

Unit 1: Radicals

1.6 Solving Radical Equations

Every time you solve a radical equation you must show 3 things:

- solve
- verify
- state restrictions on the variable

You will see two types of solving questions; one radical and two radicals.

Steps for Solving Questions with ONE RADICAL:

1. Isolate the radical using algebra.
2. Square both sides of the equation.
3. Solve for 'x'.
4. Verify using the ORIGINAL equation.
5. State restrictions on the variable.

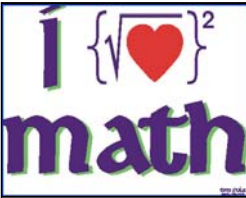

Ex.) $3\sqrt{x} = 5$
 $\frac{3\sqrt{x}}{3} = \frac{5}{3}$
 $\sqrt{x} = \left(\frac{5}{3}\right)^2$
 $x = \frac{25}{9}$

Verify: $3\sqrt{\frac{25}{9}} \stackrel{?}{=} 5$
 $3 \cdot \frac{5}{3} = 5$
 $5 = 5$
 L.S. = R.S.

Restrictions: $x \geq 0$

take radicand set it ≥ 0
 positive #

divide by neg. , switch inequality

Ex.) Solve.

a) $\sqrt{x-2} = 5$

$$\begin{array}{r} \sqrt{x-2} = 5 \\ x-2 = 25 \\ +2 \quad +2 \\ \hline x = 27 \end{array}$$

Verify $\sqrt{27-2} \stackrel{?}{=} 5$
 $5 = 5$

Restrictions
 $x-2 \geq 0$
 $x \geq 2$

b) $4\sqrt{x+1} - 5 = 3$

$$\begin{array}{r} 4\sqrt{x+1} - 5 = 3 \\ 4\sqrt{x+1} = 8 \\ \sqrt{x+1} = 2 \\ \sqrt{x+1}^2 = 2^2 \\ x+1 = 4 \\ \hline x = 3 \end{array}$$

Verify $4\sqrt{3+1} - 5 \stackrel{?}{=} 3$
 $3 = 3$

Restrictions
 $x+1 \geq 0$
 $x \geq -1$

c) $\sqrt{3x+1} = 5$

$$\begin{array}{r} \sqrt{3x+1} = 5 \\ 3x+1 = 25 \\ -1 \quad -1 \\ \hline 3x = 24 \\ \div 3 \\ \hline x = 8 \end{array}$$

Verify $\sqrt{3(8)+1} \stackrel{?}{=} 5$
 $5 = 5$

Restrictions
 $3x+1 \geq 0$
 $3x \geq -1$
 $x \geq -1/3$

d) $\sqrt{2x-1} + 5 = 2$

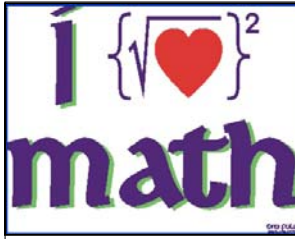
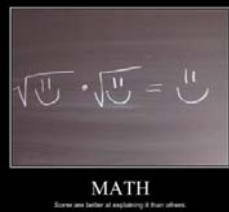
$$\begin{array}{r} \sqrt{2x-1} + 5 = 2 \\ \sqrt{2x-1} = -3 \\ \sqrt{2x-1}^2 = (-3)^2 \\ 2x-1 = 9 \\ 2x = 10 \\ \hline x = 5 \end{array}$$

Verify $\sqrt{2(5)-1} + 5 \stackrel{?}{=} 2$
 $8 \neq 2$

Restrictions
 $2x-1 \geq 0$
 $2x \geq 1$
 $x \geq 1/2$

~~$x = 5$~~ extraneous root

*Note: Extraneous roots are solutions that do not verify in the original equation. You must state this if you discover your answer is extraneous.

When you see two like radicals, simply combine like radicals first and then proceed with the previous steps.


Ex.) Solve.

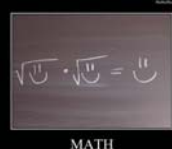
$$\begin{array}{r} 3\sqrt{x} - 4 = 2\sqrt{x} + 1 \\ -2\sqrt{x} \quad -2\sqrt{x} \\ \hline \sqrt{x} - 4 = 1 \\ +4 \quad +4 \\ \hline \sqrt{x} = 5^2 \\ \hline x = 25 \end{array}$$

Restrictions
 $x \geq 0$

Verify

$$\begin{array}{r} 3\sqrt{25} - 4 \stackrel{?}{=} 2\sqrt{25} + 1 \\ 15 - 4 \stackrel{?}{=} 10 + 1 \\ 11 = 11 \\ \checkmark \end{array}$$





Steps for Solving Questions with TWO RADICALS:

1. Rearrange so one radical is on each side of the equal sign.
2. Square both sides of the equation.
3. Solve for 'x'.
4. Verify using the ORIGINAL equation.
5. State restrictions on the variable.

Ex.) Solve. $\sqrt{x-1} = \sqrt{2x+3}$

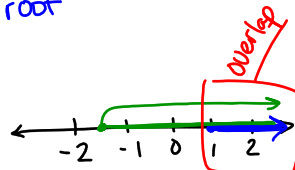
$$\begin{aligned} \sqrt{x-1} &= \sqrt{2x+3} \\ -x & \quad -x \\ x-1 &= 2x+3 \\ -1 &= x+3 \\ -3 & \quad -3 \\ -4 &= x \end{aligned}$$

~~x = -4~~ extraneous root

Restrictions


 $x-1 \geq 0$
 $x \geq 1$


$2x+3 \geq 0$
 $2x \geq -3$
 $x \geq -3/2$



Verify

 $\sqrt{-4-1} \stackrel{?}{=} \sqrt{2(-4)+3}$
 $\sqrt{-5} \stackrel{?}{=} \sqrt{-5}$
 $\sqrt{-5}$





Ex.) Solve.

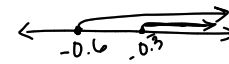
a) $\sqrt{5x+3} = \sqrt{3x+1}$

$$\begin{aligned} 5x+3 &= 3x+1 \\ -3x & \quad -3x \\ 2x+3 &= 1 \\ -3 & \quad -3 \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

~~x = -1~~ extraneous root

Verify $\sqrt{5(-1)+3} \stackrel{?}{=} \sqrt{3(-1)+1}$
 $\sqrt{-2} \stackrel{?}{=} \sqrt{-2}$

Restrictions

 $5x+3 \geq 0 \quad 3x+1 \geq 0$
 $-3 \quad -3 \quad 3x \geq -1$
 $\frac{5x}{5} \geq \frac{-3}{5} \quad x \geq -1/3$
 $x \geq -3/5$


b) $\sqrt{x+5} - \sqrt{x} = 2$

$$\begin{aligned} \sqrt{x+5} &= 2 + \sqrt{x} \\ \sqrt{x+5} &^2 = (2 + \sqrt{x})^2 \\ x+5 &= (2+\sqrt{x})(2+\sqrt{x}) \\ x+5 &= 4 + 2\sqrt{x} + 2\sqrt{x} + x \\ -4 & \quad -4 \\ 1 &= 4\sqrt{x} \\ (\frac{1}{4})^2 &= \sqrt{x}^2 \end{aligned}$$

Pg. 300 # 3, 4, 7a, 9abc, 10a.
Solving Worksheet.

 $\frac{1}{16} = x$

Verify $\sqrt{1/16 + 5} - \sqrt{1/16} \stackrel{?}{=} 2$
 $2 = 2$

$x+5 \geq 0 \quad x \geq 0$
 $x \geq -5$

