}$
$= \boxed{\frac{4^3}{x^3}}$

c) $\left(\frac{p^2}{r^4}\right)^{-3}$
$= \frac{p^{-6}}{r^{-12}}$
$= \boxed{\frac{r^{12}}{p^6}}$

d) $\left(\frac{a^{-2}}{b^{-5}}\right)^{-3}$
$= \boxed{\frac{a^6}{b^{15}}}$



1.5(d) Rational Exponents: The Meaning of $a^{1/n}$

a) $\sqrt{(5)} \times \sqrt{(5)} = 5$ $\quad 5^{(1/2)} \times 5^{(1/2)} = 5$

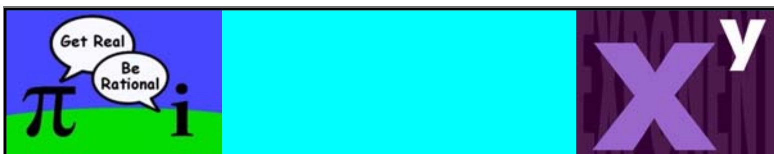
Deduce a meaning for $5^{1/2}$.

$\sqrt{x} = x^{1/2}$

b) $\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$ $\quad 2^{1/3} \times 2^{1/3} \times 2^{1/3} = 2$

Deduce a meaning for $2^{1/3}$.

$\sqrt[3]{x} = x^{1/3}$



$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $a^{\frac{m}{n}} = \sqrt[n]{a^m}$, $m \in I,\ n \in N,\ a \neq 0$

The Rule: When you have an exponent that is a fraction, the denominator becomes your index and the numerator stays with the base.

Ex 1) Write the following using an equivalent radical:

a) $r^{1/3}$ $= \sqrt[3]{r^1}$ $= \boxed{\sqrt[3]{r}}$

b) $s^{4/7}$ $= \sqrt[7]{s^4}$

c) $\frac{t^{-1/6}}{1} = \frac{1}{t^{1/6}} = \boxed{\frac{1}{\sqrt[6]{t^1}}}$

d) $\frac{v^{-3/2}}{1} = \frac{1}{v^{3/2}}$ $= \boxed{\frac{1}{\sqrt{v^3}}}$

e) $5t^{3/4}$ $= \boxed{5\sqrt[4]{t^3}}$

f) $(5t)^{3/4}$ $= \boxed{\sqrt[4]{(5t)^3}}$

g) $\frac{(-z)^{-5/3}}{1}$ $= \frac{1}{(-z)^{5/3}}$ $= \boxed{\frac{1}{\sqrt[3]{(-z)^5}}}$

Ex 2) Write the following in radical form and evaluate without using a calculator.

a) $25^{3/2}$

$= \sqrt[2]{25^3}$

$= \sqrt{25}^{\,3}$

$= 5^3$

$= 125$

b) $(-8)^{2/3}$

$= \sqrt[3]{(-8)}^{\,2}$

$= (-2)^2$

$= 4$

c) $16^{-3/4}$

$= \frac{1}{16^{3/4}}$

$= \frac{1}{\sqrt[4]{16}^{\,3}}$

$= \frac{1}{2^3}$

$= \frac{1}{8}$

d) $(3^2+4^2)^{1/2}$

$= (9+16)^{1/2}$

$= 25^{1/2}$

$= \sqrt[2]{25^1}$

$= 5$

Ex 3) Write the following with positive exponents.

a) $\sqrt[2]{x^1}$

$= \boxed{x^{1/2}}$

b) $\sqrt[3]{b^2}$

$= \boxed{b^{2/3}}$

c) $\frac{1}{}$

$= \sqrt{1}$

$= \boxed{1^{1/2}}$

d) $2\sqrt[3]{h^4}$

$= \boxed{2h^{4/3}}$

$\neq (2h)^{4/3}$

① Regular Gr.9 Exponent Laws.
② Negatives.
③ Fractional Exponents.

Ex 4) Simplify the following. Write each expression with positive exponents and then as entire radical.

a) $(x^{3/2})(x^{1})$

$= x^{5/2}$

$= \boxed{\sqrt{x^5}}$

b) $y^{1/3} \div y^{5/3}$

$= y^{-4/3} = \dfrac{1}{y^{4/3}}$

$= \boxed{\dfrac{1}{\sqrt[3]{y^4}}}$

c) $(a^{1/2})^{2/3}$

$= a^{1/3}$

$= \boxed{\sqrt[3]{a}}$

d) $(4x^{3/4})(3x^{-1/2})$

$= 12x^{1/4}$

$= \boxed{12\sqrt[4]{x}}$

e) $\dfrac{5x^{3/5}}{25x^{-3/5}}$

$= \dfrac{1x^{6/5}}{5}$

$= \dfrac{\sqrt[5]{x^6}}{5}$

$= \dfrac{1}{5}\sqrt[5]{x^6}$

f) $\left(\dfrac{x^2}{y^1}\right)^{-1/2}$

$= \dfrac{x^{-1}}{y^{-1/2}} = \dfrac{y^{1/2}}{x}$

$= \boxed{\dfrac{\sqrt{y}}{x}}$

Ex 5) Write as a power and evaluate.

a) $\sqrt{\sqrt{1296}}$

$= (1296^{1/2})^{1/2}$

$= \boxed{1296^{1/4}} = \boxed{6}$

b) $\dfrac{1}{\sqrt{169}} = \dfrac{1}{169^{1/2}}$

$= \boxed{169^{-1/2}}$

$= \boxed{\dfrac{1}{13}}$

c) $\sqrt[3]{\sqrt{64}}$

$= (64^{1/2})^{1/3} = \boxed{64^{1/6}}$

$= \boxed{2}$

Ex 6) Write each in the form ax^n.

a) $\sqrt[3]{8x^5}$

$= (8x^5)^{1/3}$

$= 8^{1/3}x^{5/3}$

$= \boxed{2x^{5/3}}$

b) $\sqrt[4]{16x^3}$

$= (16x^3)^{1/4}$

$= 16^{1/4}x^{3/4}$

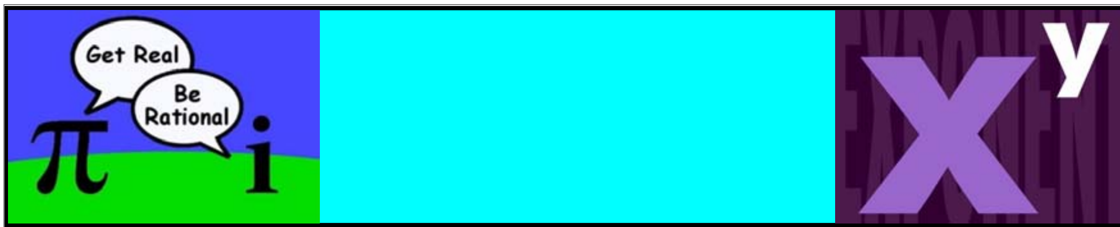
$= \boxed{2x^{3/4}}$

c) $\sqrt{900x}$

$= (900x)^{1/2}$

$= 900^{1/2}x^{1/2}$

$= \boxed{30x^{1/2}}$



To learn more about Exponent Laws check out the following link:

http://www.math10.ca/lessons/exponentsAndRadicals/exponentsOne/exponentsOne.php