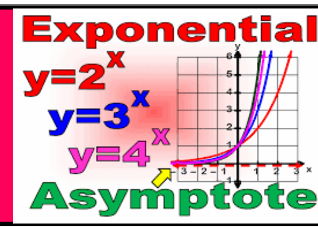
Unit 2

Exponents and Logarithms



2.6 Log Laws

Laws of Logarithms

$$\log_b(M \times N) = \log_b M + \log_b N$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_b(M^n) = n \log_b M$$

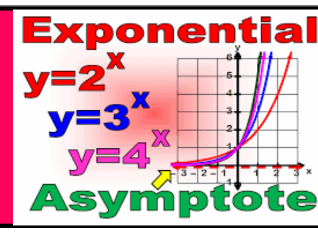
$$\log_b c = \frac{\log_a c}{\log_a b}$$

$a^2 \cdot a^3 = a^5$

Ex.) log 3 + log 5 = 1.176...
log 15 = 1.176...
×

Ex.) log 25 − log 5 = 0.698...
log 5 = 0.698...
÷

Ex.) log 3^2 = 0.954...
2log 3 = 0.954...



Ex.) Simplify the following expressions into a single logarithm:

a) $\log_2 16 - \log_2 4$

$= \log_2\left(\frac{16}{4}\right)$

$= \log_2 4$

$= \boxed{2}$

b) $\underbrace{\log_6 9 + \log_6 8}$ $- \log_6 2$

$= \log_6 72 - \log_6 2$

$= \log_6 36$

$= \boxed{2}$

OR $= \log_6\left(\frac{9 \times 8}{2}\right)$

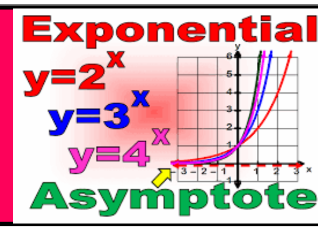
c) $\log_2 2 + \log_2 3 - \log_2 6 - \log_2 8$

$= \log_2 2 + \log_2 3 - (\log_2 6 + \log_2 8)$

$= \log_2\left(\frac{2 \times 3}{6 \times 8}\right)$

$= \log_2\left(\frac{1}{8}\right)$

$= \boxed{-3}$



d) $\log_3 x + \log_3 2x - \log_3 y - \log_3 x^2$

$= \log_3 x + \log_3 2x - \log_3 y - \log_3 x^2$

$= \log_3 x + \log_3 2x - (\log_3 y + \log_3 x^2)$

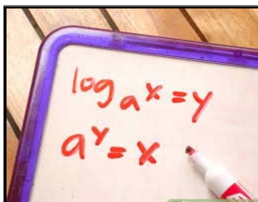
$= \log_3 \left(\frac{\cancel{x} \cdot 2\cancel{x}}{y \cdot \cancel{x^2}} \right)$

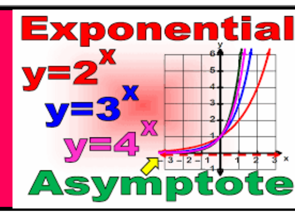
$= \boxed{\log_3 \left(\frac{2}{y} \right)}$

e) $\log_4(a+b) + \log_4(a-b)$

$= \log_4 [(a+b)(a-b)]$

$= \boxed{\log_4 (a^2 - b^2)}$





f) $\frac{1}{2}\log_2 16 - \frac{1}{3}\log_2 8$

$= \log_2 16^{1/2} - \log_2 8^{1/3}$

$= \log_2 \sqrt{16} - \log_2 \sqrt[3]{8}$

$= \log_2 4 - \log_2 2$

$= \log_2 \left(\frac{4}{2} \right)$

$= \log_2 2$

$= \boxed{1}$

g) $2\log 5 + 2\log 2$

$= 2(\log 5 + \log 2)$

$= 2(\log_{10} 10)$

$= 2 \cdot 1$

$= \boxed{2}$

h) $4\log x - \log x^3$

$= \log x^4 - \log x^3$

$= \log \left(\frac{x^4}{x^3} \right)$

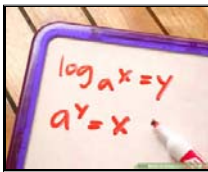
$= \boxed{\log x}$

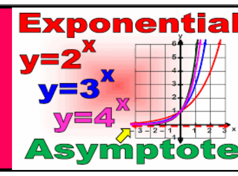
OR $= 4\log x - 3\log x$

$= 1\log x$

$= \boxed{\log x}$

Pg. 400 # 2-3.





Ex.) Expand the following expressions:

a) $\log_5(xy/z)$

$\boxed{= \log_5 x + \log_5 y - \log_5 z}$

b) $\log_7 \sqrt[3]{x} \cdot y^2$

$= \log_7 \sqrt[3]{x} + \log_7 y^2$

$= \log_7 x^{1/3} + \log_7 y^2$

$= \boxed{\frac{1}{3}\log_7 x + 2\log_7 y}$

c) $\log_3(9x^3/y\sqrt{z})$

$= \log_3(9x^3) - \log_3(y\sqrt{z})$

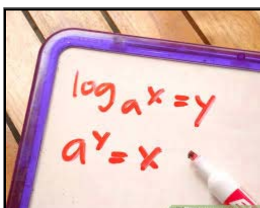
$= \log_3 9 + \log_3 x^3 - (\log_3 y + \log_3 \sqrt{z})$

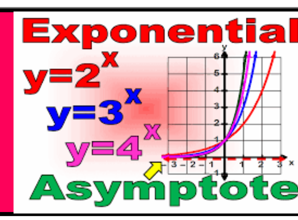
$= 2 + 3\log_3 x - \log_3 y - \log_3 z^{1/2}$

$\boxed{= 2 + 3\log_3 x - \log_3 y - \frac{1}{2}\log_3 z}$

Quiz: logs - Thur.
(2.4-2.6)

Pg. 400 # 1, 5, 7, 10-12.





More examples of Log Law questions:

Ex.) If $\log_3 8 = m$, what is the value of $\log_3 72$ in terms of m?

$= \log_3(8 \times 9)$

$= \log_3 8 + \log_3 9$

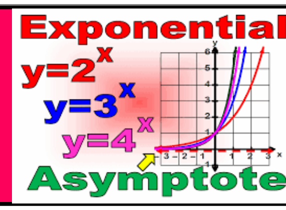
$= \boxed{m + 2}$

Ex.) If $\log_4 11 = p$, what is the value of $\log_4 44$ in terms of p?

$= \log_4(11 \times 4)$

$= \log_4 11 + \log_4 4$

$= \boxed{p + 1}$



Ex.) If log17 = k, rewrite the following in terms of *k*?

a) log170

$= \log(17 \times 10)$

$= \log 17 + \log 10 = \boxed{k+1}$

b) $\log\sqrt{17}$

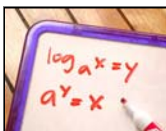
$= \log 17^{1/2}$

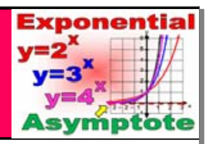
$= \frac{1}{2}\log 17 = \boxed{\frac{1}{2}k \text{ or } \frac{k}{2}}$

c) log(17/1000)

$= \log 17 - \log 1000$

$= \boxed{k-3}$





Comparison Problems:

The Richter Scale $I=I_o10^R$

Ex.) How many more times intense is an earthquake with R = 9.1 compared to R = 7.3?

$I = \frac{\cancel{I_o}10^{9.1}}{\cancel{I_o}10^{7.3}} = 10^{(9.1-7.3)} = 10^{1.8} = 63$ times more intense

"compared to"

Ex.) An earthquake has R = 7.6 another is 300 times more intense, what is its Richter scale reading?

$\frac{I_{\text{intense}}}{I_{\text{lesser}}}$

$I = \frac{\cancel{I_o}10^{R}}{\cancel{I_o}10^{7.6}}$

$300 = \frac{10^R}{10^{7.6}}$

$1.19 \times 10^{10} = 10^R$

$R = \log_{10} 1.19 \times 10^{10}$

$\boxed{R = 10.1}$

Pg. 401 # 8, 9, 15, 16.