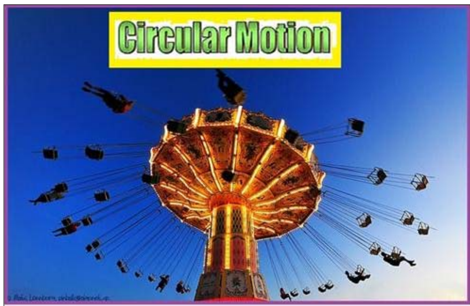
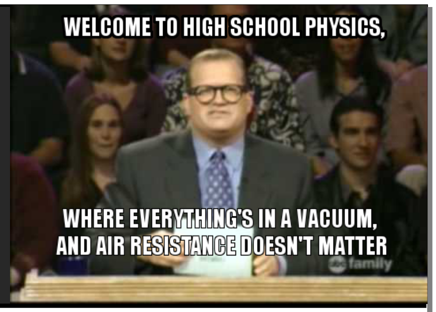
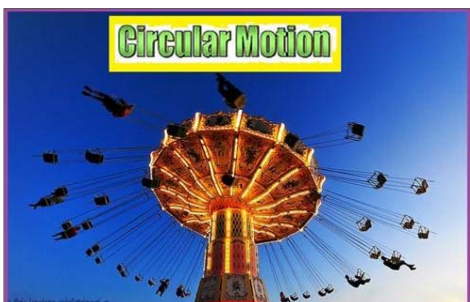


Unit 3: Circular Motion, Work and Energy

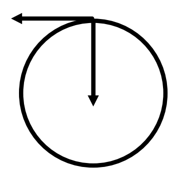


3.2 Vertical Circular Motion

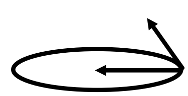
Review: In order to increase speed with which a vehicle can navigate a turn, roads are banked (angled slightly). If a typical bank is 3.0° to the horizontal, what is the maximum speed a 1500 kg car can make a 200 m radius turn(μ=0.10)?



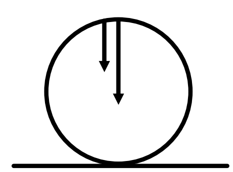
We have always drawn horizontal circular motion like this:



But it really should have been drawn like this:



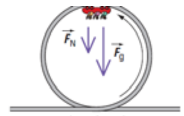
Because now we look at vertical circular motion like this:



($\vec{F}_g$ is now at play)

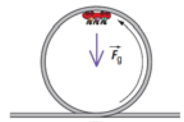
<u>Analysis of Vertical Circular Motion</u>: Why doesn't a roller coaster fall off the tracks when upside down?

▶ **Figure 5.28(a)** The first time through the loop, the speed is such that the roller coaster requires a centripetal force of 1500 N to keep it moving in a circular path. At the top of the loop, the roller coaster car will experience a centripetal force that is the sum of the force of gravity and the force exerted by the track, pushing the car inward to the centre of the circle. The centripetal force acts down, so it is −1500 N. The force of gravity is constant at 1000 N so the track pushes inward with 500 N to produce the required centripetal force. The car goes around the loop with no problem.

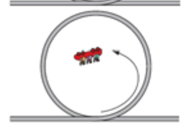


$$\vec{F}_c = \vec{F}_g + \vec{F}_N$$
$$-1500\text{ N} = -1000\text{ N} + -500\text{ N}$$

▶ **Figure 5.28(b)** Suppose the next time the car goes around the track, it is moving more slowly, so that the centripetal force required is only 1000 N. In this case, the force of gravity alone can provide the required centripetal force. Therefore, the track does not need to exert any force on the car to keep it moving on the track. There is no normal force, so the force of gravity alone is the centripetal force. The car goes around the loop again with no problem.



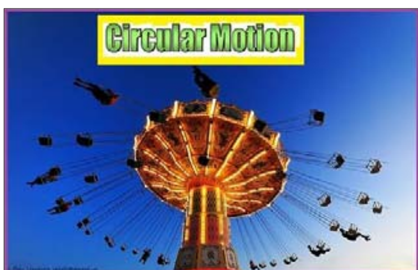
▶ **Figure 5.28(c)** Now suppose the last time the car goes around the track, it is moving very slowly. The required centripetal force is just 800 N, but the force of gravity is constant, so it is still 1000 N; that is, 200 N more than the centripetal force required to keep the car moving in a circular path with this radius. If the track could somehow pull upward by 200 N to balance the force of gravity, the car would stay on the track. This is something it can't do in our hypothetical case. Since the gravitational force cannot be balanced by the track's force, it pulls the car downward off the track.



$$\vec{F}_c = \vec{F}_g + \vec{F}_N$$
$$-800\text{ N} = -1000\text{ N} + 200\text{ N}$$

Since the normal force cannot pull upward, it cannot generate +200 N. 200 N more is needed to keep the car on a track of this radius, so the car falls off.

The centripetal force is supplied by the normal force and gravity at the top of the loop. $F_{net} = F_c$



Ex.) Neglecting friction, what is the minimum speed a Hot Wheels car must go around a vertical loop of radius $\underbrace{15.0\text{ cm}}_{0.15\text{m}}$ to keep from falling off?

$F_{net} = F_g$

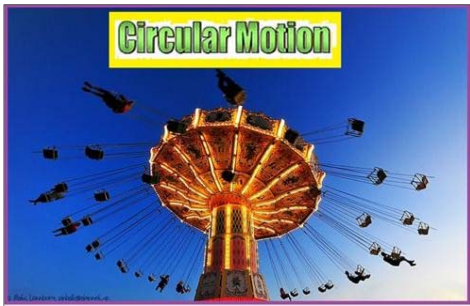
$F_c = F_g$ * minimum speed to keep car on track

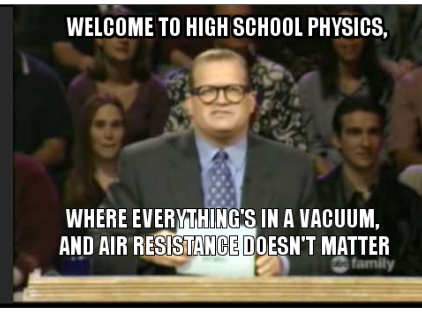
$\frac{\cancel{m}v^2}{r} = \cancel{m}g$

$v^2 = rg$

$v = \sqrt{rg} = \sqrt{(0.15)(9.81)} = \boxed{1.21\text{ m/s}}$

$v \geq 1.21\text{ m/s}$





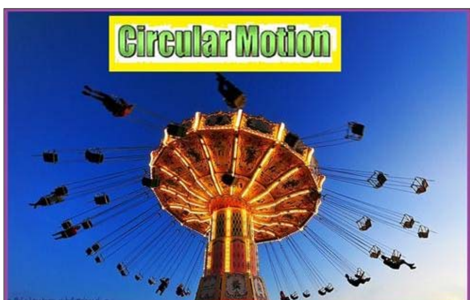
Ex.) What is the maximum radius a roller coaster loop can be if a cart with speed of 20.0 m/s is to go around safely?

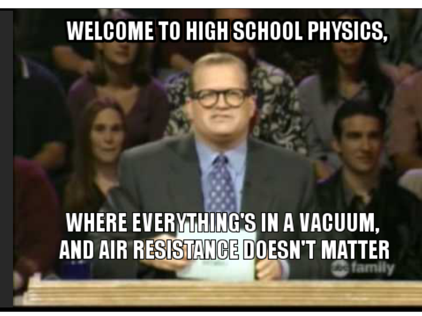
$F_{net} = F_g$

$F_c = F_g$

$\frac{\cancel{m}v^2}{r} = \cancel{m}g$

$r = \frac{v^2}{g} = \frac{20.0^2}{9.81} = \boxed{40.8\text{m}}$





Ex.) What is the force the roller coaster track is providing to a 102 kg cart traveling at 15.0 m/s around a 7.0 m loop?

$r = 3.5\text{m}$

$F_c = \frac{mv^2}{r}$

$= \frac{102(15.0)^2}{3.5}$

$= 6.6\times10^3\text{N}$

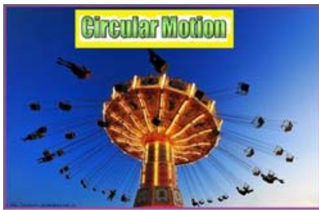
From slide 3:

$F_c = F_g + F_N$

$-6.6\times10^3 = 102(-9.81) + F_N$

$\boxed{F_N = -5.6\times10^3}$

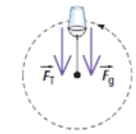
force of track on the cart



Recall the bucket of water scenario. If we now twirl the bucket of water vertically what is keeping the water in the bucket?

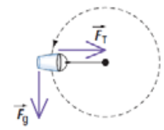
▶ **Figure 5.30(a)** The bucket is at the top of the circle. In this position, two forces are acting on the bucket: the force of gravity and the tension of the rope. Both are producing the centripetal force and are acting downward. The equation to represent this situation is:
$\vec{F}_c = \vec{F}_g + \vec{F}_T$

(a)



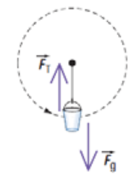
▶ **Figure 5.30(b)** When the bucket has moved to the position where the rope is parallel to the ground, the force of gravity is perpendicular to the tension. It does not contribute to the centripetal force. The tension alone is the centripetal force. We can write this mathematically as:
$\vec{F}_c = \vec{F}_T$

(b)

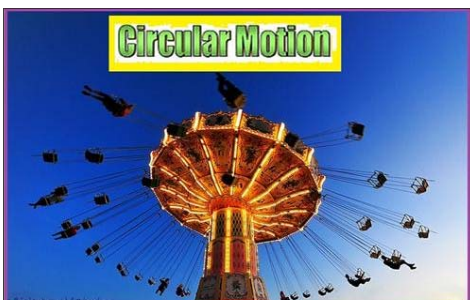


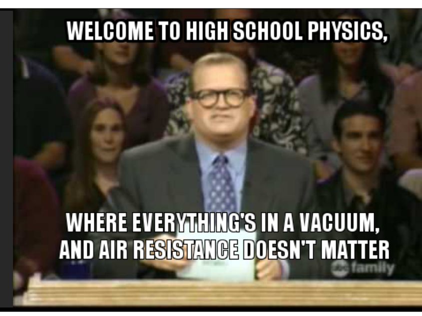
▶ **Figure 5.30(c)** As the bucket moves through the bottom of the circle, it must have a centripetal force that overcomes gravity. The tension is the greatest here because gravity is acting opposite to the centripetal force. The equation is the same as in (a) above, but tension is acting upward, so when the values are placed into the equation this time, $\vec{F}_T$ is positive and $\vec{F}_g$ is negative. The effect is demonstrated in Example 5.7.

(c)



Here, gravity and tension work to supply the centripetal force.





It would seem like the force acting on the water must be acting outward to keep the water in the bucket, but it is not. The centripetal force is pointed towards the centre of the circle. And this force keeps things in a circular pattern, not a pattern where the water will fall.

Some people will refer to the "force" keeping the water in the bucket as "centrifugal force." But we know this isn't a real force. Physicists will refer to these "made up" forces as phantom forces or fictitious forces.

Ex.) A string can hold a force of 135 N before breaking. If a 2.00 kg object is tied to the end of this string (L = 1.10 m), how fast can I spin it vertically before the string breaks?

radius

$F_{net} = 135 - F_g$

$F_c = 135 - mg$

$\frac{mv^2}{r} = 135 - mg$

$\frac{(2.00)v^2}{1.10} = 135 - (2.00)(9.81)$

$v^2 = 63.459$

$\boxed{v = 7.97 \text{ m/s}}$

Speeds that break string: $v > 7.97$ m/s

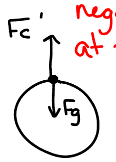
Bucket on String

Top

$F_{net} = F_c + F_g$

$0 = F_c + F_g$

negative at top

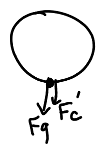


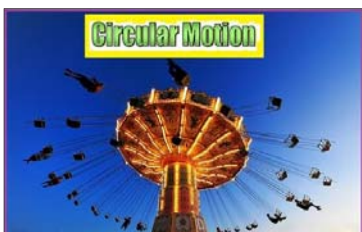
Bottom

$F_{net} = F_c + F_g$

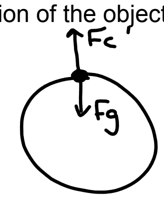
$0 = F_c + F_g$ ← positive at bottom

zero net force means circular motion maintained.





Ex.) An object is being swung in a vertical circle with r = 0.75 m. What is the minimum speed needed at the top of the swing to maintain uniform circular motion of the object?



$F_{net} = F_c - F_g$

$0 = F_c - F_g$

$F_c = F_g$

$\frac{\cancel{m}v^2}{r} = \cancel{m}g$

$|v| = \sqrt{rg} = \sqrt{(0.75)(9.81)}$

$= \sqrt{7.4} \text{ m/s} = \boxed{2.7 \text{ m/s}}$

Questions: Pg. 268 # 6-13.