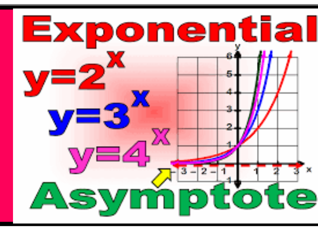
Unit 2

Exponents and Logarithms



2.2 Exponential Word Problems

Growth/Decay Formula

$$y = ab^{\frac{t}{P}}$$

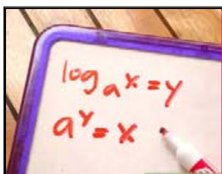
a = initial amount

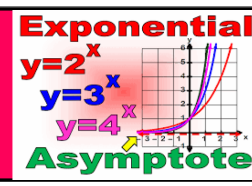
b = rate of change

t = time passed

P = time to multiply(period)

y = future amount





Ex.) A 90 mg sample of a radioactive isotope has a half-life of 5 years.

b

a) Write a function, *m(t)*, that relates the mass of the sample, *m*, to the elapsed time, *t*.

$m(t) = 90(1/2)^{t/5}$

$a = 90$
$b = 1/2$
$t = t$
$P = 5$ years
$y = m(t)$

b) What will be the mass of the sample in 6 months?

0.5 years

$y = m(0.5) = 90(1/2)^{0.5/5}$

$= \boxed{84 \text{mg}}$

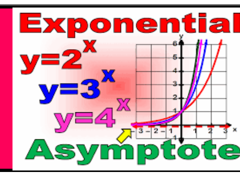
c) How long will it take for the sample to have a mass of 0.1 mg?

$0.1 = 90(1/2)^{t/5}$

$Y_1 = 0.1$
$Y_2 = 90(1/2)^{x/5}$

Intersection: $(49, 0.1)$

$\boxed{49 \text{ years}}$



Ex.) A bacterial culture containing 800 bacterial initially will double every 90 minutes.

a $b=2$ P

a) Write a function, *B(t),* that relates the number of bacteria, *B,* to the elapsed time, *t.*

$$y = B(t) = 800(2)^{t/90}$$

b) How many bacteria will exist after 8 hours? $8\cancel{h} \times \frac{60\,\text{min}}{1\cancel{h}} = 480\,\text{min}$

$$B(480) = 800(2)^{(480/90)}$$

$$= \boxed{32\,254 \text{ bacteria}}$$

c) How long ago did the culture have 50 bacteria?

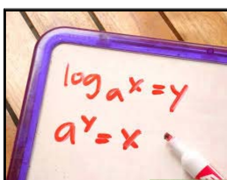
$$50 = 800(2)^{x/90}$$

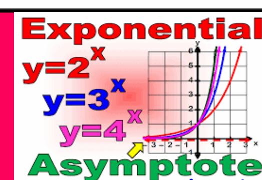
$Y_1 = 50$

$Y_2 = 800(2)^{x/90}$

Intersection: $(-360, 50)$

$\boxed{\text{360 min ago (6 hours)}}$





Ex.) A city with a population of 800,000 is projected to grow annually at a rate of 1.3% $\div 100$

a) Estimate the population of the city in 5 years. $P = 1/\text{year}$ $1 + 0.013$

$b = 1.013$

$$y = 800\,000(1.013)^x = 800\,000(1.013)^5$$

$$= \boxed{853\,370 \text{ people}}$$

b) How many years will it take for the population to double?

$$\frac{1\,600\,000}{800\,000} = \frac{800\,000(1.013)^x}{800\,000} \qquad \underset{Y_1}{2} = \underset{Y_2}{1.013^x}$$

$\boxed{x \approx 54 \text{ years}}$

c) If projections are incorrect, and the city's population actually decreases at an annual rate of 0.9%, estimate how many people will leave the city in 3 years.

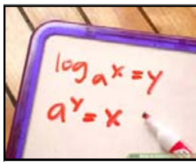
99.1% return

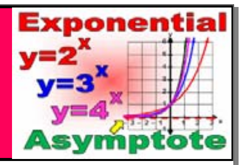
$0.991 = b$

$$y = 800\,000(0.991)^3$$

$$= \boxed{778\,594 \text{ people stay}}$$

$\therefore$ $\boxed{\text{21 406 people left}}$





Ex.) $750 is placed in a savings account that compounds interest annually at a rate of 2.5%

a   P = 1

b = 1.025

a) Write a function, *A(t)*, that relates the amount of the investment, *A*, with the elapsed time, *t*.

$$A(t) = 750(1.025)^t$$

b) How much will the investment be worth in 5 years? How much interest will have been earned?

$$A(5) = 750(1.025)^5 = \boxed{\$848.55}$$

Interest: $\boxed{\$98.55}$

c) How long does it take for the investment to double?

$$1500 = 750(1.025)^x$$

$$2 = 1(1.025)^x$$

$$x = 28.071$$

$$\boxed{x = 29 \text{ years}}$$