
3.5 Determining the Equation of a Polynomial

$$
P(x)=a(x-a)(x-b)(x-c) \ldots
$$

Here's the information you may be given in a question or on a graph in order to write the equation:

- roots ( x-int, related to factors)
- multiplicities of roots
- degree of the equation
- a point on the graph ( $x, y$ )


Ex.) A polynomial function has zeros at -3 and 4 and passes through the point $(1,15)$. The multiplicity of $(-3,0)$ is 1 and the multiplicity of $(4,0)$ is 2 . Find $P(x)$.

$$
\begin{aligned}
& P(x)=a(x+3)(x-4)^{2} \\
& 15=a(1+3)(1-4)^{2} \\
& 15=a(4)(9) \\
& \frac{15}{36}=\frac{36 a}{36} \\
& a=5 / 12
\end{aligned}
$$




Ex.) Determine the equations for the following:
a) Roots at $x=3, x=4, x=-7$ and it passes through $(2,54)$.

$$
\begin{aligned}
& P(x)=a(x-3)(x-4)(x+7) \\
& 54=a(2-3)(2-4)(2+7) \\
& 54=a(-1)(-2)(9) \quad P(x)=3(x-3)(x-4)(x+7) \\
& 54=18 a
\end{aligned}
$$

b) Roots at $x=4$ multiplicity of $3, x=1$ multiplicity of 2 , and passes through $(0,32)$.

$$
\begin{aligned}
P(x) & =a(x-4)^{3}(x-1)^{2} \\
32 & =a(0-4)^{3}(0-1)^{2} \\
32 & =-64 a \\
a & =-1 / 2 \\
P(x) & =-1 / 2(x-4)^{3}(x-1)^{2}
\end{aligned}
$$


c) Roots at $x=0,-4,2$ (mull. of 2 ) and passes through ( $-2,128$ ).

$$
\begin{aligned}
P(x) & =a(x-0)(x+4)(x-2)^{2} \\
128 & =a(-2)(2)(-4)^{2} \\
128 & =-64 a \\
a & =-2 \\
P(x) & \left.=-2 x(x+4)(x-2)^{2}\right)
\end{aligned}
$$

