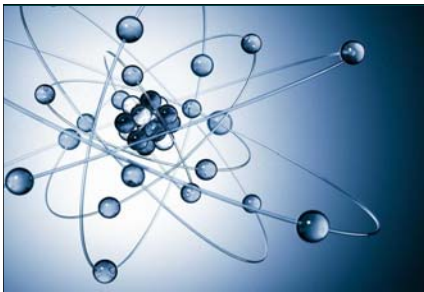
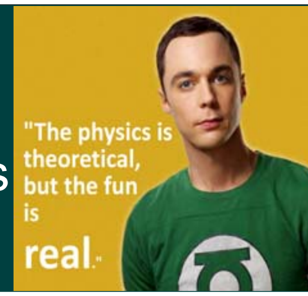
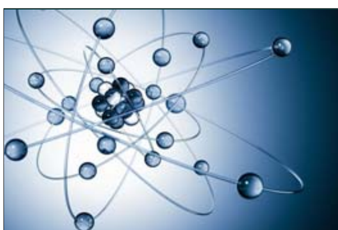


Unit 1: Kinematics



1.5 The Kinematics Equations

We derive the Kinematics Equations by substituting some old equations to form new ones. We will look at three new equations, how they were derived and examples using them to study objects with constant acceleration.



1. Displacement from velocity and time in accelerated motion.

During constant acceleration, velocity is changing, so we can't use plain old $\vec{v}$ anymore.

$\vec{d} = \vec{v}t$ only holds true if there is no acceleration. To consider a constant acceleration we need to use average velocity $\vec{v}_{ave}$.

$$\vec{v}_{ave} = 1/2(\vec{v}_f + \vec{v}_i)$$

v_f = final velocity

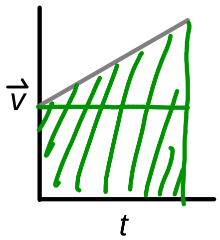
v_i = initial velocity

$$\vec{d} = \vec{v}t = 1/2(\vec{v}_f + \vec{v}_i)t$$

We sub the displacement equation into the average velocity equation:

$$\boxed{\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)t}$$

We can connect several ideas at this point. The area under the curve of a velocity-time graph is displacement:



$$A_{trapezioid} = \frac{(h_1 + h_2)b}{2} \qquad \vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)t$$

The area under the curve forms a trapezoid.

Ex.) An animal moves with a velocity of 3.00 m/s E and accelerates constantly. If the velocity after 4.70 s is 15 m/s E, what is the displacement of the object?

$\vec{v}_i = 3.00 m/s$
$t = 4.70 s$
$\vec{v}_f = 15 m/s$ [East]
$\vec{d} = ?$

$\vec{d} = \frac{(\vec{v}_f + \vec{v}_i)}{2}t = \frac{(+15 m/s + +3.00 m/s)}{2}$ 4.70 s

$\vec{d} = \boxed{4.2 \times 10^1 \text{ m [E]}}$

$= \boxed{+42 \text{ m}}$

list all variables	formula	substitute (with units) and solve

Ex.) A driver accelerates constantly to a velocity of 7.5 m/s during 4.5 s. The driver's displacement is 19 m [E]. What is the initial velocity?

$\vec{v}_f = 7.5\,m/s$

$t = 4.5\,s$

$\vec{d} = {}^{+}19\,m$

$\vec{v}_i = ?$

$$\vec{d} = \frac{(\vec{v}_f + \vec{v}_i)}{2}t$$

$$\frac{^{+}19\,m}{4.5\,s} = \frac{(7.5\,m/s + \vec{v}_i)}{2} \cdot \frac{4.5\,s}{4.5\,s}$$

$$2 \cdot 4.2\,m/s = \frac{7.5\,m/s + \vec{v}_i}{\cancel{2}} \cdot \cancel{2}$$

$$8.4\,m/s = 7.5\,m/s + \vec{v}_i$$
$$-7.5\,m/s \quad -7.5\,m/s$$

$$\boxed{{}^{+}0.94\,m/s = \vec{v}_i}$$

2. Displacement when acceleration, initial velocity, and time are known.

Sometimes, problems will not give us $\vec{v}_f$ or $\vec{v}_i$ but we can still solve for $\vec{d}$. Again we do this by combining two past equations.

$$\vec{d} = 1/2(\vec{v}_f + \vec{v}_i)t$$

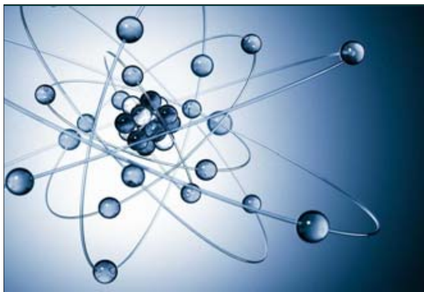
but we don't have v_f so rearrange the following for v_f:

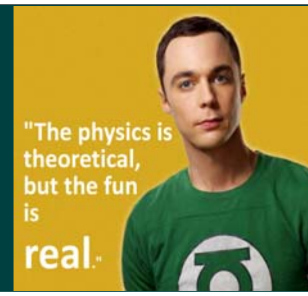
$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t} \quad \ldots \quad \vec{v}_f = \vec{v}_i + \vec{a}t$$

Now, put v_f into the first equation:

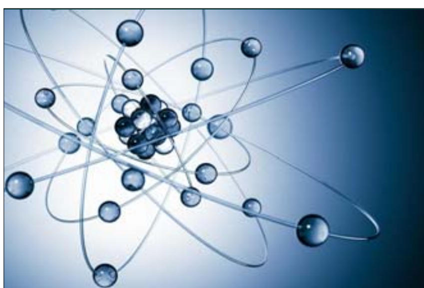
$$\vec{d} = \tfrac{1}{2}((\vec{v}_i + \vec{a}t) + \vec{v}_i)t$$
$$= \tfrac{1}{2}(2\vec{v}_i + \vec{a}t)t$$
$$= \tfrac{1}{2}(2\vec{v}_i t + \vec{a}t^2)$$

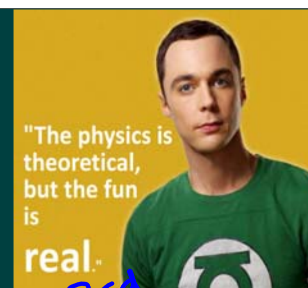
$$\boxed{\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2}$$





$$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \xrightarrow{\text{when we know } v_f \text{ instead it changes to:}} \vec{d} = \vec{v}_f t - \frac{1}{2}\vec{a}t^2$$





2 sd

Ex.) A biker passes a lightpost at the top of a hill traveling at 4.5 m/s. She accelerates down the hill at a constant rate of 0.40 m/s² for 12.0 s. How far down the hill did she move?

$\vec{d} = ?$

$\vec{a} = 0.40 \text{ m/s}^2$

$t = 12.0 \text{ s}$

$\vec{v}_i = 4.5 \text{ m/s}$

$$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$= (4.5 \text{ m/s} \cdot 12.0 \text{ s}) + \left(\frac{1}{2}\right)(0.40 \text{ m/s}^2)(12.0 \text{ s})^2$$

$$= \boxed{83 \text{ m [down the hill]}}$$

Ex.) A sheep starts from rest ($\vec{v}_i$) and accelerates at a constant rate of 3.15 m/s² forward for 28.65 s. What is the displacement during this time?

$\vec{a} = {}^{+}3.15\,m/s$

$\vec{v}_i = 0\,m/s$

$t = 28.65\,s$

$\vec{d} = ?$

$$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$\vec{d} = (0\,m/s \cdot 28.65\,s) + \frac{1}{2}(3.15\,m/s^2)(28.65\,s)^2$$

$$\vec{d} = 1292.7\,m$$

$$\vec{d} = \boxed{1.29\times10^3\ m\ [forward]}$$

3. Final Velocity if displacement, initial velocity and acceleration are known but you don't have time.

$\vec{d} = 1/2(\vec{v}_f + \vec{v}_i)t$ $\qquad$ $t = \left(\frac{\vec{v}_f - \vec{v}_i}{\vec{a}}\right)$

Sub time into displacement formula:

$$2\cdot\vec{d}\cdot\vec{a} = \cancel{2}\cdot\frac{1}{\cancel{2}}(\vec{v}_f+\vec{v}_i)\left(\frac{\vec{v}_f-\vec{v}_i}{\cancel{\vec{a}}}\right)\cdot\cancel{\vec{a}}$$

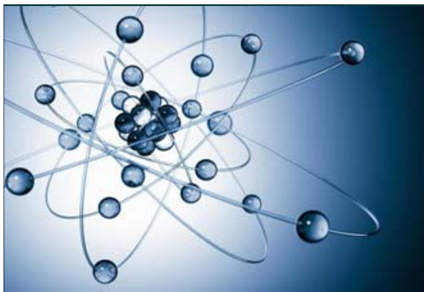
$$2\vec{d}\vec{a} = (\vec{v}_f+\vec{v}_i)(\vec{v}_f-\vec{v}_i)$$

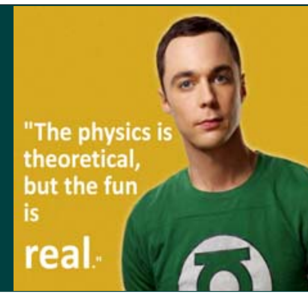
$$2\vec{a}\vec{d} = \vec{v}_f^{\,2} - \cancel{\vec{v}_f\vec{v}_i} + \cancel{\vec{v}_f\vec{v}_i} - \vec{v}_i^{\,2}$$

$$2\vec{a}\vec{d} = \vec{v}_f^{\,2} - \vec{v}_i^{\,2}$$

$$+\vec{v}_i^{\,2} \qquad\qquad +\vec{v}_i^{\,2}$$

$$\boxed{v_f^2 = v_i^2 + 2ad}$$





Ex.) A cannon is shot with initial velocity of +15 m/s. The cannon ball travels for 50 m with a constant acceleration of 2.5 m/s². What is the final velocity of the ball?

$\vec{V}_i = {}^{+}15\,m/s$

$\vec{d} = 50\,m$

$\vec{a} = 2.5\,m/s^2$

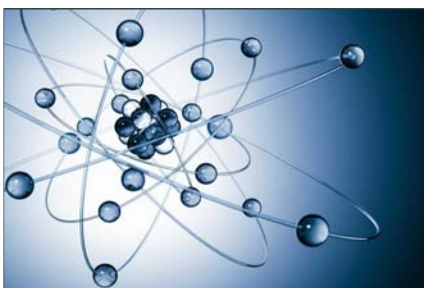
$\vec{V}_f = ?$

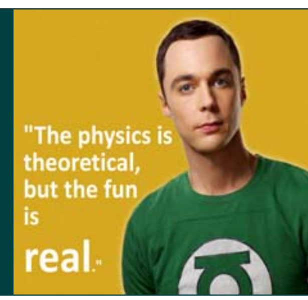
$\vec{V}_f^{\,2} = \vec{V}_i^{\,2} + 2\vec{a}\vec{d}$

$\vec{V}_f^{\,2} = ({}^{+}15\,m/s)^2 + 2(2.5\,m/s^2)(50\,m)$

$\sqrt{\vec{V}_f^{\,2}} = \sqrt{475}$

$\boxed{\vec{V}_f = {}^{+}22\,m/s}$





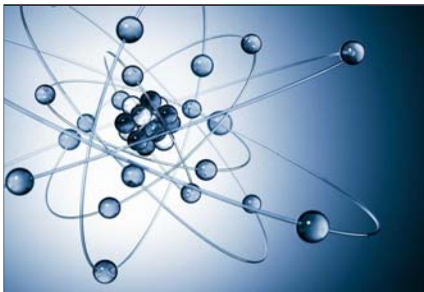
Hints:

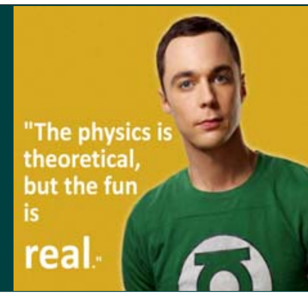
1. When you list all the variables that are given to you in the question, it should be clear which kinematics equation should be used with the information given. It will usually be the only equation that has one unknown after plugging in all your data.

2. This Unit is called Kinematics. All the equations you need to consider for Kinematics are neatly organized in a Kinematics section on your formula sheet:

Kinematics

$\vec{v}_{ave} = \frac{\Delta\vec{d}}{\Delta t}$	$\vec{d} = \vec{v}_f t - \frac{1}{2}\vec{a}t^2$
$\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t}$	$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)t$
$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$	$v_f^2 = v_i^2 + 2ad$
$\lvert\vec{v}_c\rvert = \frac{2\pi r}{T}$	$\lvert\vec{a}_c\rvert = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$





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