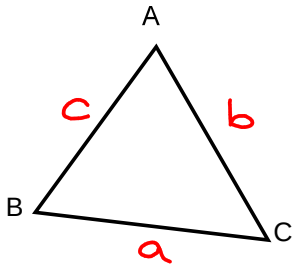
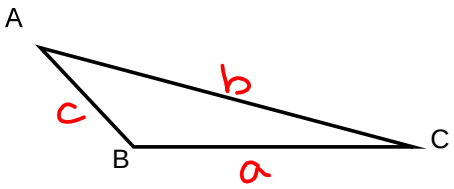
3.3 Sine Law

The Sine Law states that, for any triangle ABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ OR $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



Acute Triangle

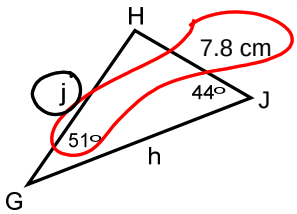


Oblique Triangle

The Sine Law can be used to determine

a missing side or angle given an "angle-side pair", a, ∠A.

Ex.) In ΔGHJ, determine the length of GH to the nearest tenth of a centimeter.



Sine Law

$\frac{j}{\sin J} = \frac{g}{\sin G}$

$\frac{j}{\sin 44°} = \frac{7.8}{\sin 51°}$

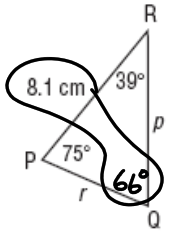
$j = \frac{(7.8 \times \sin(44°))}{\sin 51°}$

$j = 7.0\text{ cm}$

Example 1 Using the Sine Law to Determine the Length of a Side

*any time you have 2 angles, write the 3rd *

In ΔPQR, determine the length of QR to the nearest tenth of a centimetre.



$$\frac{P}{\sin P} = \frac{q}{\sin Q}$$

$$\frac{p}{\sin 75^\circ} = \frac{8.1}{\sin 66^\circ}$$ ✓

$$p = 8.6\text{ cm}$$

The Ambiguous Case of Sine Law

Always considered when calculating an ANGLE with Sine Law

The sine of supplementary angles are equivalent.

add to 180°

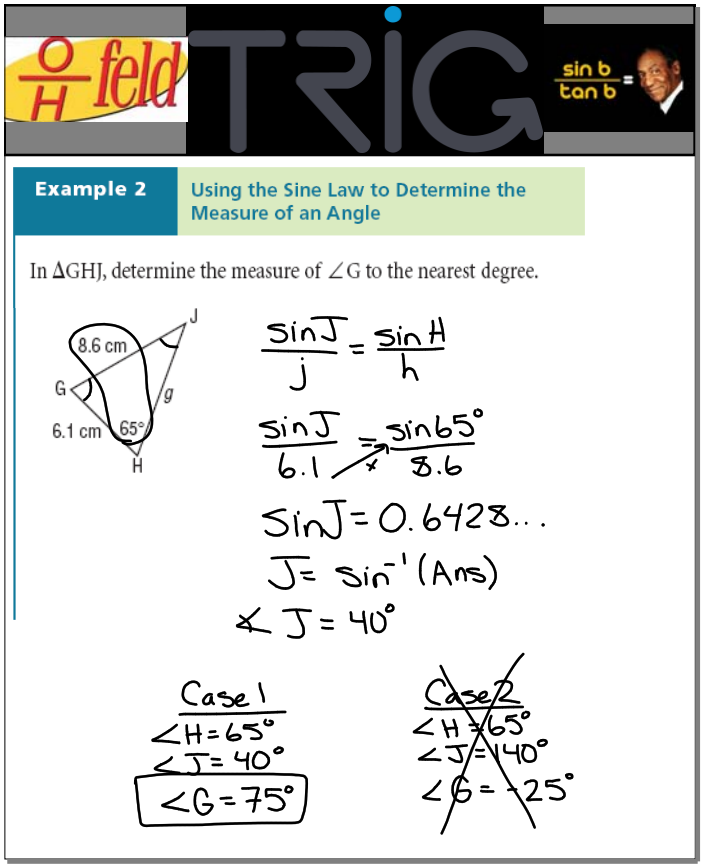
i.e. $\sin 10^\circ = \sin 170^\circ$

$\sin 110^\circ = \sin 70^\circ$

$>90^\circ$

Problem: Calculators won't calculate obtuse angles with $\sin^{-1}$.

Therefore, we consider 2 cases **everytime.**



Example 2 Using the Sine Law to Determine the Measure of an Angle

In ΔGHJ, determine the measure of ∠G to the nearest degree.

$\frac{\sin J}{j} = \frac{\sin H}{h}$

$\frac{\sin J}{6.1} = \frac{\sin 65°}{8.6}$

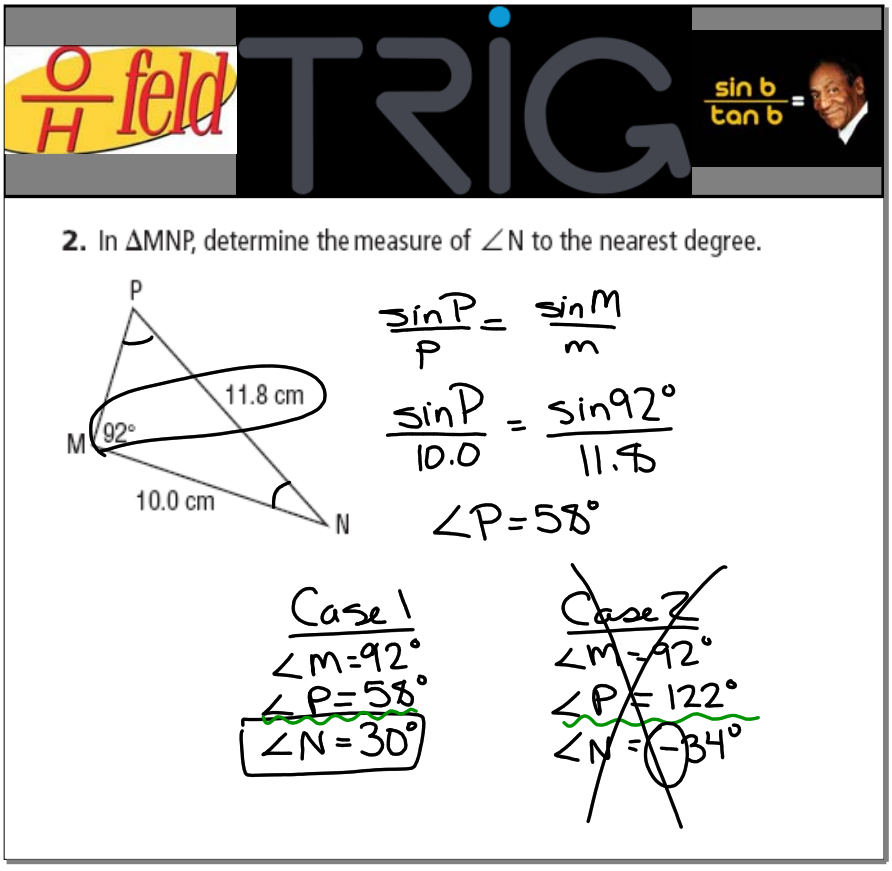
$\sin J = 0.6428...$

$J = \sin^{-1}(\text{Ans})$

$\angle J = 40°$

Case 1
$\angle H = 65°$
$\angle J = 40°$
$\angle G = 75°$

Case 2
$\angle H = 65°$
$\angle J = 140°$
$\angle G = -25°$



2. In ΔMNP, determine the measure of ∠N to the nearest degree.

$\frac{\sin P}{p} = \frac{\sin M}{m}$

$\frac{\sin P}{10.0} = \frac{\sin 92°}{11.8}$

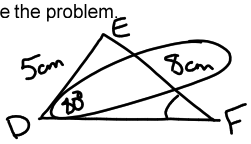
$\angle P = 58°$

Case 1
$\angle M = 92°$
$\angle P = 58°$
$\angle N = 30°$

Case 2
$\angle M = 92°$
$\angle P = 122°$
$\angle N = -34°$

In $\triangle DEF$ DE = 5 cm, $\angle D = 80°$, and FE = 8 cm.

a) Draw a triangle to illustrate the problem.



all sides
all angles

b) Solve the triangle. Give angle measures to the nearest degree and side lengths to the nearest tenth of a centimeter.

$\frac{\sin F}{f} = \frac{\sin D}{d}$

$\frac{\sin F}{5} = \frac{\sin 80°}{8}$

$\angle F = 38°$

$\frac{e}{\sin E} = \frac{d}{\sin D}$

$\frac{e}{\sin 62°} = \frac{8}{\sin 80°}$

$e = 7.2\text{cm}$

Case 1
$\angle D = 80°$
$\angle F = 38°$
$\angle E = 62°$
$d = 8\text{cm}$
$f = 5\text{cm}$
$e = 7.2\text{cm}$

~~Case 2~~
~~$\angle D = 80°$~~
~~$\angle F = 142°$~~
~~$\angle E = 142°$~~

Pg. 108 # 1-5.