Key

Pre-Calculus Math 30-1
Exponential and Logarithmic word problems

Solve the following problems algebraically, show all work including setting up the equation for the problem.

1. The population of a town changes by an exponential growth factor, b, every 4 years. If a population of 2350 grows to 7000 in 32 years, what is the value of b? Round your answer to two decimal places.

$y = ab^{t/p}$

$\frac{7000}{2350} = \frac{2350 b^{32/4}}{2350}$

$\left(\frac{140}{47}\right)^{1/8} = \left(b^8\right)^{1/8}$

$\boxed{b = 1.15}$

2. A sports car priced at \$60000, depreciates at 14% per year. How long will it take to depreciate to \$18000? Round to the nearest year.

100% - 14% = 86% of value remains

$y = ab^{t/p}$

$\frac{18000}{60000} = \frac{60000(0.86)^t}{60000}$

$0.3 = 0.86^t$

$\log_{0.86} 0.3 = t$

$\boxed{t = 8 \text{ years}}$

3. Light passing through murky water loses 30% of its intensity for every metre of water depth. At what depth will the light intensity be half of what it is at the surface? Round your answer to two decimal places.

70% intensity remains

$p = 1$

$y = ab^{t/p}$

$1 = 2(0.70)^t$

$\frac{1}{2} = 0.70^t$

$\log_{0.70}(\tfrac{1}{2}) = t$

$\boxed{t = 1.94\text{m}}$

4. A town in Saskatchewan is growing at a rate of 3.5% per annum. If it continues to grow at this rate the population will reach 40000 in 5 years. What is the current population of the town?

$b = 1.035$

$y = ab^{t/p}$

$40000 = a(1.035)^{5/1}$

$\frac{40000}{1.187\ldots} = \frac{a(1.187\ldots)}{1.187\ldots}$

$\boxed{a = 33\,679 \text{ people}}$

5. The water in a sketchy little town has 500g of pollutants. Each time the water passes through a filter 18% of the pollutants are removed. How many filters will it take to remove the mass of the pollutants to 150g?

82% remain

$b=0.82$

$y=ab^{t/p}$

$\frac{150}{500}=\frac{500(0.82)^t}{500}$

$0.3=0.82^t$

$\log_{0.82} 0.3=t$

$\boxed{t=7 \text{ filters}}$

6. A radioactive isotope has a half-life of 45 minutes. How long will it take 480 mg to decay to 15mg? Round to the nearest minute.

$y=ab^{t/p}$

$\frac{15}{480}=\frac{480(1/2)^{t/45}}{480}$

$1/32=(1/2)^{t/45}$

$45\cdot\log_{0.5}(1/32)=\frac{t}{45}\cdot 45$

$\boxed{t=225\text{ min}=3.75\text{h}}$

7. The population of a bacteria doubles every 20 minutes. How long will it take for the population to triple? Round to the nearest minute

$3=1(2)^{t/20}$

$20\cdot\log_2 3=\frac{t}{20}\cdot 20$

$\boxed{t=32\text{ min}}$

8. What is the half-life of a radioactive sample if it takes 7 years to decay from 560 grams to 35 grams? Round to the nearest year.

$P=?$

$35=560(1/2)^{7/p}$

$\frac{1}{16}=(1/2)^{7/p}$

$p\cdot\log_{0.5}(1/16)=\frac{7}{p}\cdot p$

$\frac{p\log_{0.5}(1/16)}{\log_{0.5}(1/16)}=\frac{7}{\log_{0.5}(1/16)}$

$p=1.75=\boxed{2\text{ year}}$

9. The mass of a radioactive isotope in a sample can be represented by $M=14(0.96)^x$, where x is the time in minutes.

a. What is the initial mass of the sample?

14 units

b. Determine the mass after 1 hour.

60min

$M=14(0.96)^{60}=1.2$ units

c. Determine the time for the initial mass to reduce to one-half.

$7=14(0.96)^x$

$0.5=0.96^x$

$\log_{0.96} 0.5=x$

$\boxed{x=17\text{ min.}}$

d. Rewrite the equation where the base (b) is $\frac{1}{2}$.

$y = 14(1/2)^{x/171}$

from question (c)