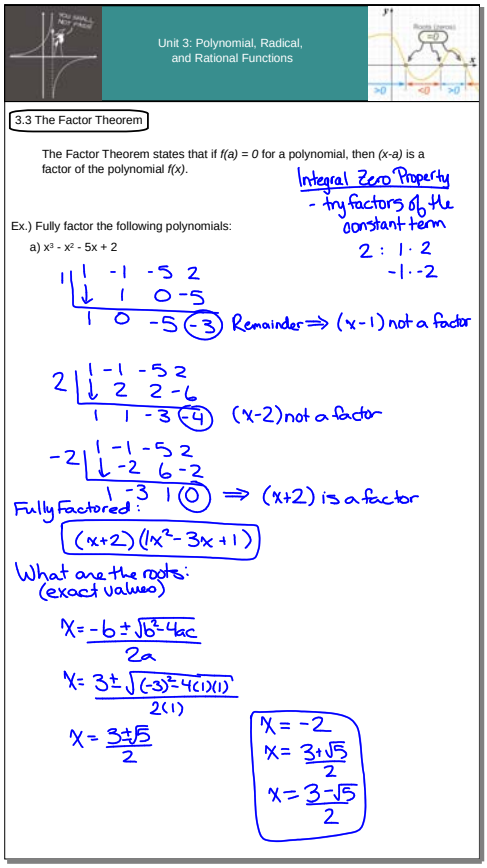
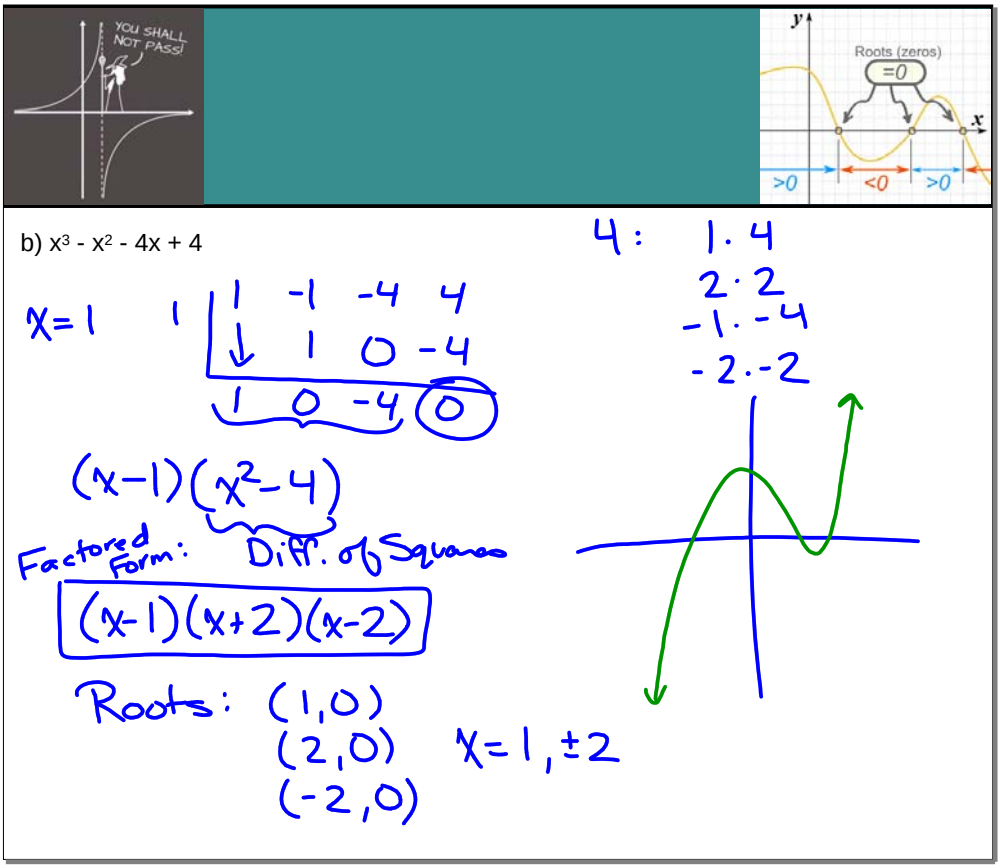

Unit 3: Polynomial, Radical, and Rational Functions
3.3 The Factor Theorem
The Factor Theorem states that if f(a) = 0 for a polynomial, then (x-a) is a factor of the polynomial f(x).
Integral Zero Property
- try factors of the constant term
Ex.) Fully factor the following polynomials:
a) x³ - x² - 5x + 2
2 : 1 · 2
-1 · -2
1 | 1 -1 -5 2
↓ 1 0 -5
1 0 -5 (-3) Remainder ⇒ (x-1) not a factor
2 | 1 -1 -5 2
↓ 2 2 -6
1 1 -3 (-4) (x-2) not a factor
-2 | 1 -1 -5 2
↓ -2 6 -2
1 -3 1 (0) ⇒ (x+2) is a factor
Fully Factored:
(x+2)(1x² - 3x + 1)
What are the roots: (exact values)
x = -b ± √(b² - 4ac) / 2a
x = 3 ± √((-3)² - 4(1)(1)) / 2(1)
x = 3 ± √5 / 2
x = -2
x = 3+√5 / 2
x = 3-√5 / 2


YOU SHALL NOT PASS!
y
Roots (zeros)
=0
x
>0
<0
>0
b) x³ - x² - 4x + 4
4 : 1 · 4
2 · 2
-1 · -4
-2 · -2
x = 1
1 | 1 -1 -4 4
↓ 1 0 -4
1 0 -4 0
(x-1)(x²-4)
Factored Form: Diff. of Squares
(x-1)(x+2)(x-2)
Roots: (1,0)
(2,0)
(-2,0)
x = 1, ±2

c) $2x^3 - 5x^2 - 4x + 3$

$x = 3$ | 2 -5 -4 3
↓ 6 3 -3
2 1 -1 0

$(x-3)(2x^2+1x-1)$

m: -2
2 ◇ -1 $\frac{2x^2}{2x} + \frac{1x}{-1} = \frac{-1x}{-1} = 1$
A: 1 $2x(x+1)(-1)(x+1)$

$(x-3)(x+1)(2x-1)$

$(x-3)(2x^2+1x-1)$

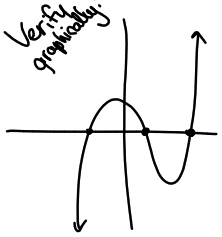
-1 | 2 1 -1
↓ -2 1
2 -1 0

$(x-3)(x+1)(2x-1)$

$x = 3, -1, \frac{1}{2}$

$-1 : -1 \cdot 1$

Verify graphically:



d) $x^4 - 5x^3 + 2x^2 + 20x - 24$

-2 | 1 -5 2 20 -24
↓ -2 14 -32 24
1 -7 16 -12 0

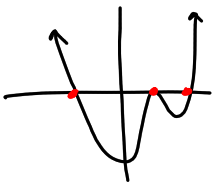
$(x+2)(x^3-7x^2+16x-12)$

2 | 1 -7 16 -12
↓ 2 -10 12
1 -5 6 0

$(x+2)(x-2)(x^2-5x+6)$ -2 ◇ -3 -5

$(x+2)(x-2)(x-2)(x-3)$

$(x-2)^2(x+2)(x-3)$



Pg. 133 # 1, 3, 5, 6, 7, 11, 12.