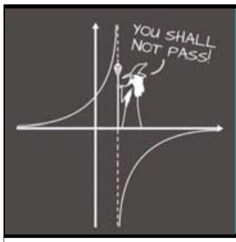
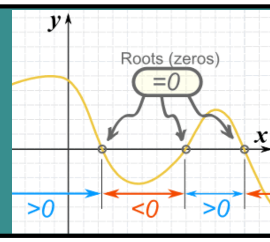


Unit 3: Polynomial, Radical, and Rational Functions



3.2 Synthetic Division and The Remainder Theorem

Recall: Long Division with Numbers

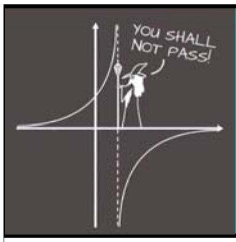
$$\begin{array}{r} 4047 \\ 12\,\overline{)\,48567} \\ -48 \\ \hline 056 \\ -48 \\ \hline 87 \\ -84 \\ \hline 3 \end{array}$$

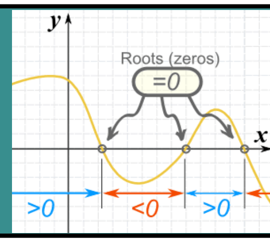
Quotient

Dividend

Divisor

Remainder





Terminology:

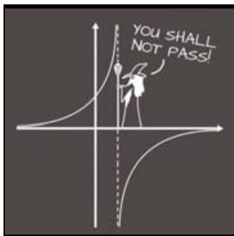
Quotient and Remainder

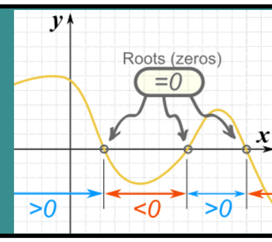
Divisor $\overline{)\text{Dividend}}$

f(x) = Dividend
g(x) = Divisor
q(x) = Quotient and Remainder

then,

f(x) = g(x) q(x) = Divisor (Quotient + [Remainder / Divisor])





Now, long division with polynomials.

$$5x^2 - 3x + 4$$

$$(x-2)\overline{)\,5x^3 - 13x^2 + 10x - 9}$$

$$-\cancel{5x^3} + 10x^2$$

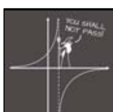
$$-3x^2 + 10x$$

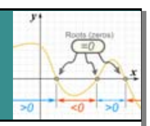
$$+\cancel{3x^2} \mp 6x$$

$$4x - 9$$

$$-\cancel{4x} + 8$$

$$-1$$





Long division is a great way to divide polynomials. However, there is a simpler way that removes all the variables and just works with the coefficients; Synthetic Division.

Synthetic Division: $(5x^3 - 13x^2 + 10x - 9) \div (x-2)$

Factor: $x-2$

$x - 2 = 0$

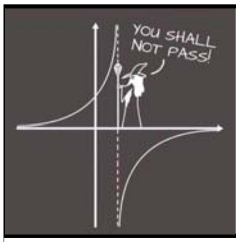
Root: $x = 2$

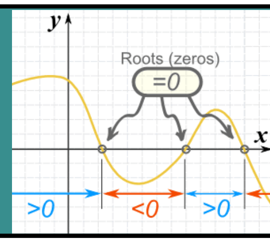
$$\begin{array}{c|cccc} 2 & 5 & -13 & 10 & -9 \\ & \downarrow & 10 & -6 & 8 \\ \hline & 5 & -3 & 4 & -1 \end{array}$$

$\underbrace{5 \quad -3 \quad 4}_{5x^2 - 3x + 4}$ -1 ↑ Remainder

$$\frac{5x^3 - 13x^2 + 10x - 9}{(x-2)} = (5x^2 - 3x + 4),\ R: -1$$

Steps:
1. Change factor to a root.
   $x - 2 = 0$
   $\underline{x = 2}$
2. Check that each subsequent degree is accounted for.
3. Write down coefficients.
4. Bring down first #.
5. Multiply, put answer in next column and <u>add</u>.
6. Repeat...
7. Answer: #'s are coefficients of the quotient (degree one less) and remainder.

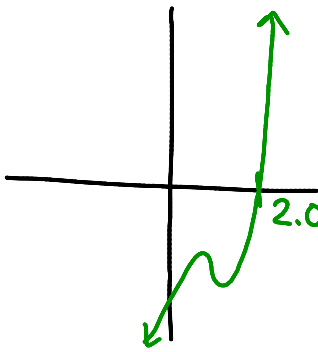




What does this tell us about polynomials?

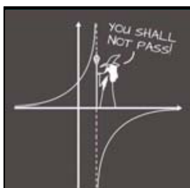
In Math 20-1: $f(x) = x^2 + 7x + 6$ factors to $(x + 6)(x + 1)$ so x-int's are -6, -1.

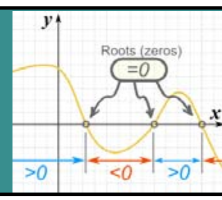
In Math 30-1: We use synthetic division to see if there is a remainder. **If there is a remainder, the divisor is not a factor of the dividend.**



2.056...

x-int $\neq$ 2 because there was a remainder





The Remainder Theorem:

If a polynomial, f(x), is divided by (x - a), the remainder is the constant f(a), and

dividend = quotient x divisor + remainder

$$f(x) = q(x) \cdot (x - a) + f(a)$$

where q(x) is a polynomials with degree one less that the degree of f(x).

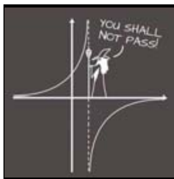
So with our quadratic example:

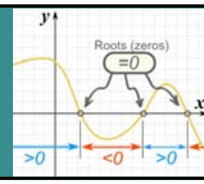
$f(x) = 5x^3 - 13x^2 + 10x - 9$

$f(2) = 5(2)^3 - 13(2)^2 + 10(2) - 9$

$f(2) = -1$ remainder

$\therefore$ $(x-2)$ is not a factor of $5x^3 - 13x^2 + 10x - 9$





Ex.) Divide $\underbrace{2x^3 + 3x^2 - 4x + 15}_{}$ by $(x+3)$.

$f(x)$

$$\begin{array}{r|rrrr} -3 & 2 & 3 & -4 & 15 \\ & \downarrow & -6 & 9 & -15 \\ \hline & 2 & -3 & 5 & 0 \end{array}$$

← Remainder = 0 means $(x+3)$ is a factor.

$$f(x) = (2x^2 - 3x + 5)(x+3)$$

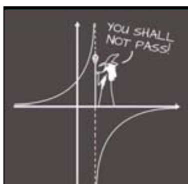
Ex.) Divide $5x^4 - 3x^2 + 2x - 7$ by $(x+2)$.

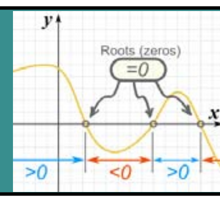
$$5x^4 + 0x^3 - 3x^2 + 2x - 7$$

$$\begin{array}{r|rrrrr} -2 & 5 & 0 & -3 & 2 & -7 \\ & \downarrow & -10 & 20 & -34 & 64 \\ \hline & 5 & -10 & 17 & -32 & 57 \end{array}$$

← remainder

$$(5x^3 - 10x^2 + 17x - 32)(x+2) + 57$$





Ex.) Use the remainder theorem to find the remainder when $P(x) = x^3 - 10x + 6$ is divided by $x-4$.

$x - 4 = 0$

$x = 4$

$P(4) = (4)^3 - 10(4) + 6$

$P(4) = 30$

Remainder: 30 ∴ $(x-4)$ not a factor

Ex.) Use synthetic division to divide $4x^5 - 3x^3 + 7x^2 - 6$ by $(x+1)$.

$x + 1 = 0$

$x = -1$

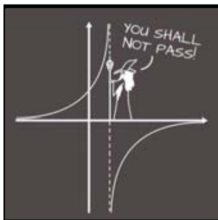
$$4x^5 + 0x^4 - 3x^3 + 7x^2 + 0x - 6$$

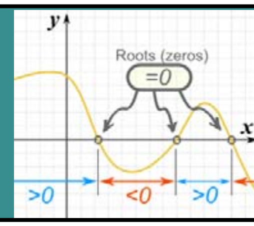
$$\begin{array}{r|rrrrrr} -1 & 4 & 0 & -3 & 7 & 0 & -6 \\ & \downarrow & -4 & 4 & -1 & -6 & 6 \\ \hline & 4 & -4 & 1 & 6 & -6 & 0 \end{array}$$

← remainder

∴ $(x+1)$ is a factor

$$\frac{4x^5 - 3x^3 + 7x^2 - 6}{(x+1)} = 4x^4 - 4x^3 + x^2 + 6x - 6$$





Ex.) When $P(x) = x^3 + 4x^2 - x + k$ is divided by $(x-1)$, the remainder is 3. What is the value of 'k'?

$$P(1) = 3$$

$$(1)^3 + 4(1)^2 - 1 + k = 3$$

$$\cancel{4} + k = 3$$

$$-\cancel{4} \qquad -4$$

$$\boxed{k = -1}$$

Pg. 124 # 3, 6, 8, 10.