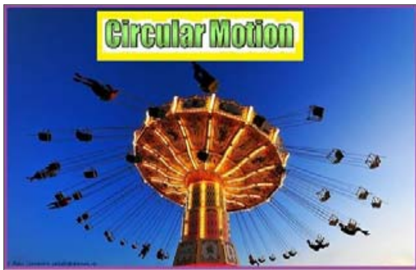


Unit 3: Circular Motion, Work and Energy

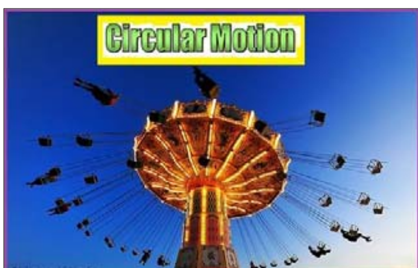
3.3 Satellite Motion

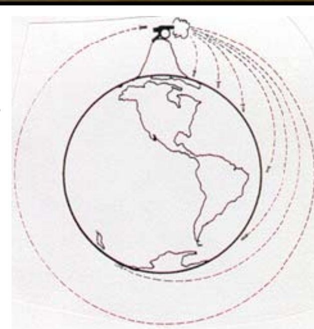
Satellite - any object that orbits around a central object (usually a planet)

Artificial Satellites - communications satellites, Space Shuttel, Hubble, ISS, space junk

Natural Satellites - moons of planets, planets orbiting the Sun

Geosynchronous Satellite - a satellite that orbits Earth once per day

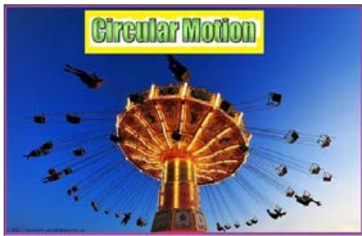




For the projectile problems we worked with there was an unstated assumption that the Earth was flat. However, we know that the Earth is in fact spherical, although not perfectly so. With this in mind, Sir Isaac Newton reasoned that some strange things would happen if one could horizontally project an object at high speeds.

At low speeds, a horizontal projectile will fall toward and hit the ground in a short time. As the speed of the horizontal projectile is increased, it will land further and further away from the starting point. For a *flat Earth* the projectile would always hit the ground; no matter how fast the projectile went, gravity would pull it down to the ground.

However, since the Earth is round, the curvature of the Earth affects where the projectile lands. As the diagram indicates, the greater the horizontal speed of the projectile, the more the Earth's curvature comes into play. Eventually, a critical speed is reached where, even though **the projectile is in constant freefall**, it would not hit the Earth, rather, it would become a **satellite** in **orbit** around the Earth.



Newton was able to calculate the speed required to put an object into an orbit which would just skim the surface of a smooth Earth. Newton realized that a satellite orbiting the Sun, the Earth, or some other body is simply a case of uniform circular motion. He reasoned that the **gravitational force would act as the centripetal force for a circular orbit**.

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

(the mass of the satellite cancels, and one r cancels as well)

$$v^2 = \frac{GM}{r}$$

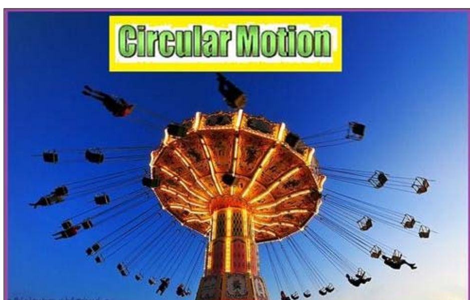
$$v = \sqrt{\frac{GM}{r}}$$

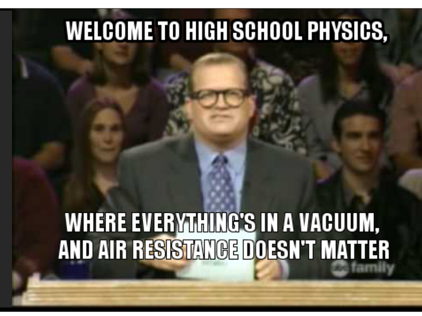
where:
M mass of body being orbited
r distance from the center of M to the satellite

Note:

1. The orbit does not depend on the mass of the satellite as long as the mass of the planet or star around which the satellite is orbiting is much, much larger than the mass of the satellite.

2. The treatment of orbits that we shall work with is quite limited. For example, to calculate the escape velocities of satellites requires that we account for gravitational potential energy. In addition, we will limit ourselves to circular and not elliptical orbits.



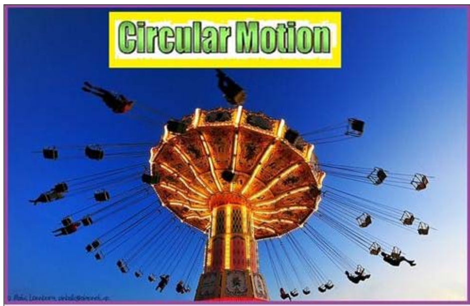


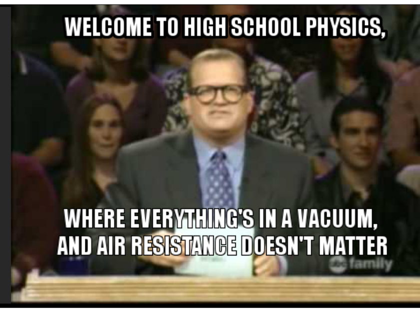
Ex.) What is the speed of orbit for a satellite orbiting Saturn if the radius of orbit is 6.43 x 10^7 m? Pg. 218.

$$V = \sqrt{\frac{Gm}{r}} = \sqrt{\frac{(6.67\times10^{-11})(5.69\times10^{26})}{(6.43\times10^{7})}}$$

$$= \boxed{2.43\times10^{4}\ \text{m/s}}$$

$$= \boxed{8.75\times10^{4}\ \text{km/h}}$$





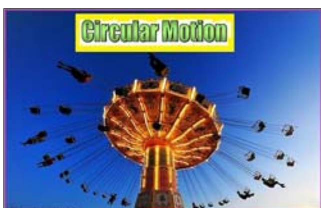
Recall that the derivation of the formula we used came from the idea that the force of gravity is supplying the centripetal force. We have more choices for these equations depending on the unknown in the problem:

$$\vec{F}_g = mg$$

$$\vec{F}_g = \frac{Gm_1m_2}{r^2}$$

$$F_c = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2}$$

In the following examples you need to choose the appropriate formulas to use based on the variables given and the unknown.



Ex.) Galileo discovered 4 moons of Jupiter, listed below. Also listed are their periods of revolution and their orbital radii (centre to centre). From this data, determine the mass of Jupiter.

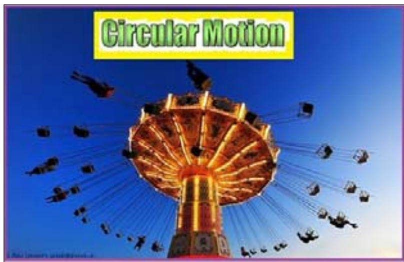
moon	period (days) T	distance (10^6 m) r
Io	1.769137786	422
Europa	3.551181041	671
Ganymede	7.154552960	1070
Callisto	16.68901840	1883

$$F_c = F_g$$

$$\frac{4\pi^2 r \cancel{m_I}}{T^2} = \frac{G m_J \cancel{m_I}}{r^2}$$

$$m_J = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (422\times10^6)^3}{(6.67\times10^{-11})(152853.5047)^2} = \boxed{1.90\times10^{27}\ \text{kg}}$$

$$1.769137786\ \text{days} \times \frac{24\ \text{h}}{1\ \text{day}} \times \frac{60\ \text{min}}{1\ \text{h}} \times \frac{60\ \text{s}}{1\ \text{min}} = 152853.5047\ \text{s}$$



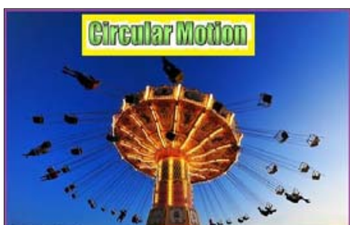
Ex.) What is the speed of the moon Io based on the information from the previous slide?

$$F_c = F_g$$

$$\frac{\cancel{m_I} v^2}{r} = \frac{G m_J \cancel{m_I}}{r^2}$$

$$v = \sqrt{\frac{G m_J \cancel{r}}{r^{2}}} = \sqrt{\frac{G m_J}{r}} = \sqrt{\frac{(6.67\times10^{-11})(1.90\times10^{27})}{(422\times10^{6})}}$$

$$= \boxed{\begin{array}{l} 1.73\times10^{4}\ m/s \\ 6.24\times10^{4}\ km/h \end{array}}$$



Ex.) The average distance from the centre of the Earth to the centre of the Moon is 3.85 x 10^8 m? What is the period of orbit of the Moon around the Earth?

$$F_c = F_g$$

$$\frac{4\pi^2 r \cancel{m_m}}{T^2} = \frac{G m_E \cancel{m_m}}{r^2}$$

$$\frac{4\pi^2 r}{G m_E} = \frac{T^2}{r^2}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{G m_E}} = \sqrt{\frac{4\pi^2 (3.85\times10^{8})^3}{(6.67\times10^{-11})(5.97\times10^{24})}} = 2.38\times10^{6}\ s$$

$$= \boxed{27.5\ days}$$

subtract r_E

Ex.) Determine the height from the surface of the Earth of a geo-sync satellite.

$$1\,\text{day} \times \frac{24\,h}{\text{day}} \times \frac{60\,\text{min}}{h} \times \frac{60\,s}{\text{min}} = 86400\,s$$

orbits once per day

$T = 86400\,s$

$$F_c = F_g$$

$$\frac{4\pi^2 r \cancel{m_s}}{T^2} = \frac{G \cancel{m_s} m_E}{r^2}$$

$$\frac{r^3}{T^2} = \frac{G m_E}{4\pi^2}$$

$$r^3 = \frac{G m_E T^2}{4\pi^2}$$

$$r = \sqrt[3]{\frac{G m_E T^2}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{(6.67\times10^{-11})(5.97\times10^{24})(86400)^2}{4\pi^2}} = \boxed{3.59\times10^{4}\,\text{km}}$$

35 857 km

$$= 4.2\times10^{7}\,m - r_E$$

$$4.2\times10^{7}\,m - 6.37\times10^{6} = 3.59\times10^{7}\,m$$

Read: Pg. 276 - 286

Questions: Pg. 286 # 9-13.