Math 30-1

Unit 1: Transformations

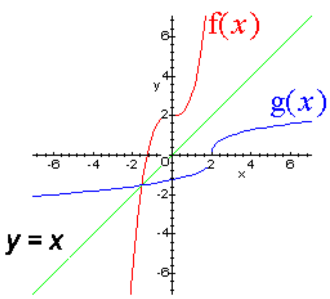
1.5 Inverses

The inverse reflects a function over the line y = x.

In terms of mapping, each point (x , y) maps to (y , x).

Invariant points lie on the line y = x.

Ex.)

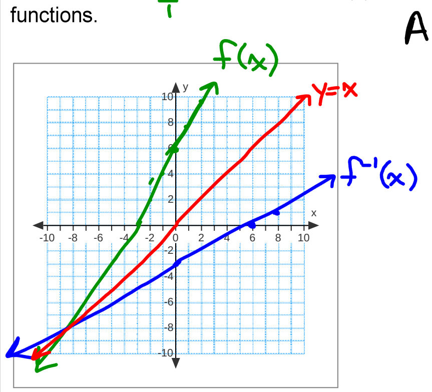


Note: Sometimes when a function is reflected across the line y = x, it no longer passes the vertical line test and is therefore not a function. The notation we use with inverses is very important because is tells us whether the resulting graph is or is not a function.

Notation:

$y = f^{-1}(x)$ is a function
$x = f(y)$ is NOT a function relation

Ex.) Given f(x) = 2x + 6, determine $f^{-1}(x)$ and state domain, range, x-int, and y-int of both functions.



Algebraically:

$$y = 2x + 6$$

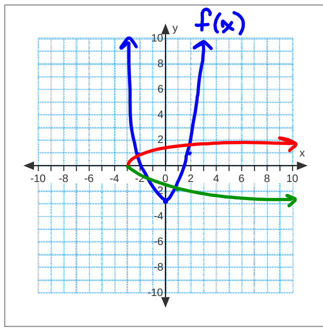
$$x = 2y + 6$$
$$-6 \qquad -6$$

$$\frac{x}{2} - \frac{6}{2} = \frac{2y}{2}$$

$$f^{-1}(x) = y = \frac{x}{2} - 3$$

	$f(x)$	$f^{-1}(x)$
domain	$(-\infty, \infty)$	$(-\infty, \infty)$
range	$(-\infty, \infty)$	$(-\infty, \infty)$
x-int	$(-3, 0)$	$(6, 0)$
y-int	$(0, 6)$	$(0, -3)$

Ex.) Find the inverse of $f(x) = x^2 - 3$ algebraically. Draw the graph of the original function and the inverse. Restrict the domain of f(x) so the inverse is a function.



$y = x^2 - 3$

$x = y^2 - 3$

$x + 3 = y^2$

$y = \pm\sqrt{x+3}$

$\left\{ \begin{array}{l} y_1 = \sqrt{x+3} \\ y_2 = -\sqrt{x+3} \\ y_3 = x^2 - 3 \end{array} \right.$

$x = f(y) \Rightarrow$ not a $f(x)$

Pg. 52 # 3, 4, 8, 9, 11, 13.

$\underbrace{\text{Domain: } x \geq 0}$

this ensures the inverse is a function

$f^{-1}(x) = \sqrt{x+3}$