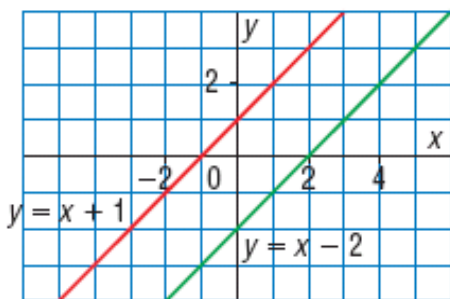




2.6 Transformations

Here is the graph of two parallel lines. Which translation would move one line so that it coincides with the other line?



translation 3 units down



Start with the equation of the quadratic function  $y = x^2$ . *mother function*

\* Graph the function in  $y_1 = x^2$

\* In  $y_2$  Graph  $y = x^2 + q$  (Try multiple values of  $q$ )

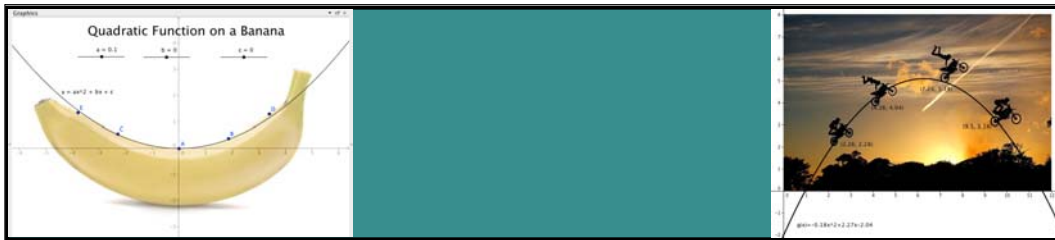
$y_2 = x^2 + 5$     $y_3 = x^2 - 7$

How does the graph of  $y = x^2 + q$  change as the value of  $q$  changes?

Vertical translation  $q$  units

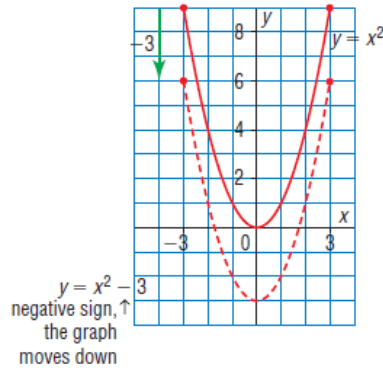
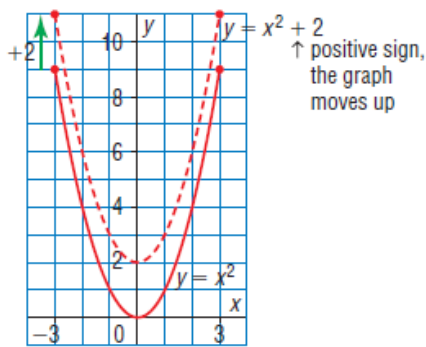
What is true for all graphs?

$q > 0$    vertical translation up  
 $q < 0$    "   "   down



The Effect of Changing  $q$  in  $y = x^2 + q$

The graph of  $y = x^2 + q$  is the image of the graph of  $y = x^2$  after a **vertical translation of  $q$  units**.



Start with the equation of the quadratic function  $y = x^2$ .

- \* Graph the function in  $y_1 = x^2$
- \* In  $y_2$  Graph  $y = (x-p)^2$  (Try multiple values of  $p$ )

How does the graph of  $y = (x-p)^2$  change as the value of  $p$  changes?

horizontal translation  $p$  units

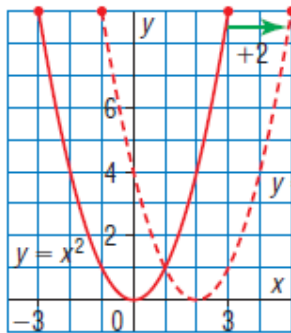
What is true for all graphs?

$p > 0$  horizontal translation left  
 $p < 0$  " " right } opposite of your intuition



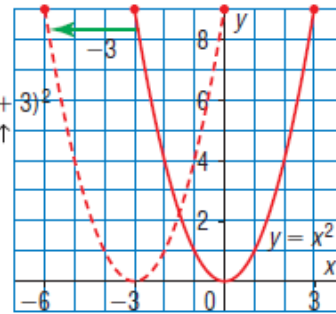
The Effect of Changing p in  $y = (x-p)^2$

The graph of  $y = (x-p)^2$  is the image of the graph of  $y = x^2$  after a horizontal translation of p units.



↑ negative sign, the graph moves right

positive sign, ↑ the graph moves left



Start with the equation of the quadratic function  $y = x^2$ .

- \* Graph the function in  $y_1 = x^2$
- \* In  $y_2$  Graph  $y = \underline{a}x^2$  (Try multiple values of a)  
(Use 0, positive and negative values)

How does the graph of  $y = ax^2$  change as the value of a changes?

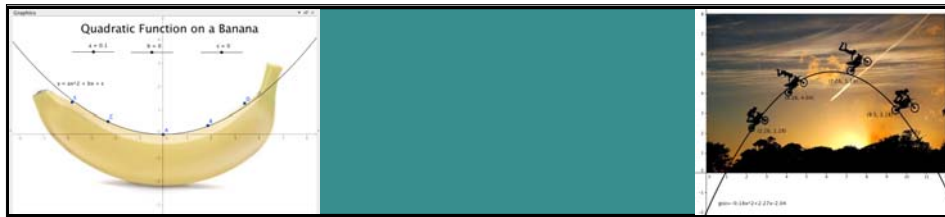
- negative : reflection about the x-axis
- Value : vertical stretch or compression of  $x^2$

What is true for all graphs?

$a > 0$  opens up  
 $a < 0$  opens down

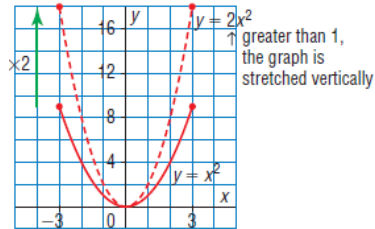
$0 < a < 1$  vertical compression  
fraction

$a > 1$  vertical stretch

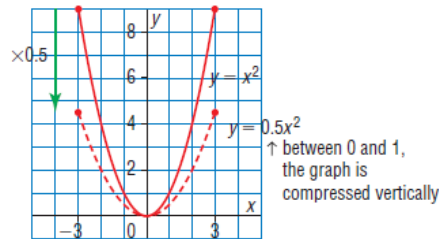


The Effect of Changing a in  $y = ax^2$

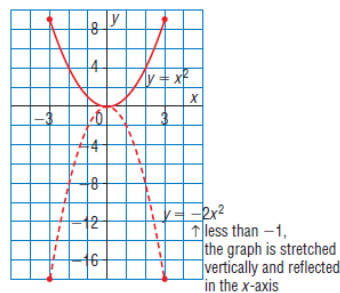
The graph of  $y = ax^2$  is the image of the graph of  $y = x^2$  after a vertical stretch of factor  $a$  when  $a > 1$ .



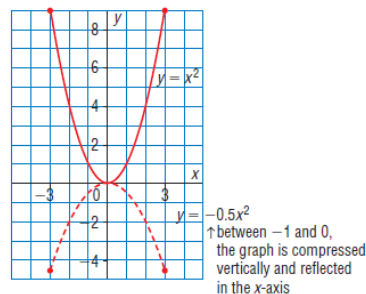
The graph of  $y = ax^2$  is the image of the graph of  $y = x^2$  after a vertical compression of factor  $a$  when  $0 < a < 1$ .

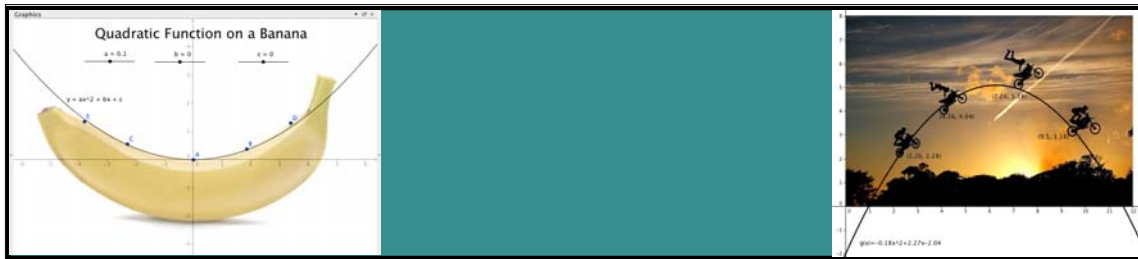


The graph of  $y = ax^2$  is the image of the graph of  $y = x^2$  after a vertical stretch of factor  $a$  and a reflection in the  $x$ -axis when  $a < -1$ .



The graph of  $y = ax^2$  is the image of the graph of  $y = x^2$  after a vertical compression of factor  $a$  and a reflection in the  $x$ -axis when  $-1 < a < 0$ .





When these three transformations are combined, the resulting equation is the ~~standard form~~ vertex form of the equation of a quadratic function:

$$y = a(x - p)^2 + q$$

Vertex Form

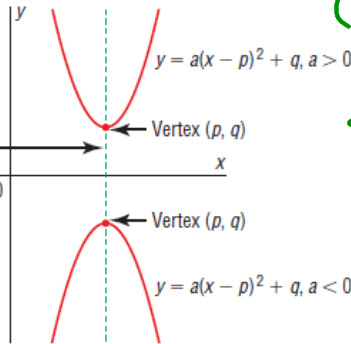
$$y = a(x - h)^2 + k$$

Axis of symmetry:

$$x = p$$

congruent to:

$$y = ax^2$$



(p, q) vertex

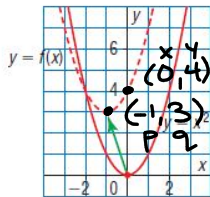
↑  
switch sign



Writing an Equation in Vertex Form



Ex.) Write an equation of the function  $y = f(x)$  in vertex form.



$$y = a(x - p)^2 + q$$

$$y = a(x + 1)^2 + 3 \quad * \text{Finding 'a'}$$

$$4 = a(0 + 1)^2 + 3$$

$$4 = \underbrace{1}_{-3} a + \underbrace{3}_{-3}$$

$$1 = a$$

$$y = 1(x + 1)^2 + 3$$