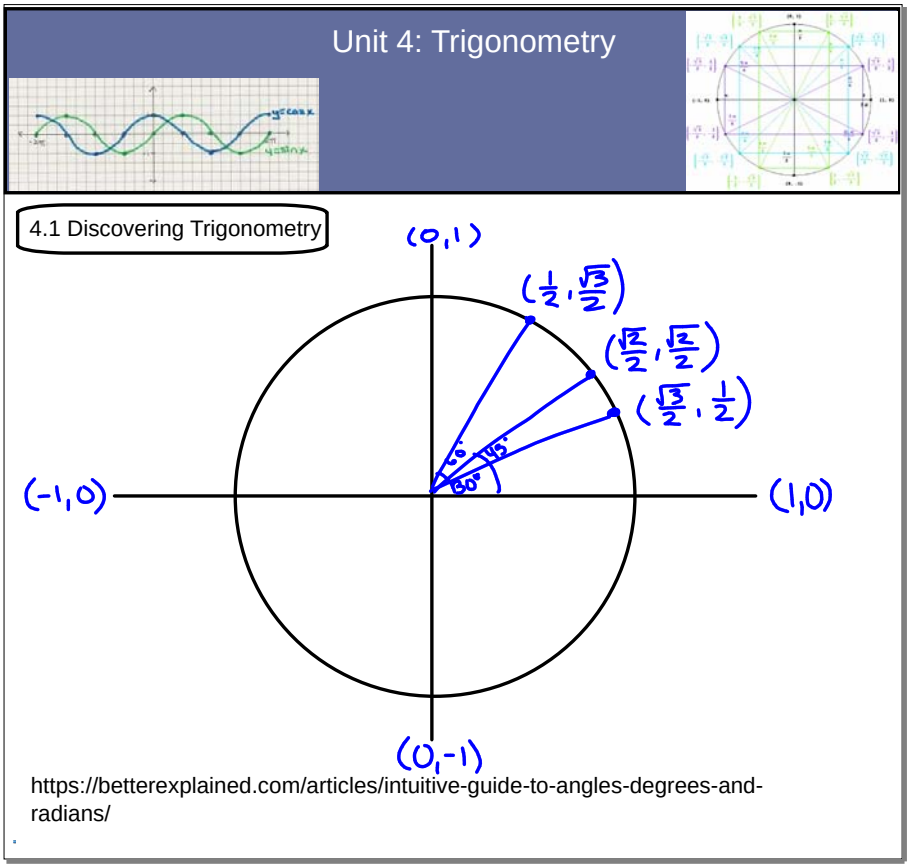
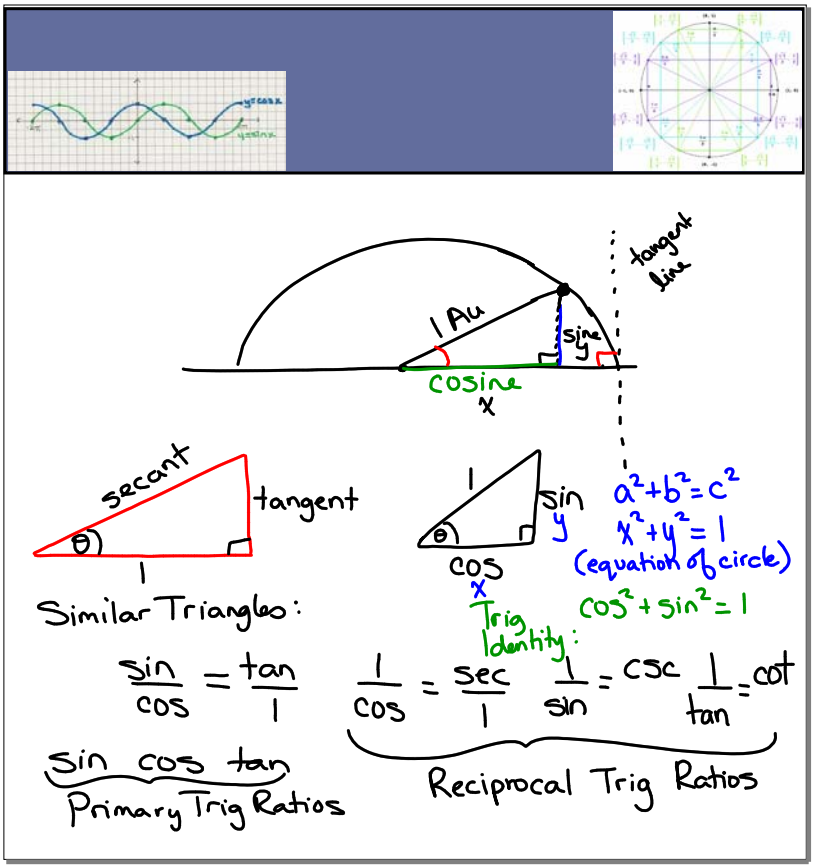
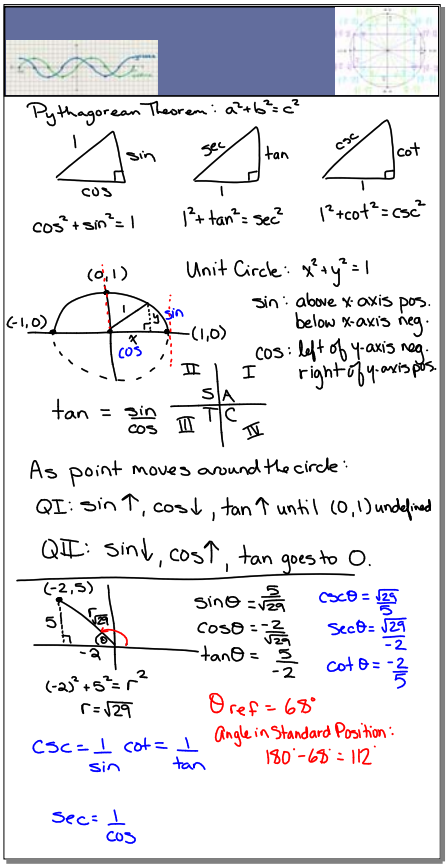
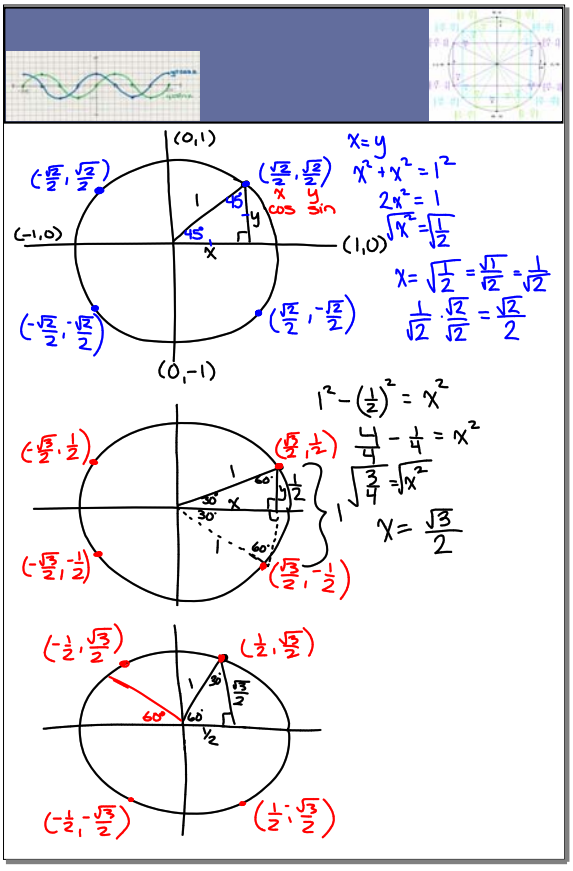
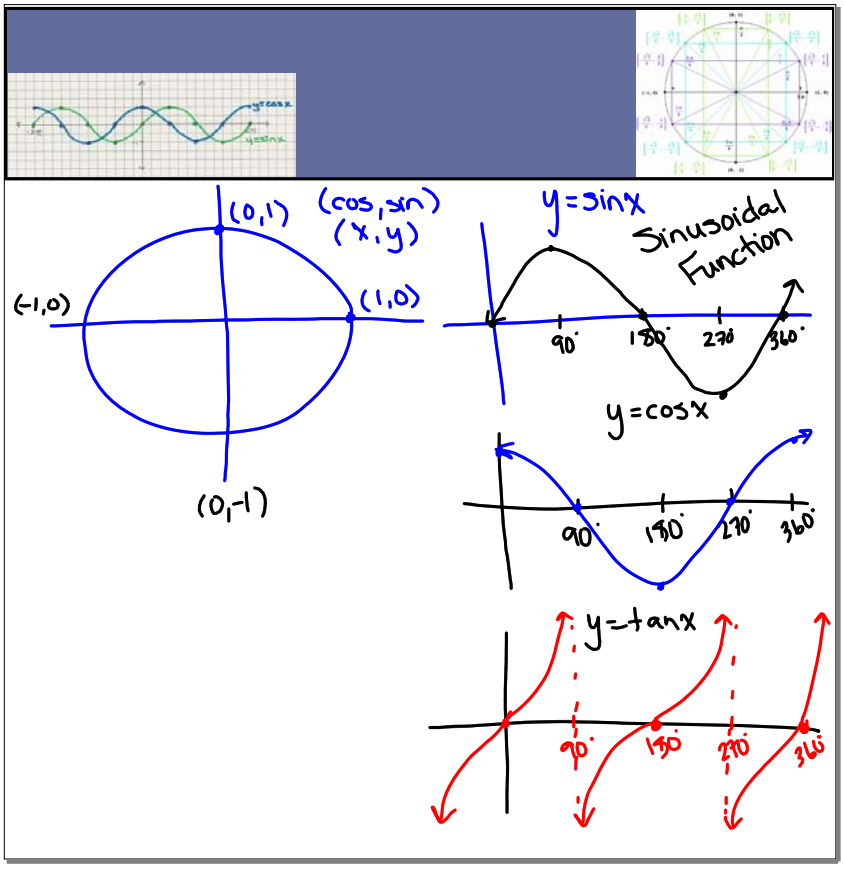
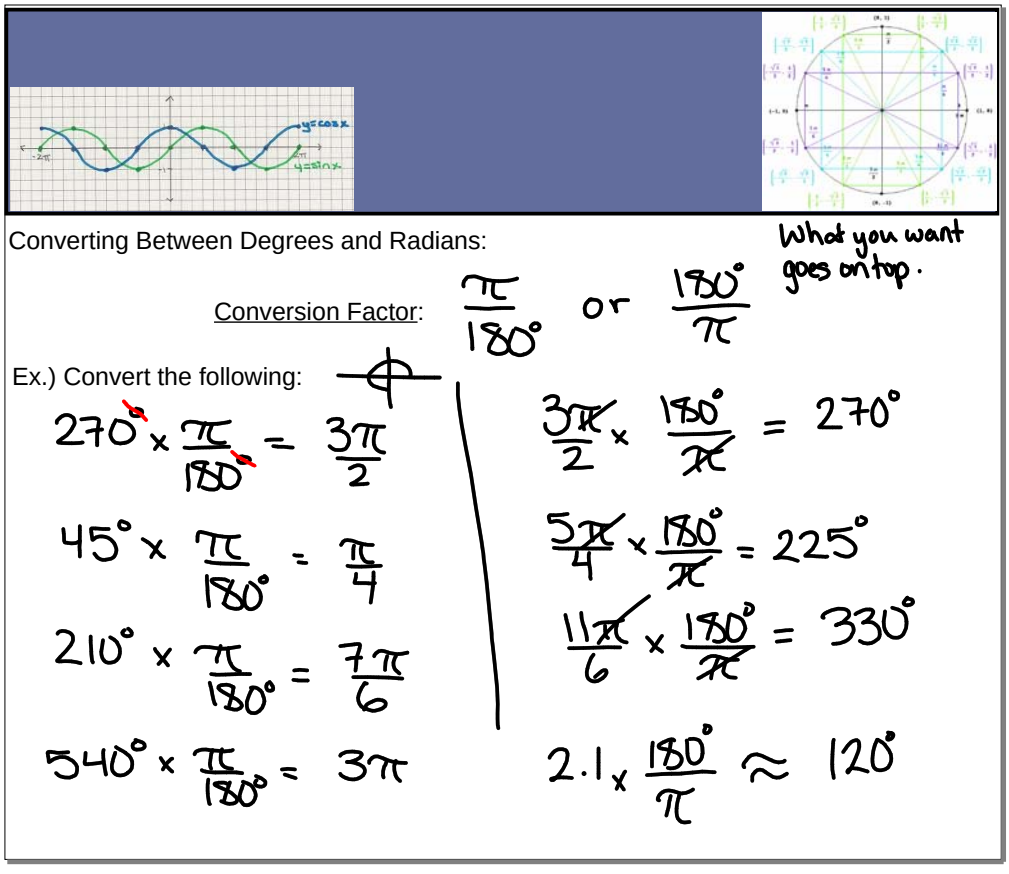

Unit 4: Trigonometry
4.1 Discovering Trigonometry
(0,1)
$(\frac{1}{2},\frac{\sqrt{3}}{2})$
$(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$
$(\frac{\sqrt{3}}{2},\frac{1}{2})$
30°
45°
60°
(-1,0)
(1,0)
(0,-1)
https://betterexplained.com/articles/intuitive-guide-to-angles-degrees-and-radians/


tangent line
1 Au
sine
y
cosine
x
secant
tangent
θ
1
Similar Triangles:
$\frac{\sin}{\cos} = \frac{\tan}{1}$
Sin Cos tan
Primary Trig Ratios
1
sin
y
θ
cos
x
$a^2+b^2=c^2$
$x^2+y^2=1$
(equation of a circle)
Trig Identity:
$\cos^2+\sin^2=1$
$\frac{1}{\cos}=\frac{\sec}{1}$ $\frac{1}{\sin}=\csc$ $\frac{1}{\tan}=\cot$
Reciprocal Trig Ratios


Pythagorean Theorem: $a^2+b^2=c^2$
sin
cos
tan
sec
cot
csc
$\cos^2+\sin^2=1$
$1^2+\tan^2=\sec^2$
$1^2+\cot^2=\csc^2$
Unit Circle: $x^2+y^2=1$
(0,1)
(-1,0)
(1,0)
sin: above x-axis pos. below x-axis neg.
cos: left of y-axis neg. right of y-axis pos.
$\tan=\frac{\sin}{\cos}$
II I III IV
S A T C
As point moves around the circle:
QI: sin↑, cos↓, tan↑ until (0,1) undefined
QII: sin↓, cos↑, tan goes to 0.
(-2,5)
$\sin\theta=\frac{5}{\sqrt{29}}$
$\cos\theta=\frac{-2}{\sqrt{29}}$
$\tan\theta=\frac{5}{-2}$
$\csc\theta=\frac{\sqrt{29}}{5}$
$\sec\theta=\frac{\sqrt{29}}{-2}$
$\cot\theta=\frac{-2}{5}$
$(-2)^2+5^2=r^2$
$r=\sqrt{29}$
$\theta_{ref}=68°$
Angle in Standard Position: $180°-68°=112°$
$\csc=\frac{1}{\sin}$ $\cot=\frac{1}{\tan}$
$\sec=\frac{1}{\cos}$


(0,1)
(-1,0)
(1,0)
(0,-1)
$(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$
$(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$
$(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2})$
$(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2})$
x=y
$x^2+x^2=1^2$
$2x^2=1$
$\sqrt{x^2}=\sqrt{\frac{1}{2}}$
$x=\sqrt{\frac{1}{2}}=\frac{\sqrt{1}}{\sqrt{2}}=\frac{1}{\sqrt{2}}$
$\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$
$(-\frac{\sqrt{3}}{2},\frac{1}{2})$
$(\frac{\sqrt{3}}{2},\frac{1}{2})$
$(-\frac{\sqrt{3}}{2},-\frac{1}{2})$
$(\frac{\sqrt{3}}{2},-\frac{1}{2})$
$1^2-(\frac{1}{2})^2=x^2$
$1-\frac{1}{4}=x^2$
$\sqrt{\frac{3}{4}}=\sqrt{x^2}$
$x=\frac{\sqrt{3}}{2}$
$(-\frac{1}{2},\frac{\sqrt{3}}{2})$
$(\frac{1}{2},\frac{\sqrt{3}}{2})$
$(-\frac{1}{2},-\frac{\sqrt{3}}{2})$
$(\frac{1}{2},-\frac{\sqrt{3}}{2})$





Converting Between Degrees and Radians:

What you want goes on top.

Conversion Factor: $\frac{\pi}{180^\circ}$ or $\frac{180^\circ}{\pi}$

Ex.) Convert the following:

$270^\circ \times \frac{\pi}{180^\circ} = \frac{3\pi}{2}$

$45^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{4}$

$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$

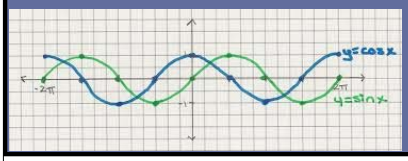
$540^\circ \times \frac{\pi}{180^\circ} = 3\pi$

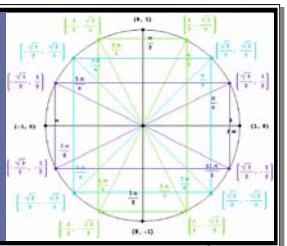
$\frac{3\pi}{2} \times \frac{180^\circ}{\pi} = 270^\circ$

$\frac{5\pi}{4} \times \frac{180^\circ}{\pi} = 225^\circ$

$\frac{11\pi}{6} \times \frac{180^\circ}{\pi} = 330^\circ$

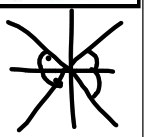
$2.1 \times \frac{180^\circ}{\pi} \approx 120^\circ$





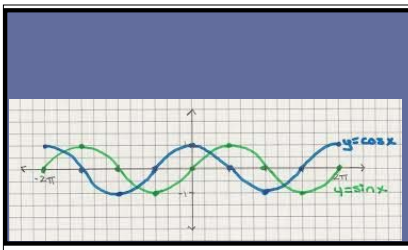
Terminology:

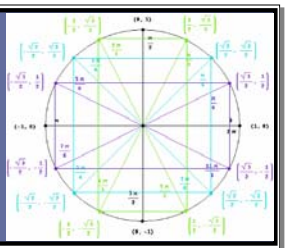
Reference Angles: how far the terminal arm is from the nearest x-axis (<90°, acute, pos.)



Principle Angles: smallest positive angle
$0 \le \theta \le 360°$
$0 \le \theta \le 2\pi$

Co-terminal Angles:
principle angle $+ 360°n$
" $+ 2\pi n$, $n \in I$

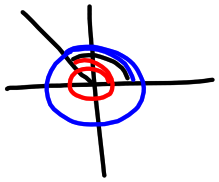




Ex.) Determine one positive and one negative co-terminal angle:

a) $\theta = 120°$

$360° + 120° = 480°$
$120° - 360° = -240°$



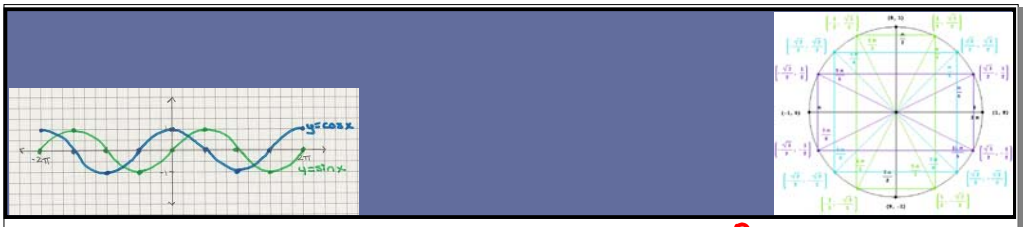
b) $\theta = \frac{5\pi}{4}$

$\frac{5\pi}{4} + \frac{8\pi}{4} = \frac{13\pi}{4}$

$\frac{5\pi}{4} - \frac{8\pi}{4} = -\frac{3\pi}{4}$

$2\pi \cdot \frac{4}{4}$

$\frac{5\pi}{4} \cdot \frac{180}{\pi} = 225°$



Ex.) Given the following angles, determine the reference angle: θ_{ref}, ref. $\measuredangle$

a) 315° $360^\circ - 315^\circ = \boxed{45^\circ}$

b) 470° principle $\measuredangle$: $470^\circ - 360^\circ = 110^\circ$ $180^\circ - 110^\circ = 70^\circ$

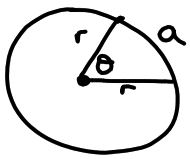
c) $-\frac{5\pi}{6}$ (-150°) $\frac{6\pi}{6} - \frac{5\pi}{6} = \frac{\pi}{6}$

d) $\frac{5\pi}{3}$ (300°) $2\pi - \frac{5\pi}{3} = \frac{6\pi}{3} - \frac{5\pi}{3}$

$= \boxed{\frac{\pi}{3}}$

Arc Length:

$$\theta = \frac{a}{r}$$



* θ must be in radians.

Ex.) A circle with radius 7 cm, has a central angle of 160° that subtends an arc. What is the length of the arc?

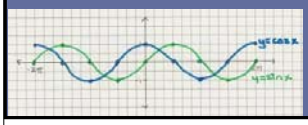
$160^\circ \times \frac{\pi}{180^\circ} = \frac{8\pi}{9}$

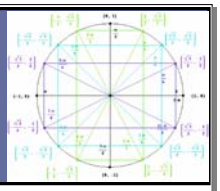
$\theta = \frac{a}{r}$

$a = \theta \cdot r$

$a = \frac{8\pi}{9} \cdot 7\text{cm}$

$\boxed{a = 19.5\text{cm}}$





Ex.) An angle of 1.8 subtends an arc 4.5 mm. What is the radius of the circle?

$\theta = \frac{a}{r}$

$r = \frac{a}{\theta} = \frac{4.5mm}{1.8} = \boxed{2.5mm}$

Ex.) A circle with an arc length of 25 m has a radius of 11 m. What is the central angle, to the nearest degree?

$\theta = \frac{a}{r} = \frac{25m}{11m} \approx 2.27...$

$2.27... \times \frac{180^\circ}{\pi}$

$\boxed{\theta = 130^\circ}$

Pg. 175 # 2, 4, 6, 7, 8, 9, 12ab, 13, 14a, 16.