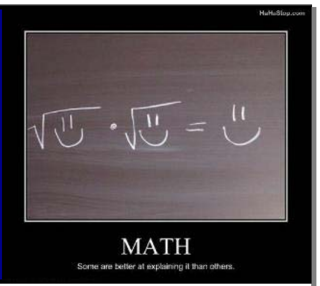
Unit 1: Radicals



1.5 Rationalizing the Denominator

For a radical to be in simplest form, the denominator cannot be irrational (ie. it cannot have a radical in it).

Example: $\frac{1}{\sqrt{2}}$ has an Irrational Denominator

There are two types of rationalizing questions we deal with in Math 20-1:

Monomial Denominators

Eg. $\frac{1}{3\sqrt{2}}$

Binomial Denominators

Eg. $\frac{3}{\sqrt{5}-7}$

Monomial Denominators

Rule: Multiply top and bottom by the simplified radical in the denominator (it isn't necessary to multiply by the coefficient).

Ex.) a) $\frac{(4\sqrt{5}-2)}{\sqrt{5}} \cdot \frac{1\sqrt{5}}{\sqrt{5}}$ = 1

$= \frac{4\overset{5}{\cancel{\sqrt{25}}}-2\sqrt{5}}{\sqrt{25}}$

$= \boxed{\frac{20-2\sqrt{5}}{5}}$

b) $\frac{9\sqrt{2}-3\sqrt{5}}{\underset{4\cdot 3}{\sqrt{12}}}$

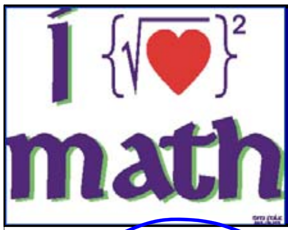
$= \frac{(9\sqrt{2}-3\sqrt{5})}{2\sqrt{3}} \cdot \frac{1\sqrt{3}}{1\sqrt{3}}$

$= \frac{9\sqrt{6}-3\sqrt{15}}{2\sqrt{9}}$

$= \frac{\overset{3}{\cancel{9}}\overset{\div 3}{}\sqrt{6}-\overset{1}{\cancel{3}}\overset{\div 3}{}\sqrt{15}}{\underset{\div 3}{\cancel{6}}2}$

do not divide radicands

$= \boxed{\frac{3\sqrt{6}-\sqrt{15}}{2}}$



c) $\frac{(5\sqrt{8}-2\sqrt{5})}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$ ✓

$= \frac{5\sqrt{48}-2\sqrt{30}}{6}$

$= \frac{5\sqrt{16\cdot 3}-2\sqrt{30}}{6}$

6

$= \frac{\overset{10}{\cancel{20}}\sqrt{3}-\overset{1}{\cancel{2}}\sqrt{30}}{\cancel{6}3}$

$= \boxed{\frac{10\sqrt{3}-\sqrt{30}}{3}}$ ✓

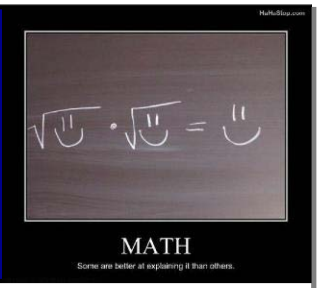
d) $\frac{(-3\sqrt{12}+2\sqrt{3})}{\underbrace{\sqrt{18}}_{3\sqrt{2}}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{-3\sqrt{24}+2\sqrt{6}}{3\sqrt{4}}$

$= \frac{-3\sqrt{4\cdot 6}+2\sqrt{6}}{3(2)}$

$= \frac{-6\sqrt{6}+2\sqrt{6}}{6}$

$= -\frac{4\sqrt{6}}{6} = \boxed{\frac{-2\sqrt{6}}{3}}$



Binomial Denominators

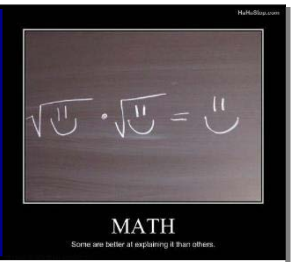
Rule: Multiply top and bottom by the **conjugate**.

The **conjugate** is where you change the sign in the middle of two terms:

$(\sqrt{3}-5)$
Expression

$(\sqrt{3}+5)$
Conjugate

This works because when you FOIL these binomials out, the middle two terms always cancel, meaning the radical terms cancel and disappear.



Remember it will be easier if you simplify first!

Ex.) Rationalize the denominator. $\sqrt{4\cdot 2}$

$$\frac{5\sqrt{8}-3\sqrt{2}}{\sqrt{32}+3\sqrt{2}}$$

$\sqrt{16\cdot 2}$

$$=\frac{10\sqrt{2}-3\sqrt{2}}{4\sqrt{2}+3\sqrt{2}}=\frac{7\sqrt{2}}{7\sqrt{2}}=\boxed{1}$$

Ex.) Rationalize the denominator.

a) $\frac{-\sqrt{5}}{(\sqrt{7}-3)}\cdot\frac{(\sqrt{7}+3)}{(\sqrt{7}+3)}$

$$=\frac{-1\sqrt{35}-3\sqrt{5}}{7+\cancel{3\sqrt{7}}-\cancel{3\sqrt{7}}-9}$$

$$=\boxed{\frac{\sqrt{35}+3\sqrt{5}}{2}}$$

b) $\frac{(5\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})}\frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$

$$=\frac{5(3)+5\sqrt{6}+\sqrt{6}+2}{3-2}$$

$$=\boxed{17+6\sqrt{6}}$$

c) $\frac{(6\sqrt{3}-2)}{(5+4\sqrt{2})}\cdot\frac{(5-4\sqrt{2})}{(5-4\sqrt{2})}$

$$=\frac{30\sqrt{3}-24\sqrt{6}-10+8\sqrt{2}}{25-16(2)}$$

$$=\frac{30\sqrt{3}-24\sqrt{6}-10+8\sqrt{2}}{-7}$$

$$=\boxed{\frac{-30\sqrt{3}+24\sqrt{6}+10-8\sqrt{2}}{7}}$$

Pg. 290 # 8-10, 13.