

Rational Expression
 Numerator → Polynomial
 Denominator → Polynomial
 7x+14

4.1 NPV's and Simplification

Rational Expressions Review

What are Rational Numbers & Rational Expressions?

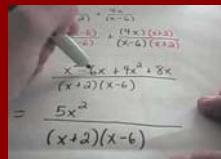
When the numerator and denominator of a fraction are integers, the fraction is a rational number.
 Ex. $\frac{1}{2}$ $-\frac{3}{4}$


When the numerator and denominator of a fraction are polynomials, the fraction is a rational expression.
 Ex.

$\frac{2x + 3}{5x + 4}$
 $\frac{x^2 - 6}{3xy}$
 $\frac{x^2 - 9}{x^2 + 8x + 15}$

A rational expression cannot contain roots of variables, or variables as exponents. These expressions are **not** rational expressions.

$\frac{\overset{\circ}{x} + 3}{4x - 2}$
 $\frac{x^2 + 4}{2\sqrt{\overset{\circ}{x}}}$





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Non-Permissible Values

Rational expressions are not defined for values of the variable that make the denominator 0. These values are called **non-permissible values**.

The expression $\frac{x + 5}{x - 3}$ is not defined for $x = 3$. So, $x = 3$ is a non-permissible value of $\frac{x + 5}{x - 3}$.

$x \neq 3$

What are the non-permissible values for:

$\frac{x^2 - 4}{(x + 7)(x - 3)}$

$x \neq -7, 3$





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Ex.) Determine the non-permissible values for each rational expression.

a) $\frac{x^2 + 2}{x^2 - x - 6}$ $-3 \begin{matrix} \diamond \\ -1 \end{matrix} 2$

$(x-3)(x+2)$

$x \neq -2, 3$

b) $\frac{x}{x^2 + 1}$

$x \in \mathbb{R}$

a) $\frac{5x}{x^2 - 9}$

$(x-3)(x+3)$

$x \neq \pm 3$

b) $\frac{3x + 2}{x^2 - 8x + 16}$ $-4 \begin{matrix} \diamond \\ -8 \end{matrix} -4$

$(x-4)^2$

$x \neq 4$





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How do you create equivalent rational expressions?

To write an equivalent form of a rational number, multiply or divide the numerator and denominator by the same number. For example, $= 1$

$$\frac{12}{18} = \frac{12 \times 2}{18 \times 2} = \frac{24}{36}$$

$$\frac{12}{18} = \frac{12 \div 3}{18 \div 3} = \frac{4}{6} \stackrel{\div 2}{=} \frac{2}{3}$$

$\frac{12}{18}$, $\frac{24}{36}$, and $\frac{4}{6}$ are equivalent rational numbers.

The same strategy is used to write an equivalent form of a rational expression: multiply or divide the numerator and denominator by the same monomial or binomial.

For example,


$$\frac{20xy}{8x^2} = \frac{20xy \cdot x}{8x^2 \cdot x} = \frac{20x^2y}{8x^3}$$


$$\frac{20xy}{8x^2} = \frac{20xy \div 2x}{8x^2 \div 2x} = \frac{10y}{4x}$$

Each expression has $x = 0$ as a non-permissible value.

So, $\frac{20xy}{8x^2}$, $\frac{20x^2y}{8x^3}$, and $\frac{10y}{4x}$ are equivalent rational expressions for $x \neq 0$.

When stating that rational expressions are equivalent, the values of the variable for which the expressions are undefined must be identified.





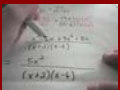
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1. Use multiplication and division to write two equivalent forms of the rational expression $\frac{3(x+2)}{(x+2)(x-4)}$

$\frac{6(x+2)}{2(x+2)(x-4)}$ $\frac{3}{(x-4)}$

2. Use multiplication and division to write two equivalent forms of the rational expression $\frac{(x+5)(x-1)}{2(x-1)}$

$\frac{(x+5)(x-1)(x+3)}{2(x-1)(x+3)}$ $\frac{x+5}{2}$





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Simplifying Rational Expressions

Steps

- Factor like crazy!
- State NPV's.
- Factors that are common to the numerator and denominator cancel.

Ex.) a) $\frac{15x^2z^2}{20xz^2}$ *State NPV's from original factored form*
 $\frac{3x^2z^2}{4xz^2}$ $x, y, z \neq 0$

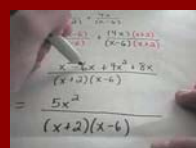
b) $\frac{3x^2 - 8x}{2x} = \frac{x(3x-8)}{2x}$
 $x \neq 0$
 $\frac{3x-8}{2}$

c) $\frac{x^2 + 6x + 8}{x^2 - 4}$ $x \neq \pm 2$
 $\frac{(x+4)(x+2)}{(x-2)(x-2)}$
 $\frac{(x+4)}{(x-2)}$

d) $\frac{2x^2 - 11x + 12}{2x^2 - 8x + 12}$ **memorize -1 trick**
 $\frac{-1(-16 + x^2)}{2x^2 - 8x + 12}$
 $\frac{-1(x^2 - 16)}{2x(x-4) - 3(x-4)}$
 $\frac{-1(x+4)(x-4)}{(x-4)(2x-3)}$
 $\frac{-1(x+4)}{(2x-3)}$
 $x \neq 4, \frac{3}{2}$

NO

Don't be a happy canceller.



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Identify & Correct Any Errors

12. Here is a student's solution for simplifying a rational expression. Identify the error in the solution. Write a correct solution.

$$\frac{3x - 12}{x^2 + x - 20} = \frac{3(x-4)}{(x+5)(x-4)}$$

$$= \frac{3}{x+5}, x \neq -5, 4.$$

-20
 5 \diamond -4
 1

$$\frac{3(x-4)}{(x+5)(x-4)}$$

13. A student claims that the expressions $\frac{12x^2}{15x}$ and $\frac{12x(x-3)}{15x(x-3)}$ are equivalent. Is the student correct? Explain.

Yes, they multiplied top and bottom by the same expression (=1).



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