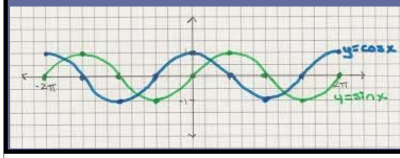
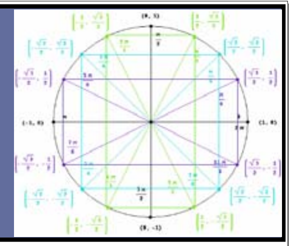


Unit 4: Trigonometry



4.10 Sum and Difference Identities

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

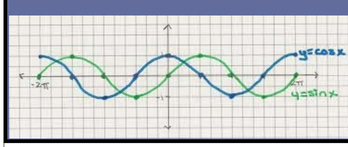
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

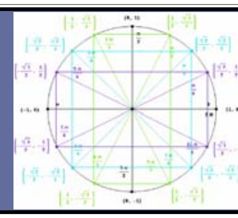
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$





Ex.) Determine the exact value of angles that are not multiples of the standard reference angles (ie. 0, 30, 45, 60, 90, etc.).

* from unit circle.

a) cos (105°) $= \cos(\underset{\alpha}{60^\circ}+\underset{\beta}{45^\circ})$

$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

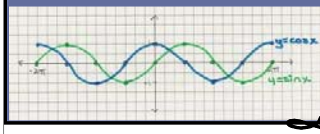
$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

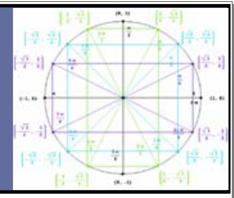
$\boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

b) sin 15° $= \sin(\underset{\alpha}{45^\circ}-\underset{\beta}{30^\circ})$

$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$





c) tan 15° * $= \tan(\overset{\alpha}{60^\circ} - \overset{\beta}{45^\circ})$

$= \dfrac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \tan 45^\circ}$

rationalize denominator * $= \boxed{\dfrac{(\sqrt{3} - 1)}{(1 + \sqrt{3})}} \cdot \dfrac{(1 - \sqrt{3})}{(1 - \sqrt{3})}$

$= \dfrac{\sqrt{3} - \sqrt{9} - 1 + \sqrt{3}}{1 - \sqrt{9}}$

$= \dfrac{2\sqrt{3} - 4}{-2} = \dfrac{\cancel{-2}(-\sqrt{3} + 2)}{\cancel{-2}}$

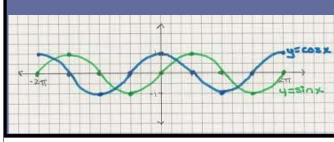
* $= \boxed{-\sqrt{3} + 2}$

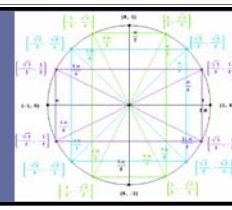
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\tan 60^\circ = \dfrac{\sin 60^\circ}{\cos 60^\circ}$

$= \dfrac{\sqrt{3}}{2} \div \dfrac{1}{2} = \sqrt{3}$

$\tan 45^\circ = \dfrac{\sin 45^\circ}{\cos 45^\circ} = 1$





Ex.) Simplify the following to a single trig function:

a) sin 35 cos 40 + cos 35 sin 40

$= \sin\alpha \cos\beta + \cos\alpha \sin\beta$

$= \sin(\alpha + \beta)$

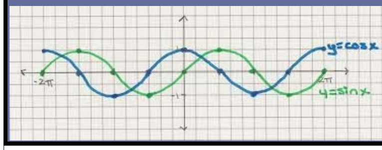
$= \boxed{\sin 75^\circ}$

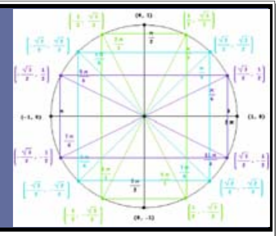
b) $\dfrac{\tan 10 + \tan 35}{1 - \tan 10 \tan 35}$

$= \tan(10^\circ + 35^\circ)$

$= \boxed{\tan 45^\circ}$ simplify

$= \boxed{1}$ evaluate





Ex.) Prove the following identity:

$$\sin(\underbrace{\pi/2}_{\alpha} + \underbrace{M}_{\beta}) = \cos M$$

$\sin\frac{\pi}{2}\cos M + \cos\frac{\pi}{2}\sin M$

$1\cdot\cos M + 0\cdot\cancel{\sin M}$

$\cos M$

LS = RS.

Pg. 306 # 1abd, 2bd, 7, 8, 10, 11c, 17, 19, 20ab.