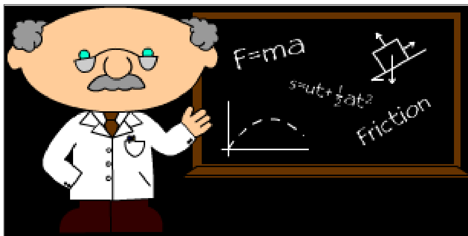
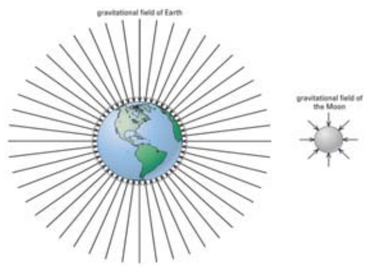


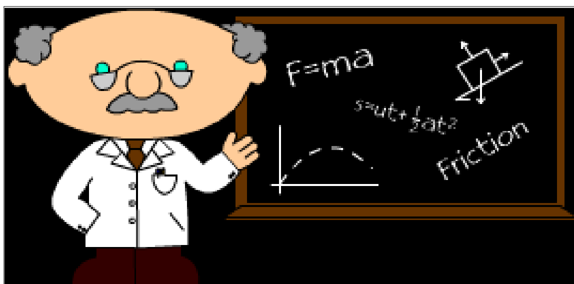
Unit 2: Dynamics

2.10 Field Theory and Universal Gravitation

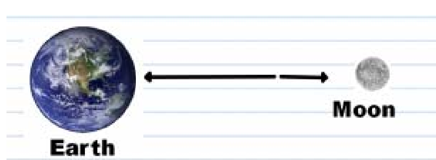
Field Theory

Invisible gravitational fields surround all objects at all times. Every mass in the universe creates its own gravitational field. A gravitational field is made up of field lines. These lines are vectors which point towards the center of the Earth or other planetary body. The closer the field lines are together, the stronger the gravitational field.





Field lines can add together like other vectors:



Ex.) What is the direction of the resultant gravity field between the Earth and the Moon?

R why Moon Orbits Earth

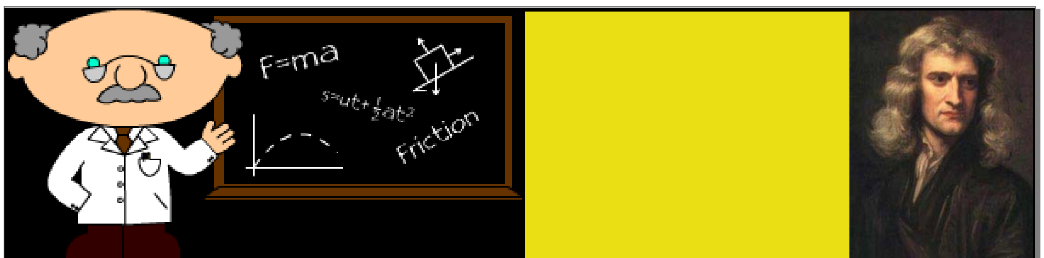
We know the direction field lines can move in so now we need to figure out the magnitude. From the *Dynamics* section of the formula sheet:

$$\boxed{\vec{F}_g = mg}$$

Where:

- g - the strength of the gravitational field
- F_g - force of gravity on test object
- m - mass of the object

*Note: This is just Newton's Second Law revisited.



If we release a test object in a gravitational field, it will accelerate in the direction of the field with a force proportional to the mass of the object.

Ex.) If an object with a mass of 175 kg experiences a force of 1480 N towards the Earth, what is the strength of the field at the object?

g

$\vec{F}_g = mg$

$1480 = 175g$

$g = 8.46$ N/kg [towards Earth's center]

field strength

Ex.) A test object of mass, m, experiences a field strength of g. If the mass doubles and the force acting on the object stays the saem, what must happen to the field of strength?

$$\vec{F_g} = mg \qquad F_g = 2mg$$

$$g = \frac{F_g}{m} \qquad g = \frac{F_g}{2m}$$

$$g = \frac{1}{2} \cdot \frac{F_g}{m}$$

g is half as big.

So we just saw that field strength is directly proportional to mass:

$$\vec{g} \propto m$$

It was also found through experimental means(page 203) that the value was also inversely proportional to the square of the distance between the test object and the gravity producing object.

$$\vec{g} \propto \frac{1}{r^2}$$

Combining these two proportions together, we get one statement:

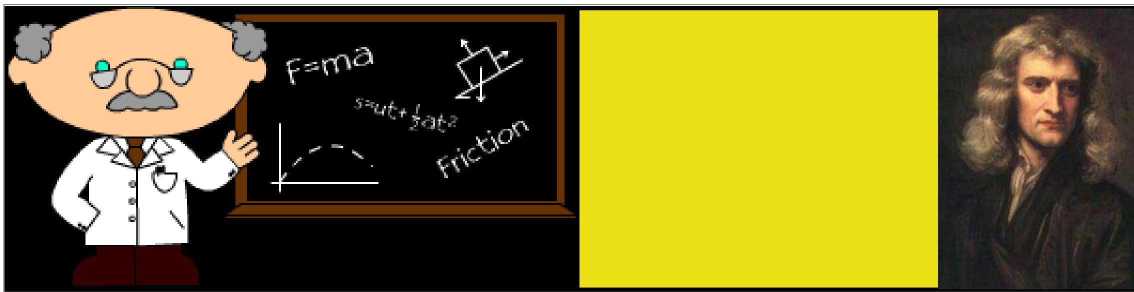
$$\vec{g} \propto m \qquad \vec{g} \propto \frac{1}{r^2}$$

$$\vec{g} \propto \frac{m}{r^2}$$

By removing the proportional symbol we get the following equation:

$$\vec{g} = \frac{Gm}{r^2}$$ *On formula sheet

$\vec{g}$ = gravitational field (m/s²) N/kg
G = universal gravitational constant
m = mass of gravity producing object
r = distance between test object and gravity producing object



G(the constant of proportionality) = $6.67 \times 10^{-11}\ Nm^2/kg^2$

* this is on your formula sheet

** it is the same everywhere in the Universe

*** it is a constant and is no subject to sig digs

This value was calculated through the *Cavendish Experiment.*


F=ma
s=ut+½at²
Friction

Ex.) Calculate the gravitational field strength on the surface of the Earth.

$$g = \frac{Gm}{r^2} = \frac{(6.67\times10^{-11})(5.97\times10^{24})}{(6.37\times10^{6})^2}$$

$$g = ((6.67\times10^{-11})(5.97\times10^{24})) \div \text{Ans}$$

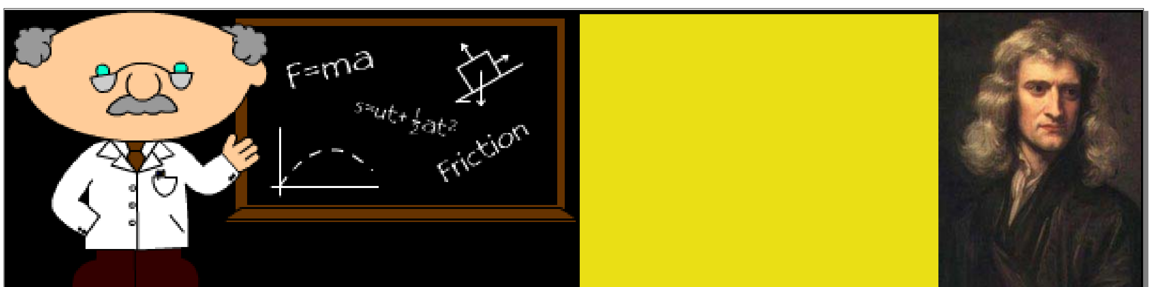
$$\boxed{g = 9.81\ N/kg}$$

Ex.) Calculate the gravitational field strength on the highest peak of Everest (8848 m above the Earth.)

$$g = \frac{Gm}{r^2} = \frac{(6.67\times10^{-11})(5.97\times10^{24})}{(6.37\times10^6 + 8848)^2} = \boxed{9.79\ N/kg}$$

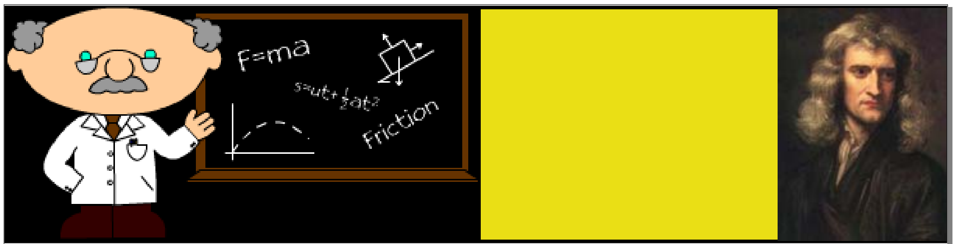
Ex.) Calculate the gravitational field strength on the lowest point of the Marianas Trench (11034 m below the surface of Earth.)

$$g = \frac{Gm}{r^2} = \frac{(6.67\times10^{-11})(5.97\times10^{24})}{(6.37\times10^6 - 11034)^2} = \boxed{9.85\ N/kg}$$



Ex.) Calculate the gravitational field strength on a planet whose mass is 1/8 of that on Earth and whose radius is three times larger.

$$g = \frac{Gm}{r^2} = \frac{(6.67\times10^{-11})\left(\frac{5.97\times10^{24}}{8}\right)}{[(3)(6.37\times10^6)]^2} = \boxed{0.136\ N/kg}$$



Newton's Law of Universal Gravitation

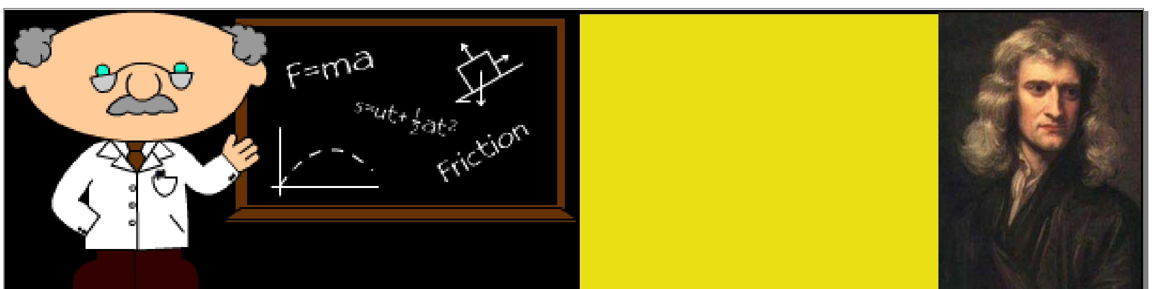
Newton not only worked out a formula for field strength but he also combined this with his second law to determine the Law of Universal Gravitation:

$\vec{g} = \frac{Gm}{r^2}$ and $\vec{F}_g = m\vec{g}$

$$\vec{F}_g = \frac{Gm_1m_2}{r^2}$$

where:

F_g = force of gravity between objects 1 and 2
G = Universal Gravitational Constant
m_1 = mass of object 1
m_2 = mass of object 2
r = distance between centres of objects



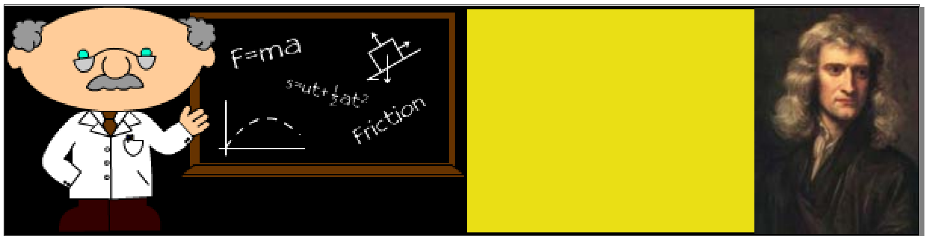
Ex.) What is the force of gravity between two neutrons placed $\underbrace{150\text{ pm}}$ apart?

150×10^{-12}

1.50×10^{-14}

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$= \frac{(6.67\times10^{-11})(1.67\times10^{-27})^2}{(150\times10^{-12})^2} = \boxed{8.27\times10^{-45}\text{ N}}$$



Ex.) If the distance between two objects doubles, and the mass of the objects stay the same, what can be said of the force of gravity between them?

$$F_g = \frac{Gm_1m_2}{r^2} \qquad \frac{G\,m_1m_2}{(2r)^2} = \frac{Gm_1m_2}{4r^2}$$

F_g reduced by a factor of 1/4. $= \frac{1}{4} \cdot \frac{Gm_1m_2}{r^2}$

Ex.) If the distance between two objects halves, and the mass of one of the objects stays the same while the other triples, what can be said about the force of gravity between them?

$$\frac{Gm_1(3m_2)}{(\frac{1}{2}r)^2} = \frac{3 \cdot Gm_1m_2}{\frac{1}{4} \cdot r^2} = 12 \cdot \frac{Gm_1m_2}{r^2}$$

F_g is 12 times larger.

Read: Pg. 203-214 (Pay close attention to Torsion Balance, Tides and Gravity Assist)

Questions: Pg. 215 # 1-6.

Watch video

Apollo 13