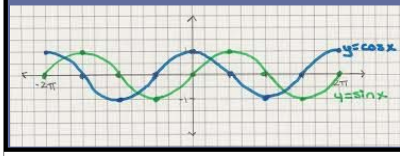
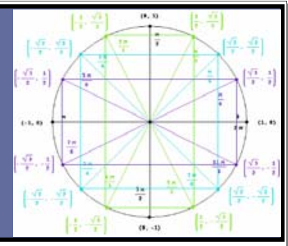
Unit 4: Trigonometry



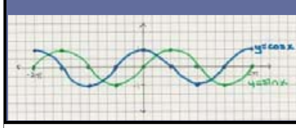


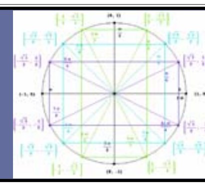
4.11 Double Angle Identities

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\left.\begin{aligned}\cos(2\alpha) &= \cos^2\alpha - \sin^2\alpha \\ \cos(2\alpha) &= 2\cos^2\alpha - 1 \\ \cos(2\alpha) &= 1 - 2\sin^2\alpha\end{aligned}\right\}$$

$$\tan(2\alpha) = \frac{2\tan\alpha}{1-\tan^2\alpha}$$





Ex.) Simplify:

a) $\cos^2\left(\frac{\pi}{3}\right) - \sin^2\left(\frac{\pi}{3}\right)$

$= \cos^2\alpha - \sin^2\alpha = \cos(2\alpha)$

$= \cos\left(2\cdot\frac{\pi}{3}\right) = \boxed{\cos\left(\frac{2\pi}{3}\right)} = \boxed{\frac{-1}{2}}$

simplified evaluated

b) $\frac{2\tan 15^\circ}{1-\tan^2 15^\circ} = \tan(2\cdot 15^\circ)$

$= \boxed{\tan 30^\circ} = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{3}}$

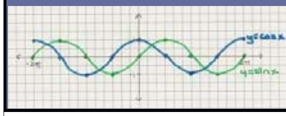
$= \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

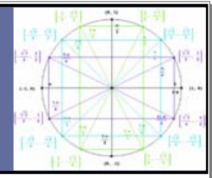
c) $2\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)$

$= \sin\left(2\cdot\frac{\pi}{4}\right)$

$= \boxed{\sin(\pi/2)}$

$= \boxed{1}$





Ex.) Prove:

a) $\dfrac{1+\cos(2x)}{\sin(2x)} = \cot x$

$$\frac{1+\cos^2 x-\sin^2 x}{2\sin x\cos x} \qquad \frac{\cos x}{\sin x}$$

$$\frac{\cos^2 x+\cos^2 x}{2\sin x\cos x}$$

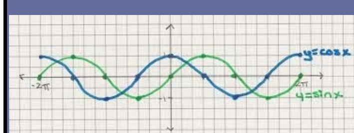
$$\frac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x\cancel{\cos x}}$$

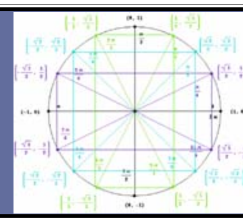
$$\frac{\cos x}{\sin x}$$

b) $\dfrac{2\sin x}{\sin(2x)} = \sec x$

$$\frac{\cancel{2\sin x}}{\cancel{2\sin x}\cos x} \qquad \frac{1}{\cos x}$$

$$\frac{1}{\cos x}$$





c) $\dfrac{1}{\sin(2x)} + \cot(2x) = \cot x$

$$\frac{1}{2\sin x\cos x} + \frac{\cos(2x)}{\sin(2x)} \qquad \frac{\cos x}{\sin x}$$

$$\frac{1+\cos^2 x-\sin^2 x}{2\sin x\cos x}$$

$$\frac{\cos^2 x+\cos^2 x}{2\sin x\cos x}$$

$$\frac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x\cancel{\cos x}}$$

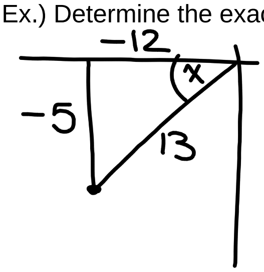
$$\frac{\cos x}{\sin x}$$

Ex.) Determine the exact value of cos (2x) when tan x = 5/12 and cos x < 0.

neg.

S | A

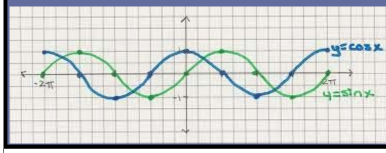
T | C

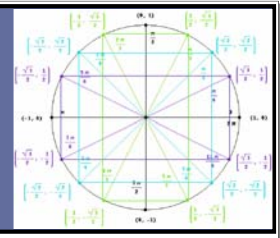


$$\cos(2x)$$

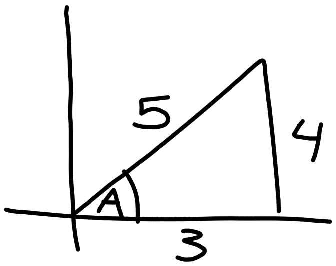
$$= \cos^2 x - \sin^2 x$$

$$= \left(\frac{-12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$$





Ex.) Determine the exact value of tan 2A when cos A = 3/5 in quadrant I.



$$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2} = \frac{\frac{8}{3}}{\frac{9}{9}-\frac{16}{9}} = \frac{8}{3}\cdot\frac{9}{-7} = \boxed{-\frac{24}{7}}$$

Pg. 306 # 1c, 2c, 4, 5, 11ab, 20cd.