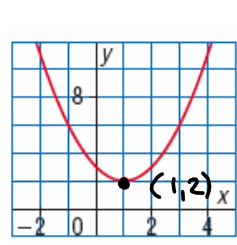




2.7 Analyzing Quadratics in Vertex Form

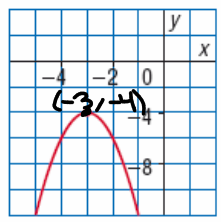
Here are the graphs of two parabolas.  
 What are the coordinates of the vertex of each graph?  
 What are the equations in vertex form?



$y = x^2 - 2x + 3$

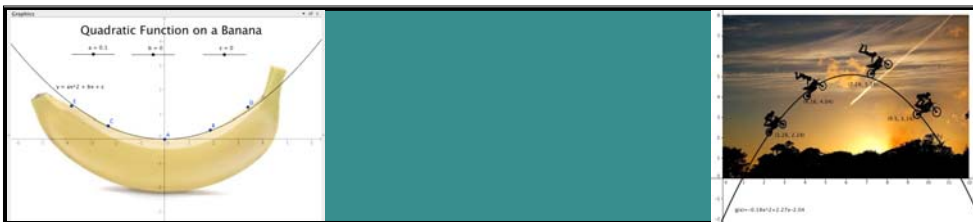
$y = (x - 1)^2 + 2$

$y = a(x - p)^2 + q$



$y = -2x^2 - 12x - 22$

$y = -2(x + 3)^2 - 4$



Ex.) Identify the coordinates of the vertex of the graph of each function.

a)  $y = (x - 1)^2 + 2$

$(1, 2)$

b)  $y = -2(x + 3)^2 + 6$

$(-3, 6)$

c)  $y = -\frac{1}{2}(x - 4)^2 - 7$

$(4, -7)$

d)  $y = 3x^2 + 5$

$y = 3(x - 0)^2 + 5$

$(0, 5)$

e)  $y = 4(x - 8)^2 + 0$

$(8, 0)$

f)  $y = \frac{2}{3}(x + 1)^2 + 0$

$(-1, 0)$



Vertex Form

$y = a(x - p)^2 + q$                       or                       $y = a(x - h)^2 + k$

In this Form what do the parameters **a,p(h)** and **q(k)** tell us?

**a** = negative → reflection  
 vertical stretch or compression  
 (whole #)                      (fraction)

**p/h** = x-value @ vertex  
 horizontal translation (opposite direction of sign)

**q/k** = y-value @ vertex  
 vertical translation



Analyzing Quadratics  
 in Vertex Form

Direction of Opening:  $y = 3(x - 0)^2 + 0$   
**Up or Down**

	Equation	Vertex	Max/Min Value	Equation of Axis of Symmetry	Domain	Range
Up	$y = 3x^2$	(0,0)	Min.	$x = 0$	$x \in \mathbb{R}$	$y \geq 0$
Up	$y = 2x^2 + 1$	(0,1)	Min.	$x = 0$	$x \in \mathbb{R}$	$y \geq 1$
Down	$y = -(x + 7)^2$	(-7,0)	Max.	$x = -7$	$x \in \mathbb{R}$	$y \leq 0$
Up	$y + 18 = (x + 5)^2 + 10$	(-5,10)	Min.	$x = -5$	$x \in \mathbb{R}$	$y \geq 10$
Up	$y + \frac{1}{3} = 3(x - 1)^2 + \frac{2}{3}$	(1,-1)	Min.	$x = 1$	$x \in \mathbb{R}$	$y \geq -1$



### How Do We Solve For "a"?

To solve for "a" you must select a point on the graph and substitute in for x and y in the equation, then, use algebra to solve for "a".

Ex.) Determine an equation of a quadratic function with x-intercepts of -2 and 4, that passes through the point  $(2, -16)$ .

$$y = a(x-r)(x-s)$$

$$y = a(x+2)(x-4) \rightarrow \boxed{y = 2(x+2)(x-4)}$$

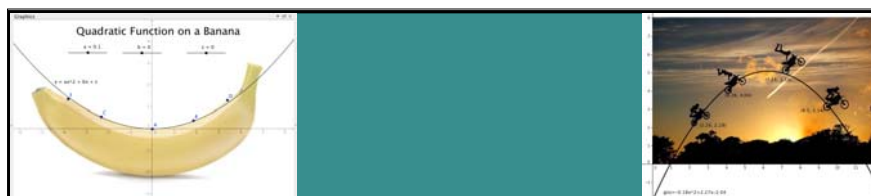
$$-16 = a(2+2)(2-4)$$

$$-16 = a(4)(-2)$$

$$-16 = a(-8)$$

$$\frac{-16}{-8} = \frac{-8a}{-8}$$

$$2 = a$$



### How Do We Find the Y-Intercept in Vertex Form?

Ex.) An equation of a quadratic function with a Vertex of  $(2, -5)$  and  $a = -2$ , find the y-intercept.

$$y = a(x-h)^2 + k$$

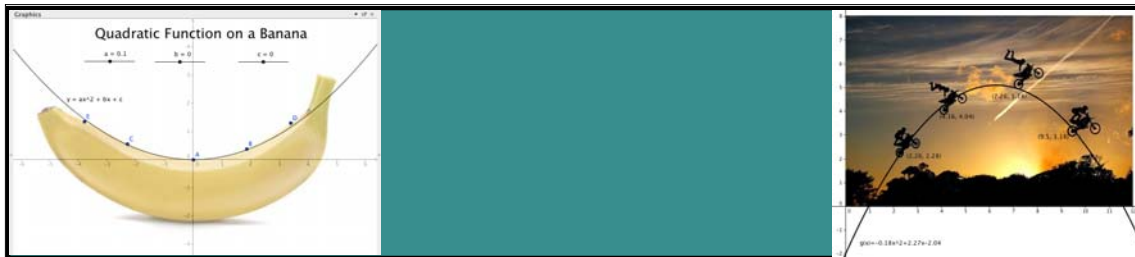
$$y = -2(x-2)^2 - 5$$

$$\underline{x=0} \quad y = -2(0-2)^2 - 5$$

$$y = -8 - 5$$

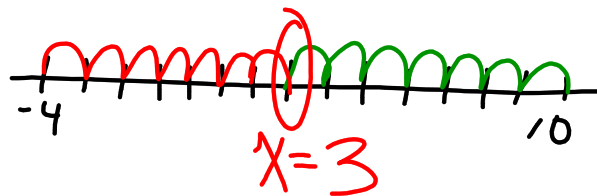
$$\underline{y = -13}$$

$$\boxed{(0, -13)}$$



How do we find the **Axis of Symmetry** when given two x-intercepts and no graph?

Ex.) Determine the axis of symmetry of a quadratic equation with x-intercepts (-4,0) and (10,0).



Ex.) Determine an equation for these graphs of quadratic functions.

a)

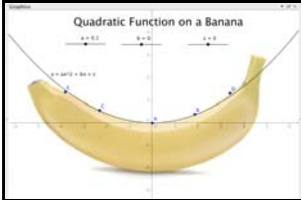
$y = a(x-h)^2 + k$   
 $y = a(x-3)^2 + 2$   
 $4 = a(1-3)^2 + 2$   
 $4 = 4a + 2$   
 $2 = 4a$   
 $\frac{1}{2} = a$


$y = \frac{1}{2}(x-3)^2 + 2$

b)

$y = a(x-h)^2 + k$   
 $y = a(x-4)^2 - 2$   
 $-14 = a(2-4)^2 - 2$   
 $-14 = 4a - 2$   
 $-12 = 4a$   
 $-3 = a$

$y = -3(x-4)^2 - 2$





For the quadratic function  $y = -2(x + 2)^2 - 3$

a) Identify:

- i) the direction of opening **down**
- ii) the vertex  **$(-2, -3)$**
- iii) the equation of the axis of symmetry  **$x = -2$**
- iv) the intercepts  **$(0, -11)$  no x-int**
- v) the domain and range of the function  
 **$x \in \mathbb{R}$     $y \leq -3$**

b) Sketch a graph.

$$0 = -2(x+2)(x+2) - 3$$

$$0 = -2(x^2 + 4x + 4) - 3$$

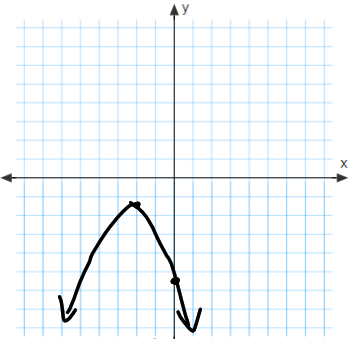
$$0 = -2x^2 - 8x - 8 - 3$$

$$0 = -2x^2 - 8x - 11$$

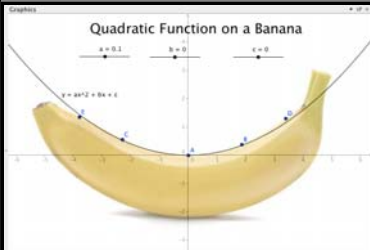
$$b^2 - 4ac$$

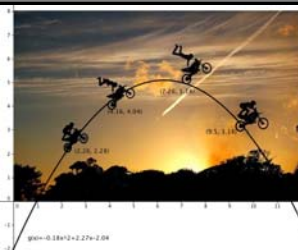
$$(-8)^2 - 4(-2)(-11)$$

$$- \#$$



$y = -2(0+2)^2 - 3$   
 $y = -11$





Ex.) The equation of the axis of symmetry of the graph of a quadratic function is  $x = -1$ . The graph passes through the points  $A(0, 3)$  and  $B(-3, 9)$ . Determine an equation of the function.

$$y = a(x+1)^2 + k \rightarrow \boxed{y = 2(x+1)^2 + 1}$$

$$\begin{aligned} 3 &= a + k \\ 3 &= 2 + k \\ \underline{\quad} & \quad \underline{k=1} \end{aligned}$$

$$3 = a(0+1)^2 + k$$

$$3 = a + k$$

$$9 = a(-3+1)^2 + k$$

$$9 = 4a + k$$

$$3 = a + k$$

$$-(9 = 4a + k)$$


---


$$-6 = -3a$$

$$\underline{2 = a}$$



A 12-m long suspension bridge is made with rope and logs. The rope forms a parabola with its lowest point 2 m above the centre of the bridge. The ropes are attached 6.5 m above the bridge. Determine an equation to model this parabola.

What assumptions are made?

$$y = a(x-h)^2 + k$$

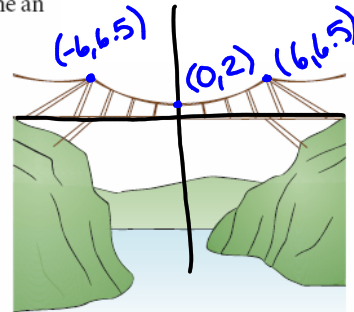
$$y = a(x-0)^2 + 2$$

$$6.5 = a(6-0)^2 + 2$$

$$6.5 = 36a + 2$$

$$4.5 = 36a$$

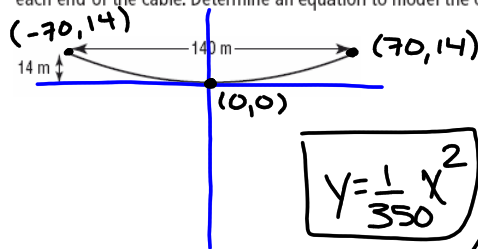
$$\frac{1}{8} = a$$



$$y = \frac{1}{8}x^2 + 2$$



4. A cable that supports a suspension bridge is parabolic. The horizontal distance between the ends of the cable is 140 m. The midpoint of the cable is 14 m below each end of the cable. Determine an equation to model the cable.



$$y = \frac{1}{350}x^2$$

$$y = a(x-p)^2 + q$$

$$y = a(x-0)^2 + 0$$

$$y = ax^2$$

$$14 = a(70)^2$$

$$14 = 4900a$$

$$\frac{1}{350} = a$$