

2.1 NPV's and Simplification

Rational Expressions Review


What are Rational Numbers & Rational Expressions?

When the numerator and denominator of a fraction are integers, the fraction is a rational number.
 Ex. $\frac{3}{4}$, $-\frac{5}{6}$

When the numerator and denominator of a fraction are polynomials, the fraction is a rational expression.
 Ex. $\frac{2x + 3}{5x + 4}$, $\frac{x^2 - 6}{3xy}$, $\frac{x^2 - 9}{x^2 + 8x + 15}$

A rational expression cannot contain roots of variables, or variables as exponents. These expressions are not rational expressions.
 $\frac{2^x + 3}{4x - 2}$, $\frac{x^2 + 4}{2\sqrt{x}}$

Apr 6-11:55 AM



Non-Permissible Values

$\frac{3}{0} \leftarrow$

Rational expressions are not defined for values of the variable that make the denominator 0. These values are called non-permissible values.
 The expression $\frac{x + 5}{x - 3}$ is not defined for $x = 3$. So, $x = 3$ is a non-permissible value of $\frac{x + 5}{x - 3}$. $x \neq 3$

$x - 3 = 0$
 $+3 \quad +3$
 $x \neq 3$


$3 - 3 = 0$

What are the non-permissible values for:

$\frac{x^2 - 4}{(x + 7)(x - 3)}$

$x + 7 = 0$ $x - 3 = 0$
 $-7 \quad -7$ $+3 \quad +3$
 $x \neq -7$ $x \neq 3$

Apr 6-12:53 PM




Ex.) Determine the non-permissible values for each rational expression.

a) $\frac{5x}{\sqrt{x^2-9}}$
 $= \frac{5x}{(x-3)(x+3)}$
 $x \neq 3, -3$
 $x \neq \pm 3$

b) $\frac{x}{\sqrt{x^2+1}}$ $(-1)^2$
 $\text{pos. } x^2 + 1 = 0$
 $\sqrt{x^2} = \sqrt{-1}$ not possible
 $x =$
 $x \in \mathbb{R}$

Apr 6-1:35 PM



How do you create equivalent rational expressions?

To write an equivalent form of a rational number, multiply or divide the numerator and denominator by the same number. For example,

$$\frac{12}{18} = \frac{12 \times \frac{2}{2}}{18 \times \frac{2}{2}} = 1 \qquad \frac{12}{18} = \frac{12 \div 3}{18 \div 3}$$

$$= \frac{24}{36} \qquad = \frac{4}{6}$$

$\frac{12}{18}$, $\frac{24}{36}$, and $\frac{4}{6}$ are equivalent rational numbers.

The same strategy is used to write an equivalent form of a rational expression: multiply or divide the numerator and denominator by the same monomial or binomial.

For example,

$$\frac{20xy}{8x^2} = \frac{20xy \cdot x}{8x^2 \cdot x} \qquad \frac{20xy}{8x^2} = \frac{20xy \div 2x}{8x^2 \div 2x}$$


$$= \frac{20x^2y}{8x^3} \qquad = \frac{10y}{4x}$$

Each expression has $x = 0$ as a non-permissible value.

So, $\frac{20xy}{8x^2}$, $\frac{20x^2y}{8x^3}$, and $\frac{10y}{4x}$ are equivalent rational expressions for $x \neq 0$.

When stating that rational expressions are equivalent, the values of the variable for which the expressions are undefined must be identified.

Apr 6-1:37 PM




1. Use multiplication and division to write two equivalent forms of the rational expression $\frac{3(x+2)}{(x+2)(x-4)} \div (x+2)$ = $\frac{3}{(x-4)}$

$\frac{6(x+2)}{2(x+2)(x-4)}$

2. Use multiplication and division to write two equivalent forms of the rational expression $\frac{(x+5)(x-1)}{2(x-1)} \times 5 \div (x-1)$ = $\frac{(x+5)}{2}$

$\frac{5(x+5)(x-1)}{10(x-1)}$

Apr 6-1:37 PM



Simplifying Rational Expressions


Steps

1. Factor like crazy!
2. State NPV's.
3. Factors that are common to the numerator and denominator cancel.

a) $\frac{3 \cancel{15}x^2 \cancel{z^3}}{4 \cancel{20}x \cancel{z^1}}$ = $\frac{3xz^2}{4}$
 $x, y, z \neq 0$

b) $\frac{3x^2 - 8x}{2x}$ = $\frac{\cancel{x}(3x-8)}{2\cancel{x}}$ = $\frac{(3x-8)}{2}$
 $x \neq 0$

Apr 6-1:38 PM



Identify & Correct Any Errors

12. Here is a student's solution for simplifying a rational expression. Identify the error in the solution. Write a correct solution.

$$\frac{3x - 12}{x^2 + x - 20} = \frac{3(x-4)}{(x+5)(x-4)}$$

$$= \frac{3}{x+5}, x \neq -5, 4$$

13. A student claims that the expressions $\frac{12x^2(x-3)}{15x(x-3)}$ and $\frac{12x^2(x-3)}{15x(x-3)}$ are equivalent. Is the student correct? Explain.

Yes, they are equivalent because $\frac{(x-3)}{(x-3)} = 1$ was multiplied.

Pg. 223 # 3-5, 9.
Pg. 229 # 2-8.

Apr 6-1:41 PM