



## 2.8 Completing the Square

### Review: Perfect Square Trinomials

Factor each trinomial.

$$x^2 - 10x + 25$$

$$x^2 + 18x + 81$$

In each trinomial, how is the third term related to the second term?

$$(x-5)^2$$

$$(x+9)^2$$



### Steps for Completing the Square

Complete the Square:

1. Take the "c" term to the right.
2. Divide the "b" term by 2 & square that answer this term (or this term times the Common Factor) is added to both sides of the EQN ("the business with the b").
3. Add like terms on the right side of the EQN.
4. Factor the perfect square trinomial on the left.
6. Move the constant on the right back to the left side of the equation after like terms are added.

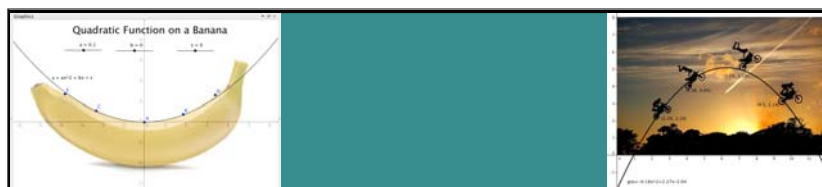


Ex.) Convert the following to vertex form. \*Complete the Square\*

$$\begin{aligned}
 & \bullet y = x^2 - 4x + 10 \\
 & \quad -10 \qquad \quad -10 \qquad \text{"business with the 'b'"} \\
 & y - 10 = x^2 - 4x + 4 \\
 & \quad +4 \qquad \text{perfect square trinomial} \left(\frac{-4}{2}\right)^2 = 4 \\
 & y - 6 = (x - 2)^2 + 6 \\
 & \bullet \boxed{y = (x - 2)^2 + 6}
 \end{aligned}$$

Ex.) Determine the coordinates of the vertex of the parabola with the equation.

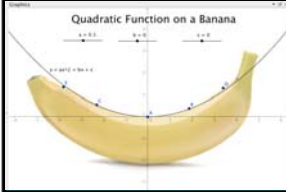
$$\begin{aligned}
 & y = 2x^2 + 16x + 24 \\
 & \quad -24 \qquad \quad -24 \qquad \left(\frac{8}{2}\right)^2 = 16 \\
 & y - 24 = 2(x^2 + 8x + 16) \\
 & \quad +32 \\
 & y + 8 = 2(x + 4)^2 - 8 \\
 & \boxed{y = 2(x + 4)^2 - 8}
 \end{aligned}$$




Ex.) Complete the Square.

$$\begin{aligned}
 & \text{a) } y = 3x^2 - 12x + 7 \\
 & y - 7 = 3(x^2 - 4x + 4) \\
 & \quad +12 \\
 & y + 5 = 3(x - 2)^2 \\
 & \boxed{y = 3(x - 2)^2 - 5}
 \end{aligned}
 \qquad \left(\frac{-4}{2}\right)^2 = 4$$

$$\begin{aligned}
 & \text{b) } y = -x^2 + 12x - 5 \\
 & y + 5 = -1(x^2 - 12x + 36) \\
 & \quad -36 \\
 & y - 31 = -1(x - 6)^2 \\
 & \boxed{y = -(x - 6)^2 + 31}
 \end{aligned}
 \qquad \left(\frac{-12}{2}\right)^2 = 36$$





c)  $y = -4x^2 + 9x - 2$

$$y + 2 = -4 \left( x^2 - \frac{9}{4}x + \frac{81}{16} \right) - \frac{81}{16}$$

$$y - \frac{49}{16} = -4 \left( x - \frac{9}{8} \right)^2$$

$y = -4 \left( x - \frac{9}{8} \right)^2 + \frac{49}{16}$

$$\left( \frac{-9/4}{2} \right)^2$$

$$-\frac{9}{4} \times \frac{1}{2} = \left( \frac{-9}{8} \right)^2 = \frac{81}{64}$$

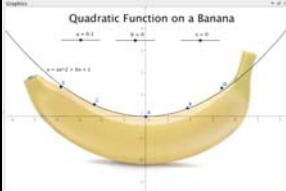
d)  $y = -2x^2 + 5x - 3$


$$y + 3 = -2 \left( x^2 - \frac{5}{2}x + \frac{25}{16} \right) - \frac{25}{8}$$

$$y - \frac{1}{8} = -2 \left( x - \frac{5}{4} \right)^2$$

$y = -2 \left( x - \frac{5}{4} \right)^2 + \frac{1}{8}$

$$\left( \frac{-5/2}{2} \right)^2 = \frac{25}{16}$$





e)  $y = -\frac{1}{2}x^2 - 3x - 8$

$$y + 8 = -\frac{1}{2} \left( x^2 + 6x + 9 \right) - \frac{9}{2}$$

$$y + 7\frac{1}{2} = -\frac{1}{2} \left( x + 3 \right)^2$$

$y = -\frac{1}{2} \left( x + 3 \right)^2 - 7\frac{1}{2}$

$$\left( \frac{6}{2} \right)^2 = 9$$

f)  $y = \frac{1}{5}x^2 + 2x - 1$

$$y + 1 = \frac{1}{5} \left( x^2 + 10x + 25 \right) - 5$$

$$y + 6 = \frac{1}{5} \left( x + 5 \right)^2$$

$y = \frac{1}{5} \left( x + 5 \right)^2 - 6$

$$\left( \frac{10}{2} \right)^2 = 25$$

Pg. 192 #2, 3.