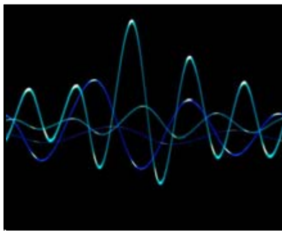


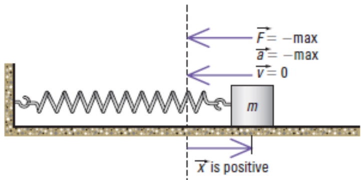
Unit 4: Oscillatory Motion and Mechanical Waves

4.2 Position, Velocity, Acceleration and Time of SHM

Today we want to use the two systems we studied last day (a mass-spring system and a pendulum) to mathematically approximate the position, velocity, acceleration and period of these movements.

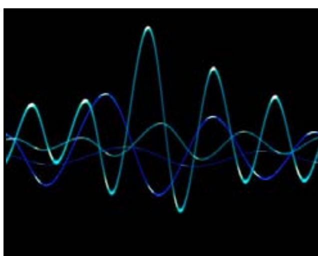
1. Mass-Spring System

Finding Acceleration



We know that the max. acceleration occurs when the mass is at its amplitude (max. displacement).

Note: these same principles would apply to a vertical mass-spring system



When the mass is at its amplitude, we can make the restoring force in the spring equal to Newton's Second Law:

$$\vec{F}_s = \vec{F}$$

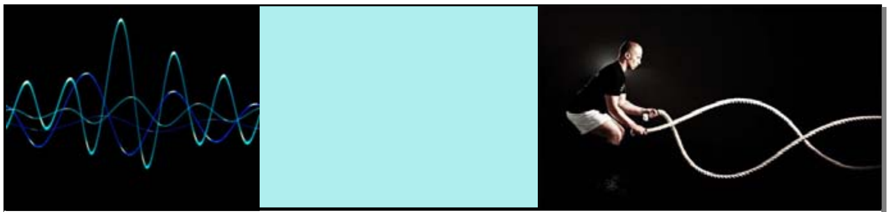
$$-k\vec{x} = m\vec{a}$$

$$\boxed{\vec{a} = \frac{-k\vec{x}}{m}}$$

where:

$\vec{a}$ = maximum acceleration of the block (m/s²)
$\vec{x}$ = maximum displacement of the block (m)
k = spring constant (N/m)
m = mass of block (kg)

Note: only applies to max. acceleration because it is uniform. Acceleration at other points is not uniform and is beyond the scope of Physics 20.



Ex.) In a mass-spring system, a 1.55 kg mass oscillates horizontally when attached to a spring of k = 15 N/m. If the amplitude of the oscillations is 0.75 m, what is the:

greatest $\vec{x}$

a) Magnitude of the max. acceleration of the mass?

b) Direction of acceleration?

c) Max. restoring force acting on the mass?

a) $\vec{a} = \frac{-k\vec{x}}{m} = \frac{-15(0.75)}{1.55} = \boxed{-7.3\ m/s^2}$

b) [toward equilibrium]

c) $F_s = F_r = kx = 15(0.75) = \boxed{11N}$

Finding Velocity

- max. velocity occurs when the mass is at equilibrium and the force is zero.

When the mass is pulled back, all the energy is PE:

$PE_{spring} = 1/2kx^2$

When the mass is at equilibrium, all the PE has turned into KE:

$KE = 1/2mv^2$

Solve: $E_{pspring} = E_k$

$\frac{\cancel{\frac{1}{2}}kx^2}{m} = \frac{\cancel{\frac{1}{2}}\cancel{m}v^2}{\cancel{m}}$

$\frac{kx^2}{m} = v^2$

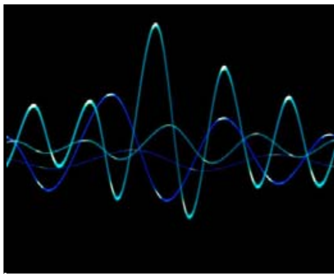
$\sqrt{\frac{kx^2}{m}} = v$

$\boxed{x\sqrt{\frac{k}{m}} = v}$

$\vec{v}$ = maximum velocity of mass (m/s)
$\vec{x}$ = maximum displacement (amplitude) (m)
k = spring constant (N/m)
m = mass (kg)

$V = A\sqrt{\frac{k}{m}}$

↑ amplitude




Ex.) Continued...In the previous system, what will; the max. speed of the mass be?

$$V = A\sqrt{\frac{k}{m}}$$

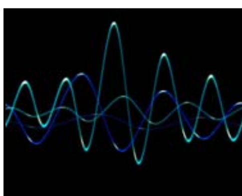
m = 1.55 kg

k = 15 N/m

x = 0.75 m

$$V = 0.75\sqrt{\frac{15}{1.55}}$$

$$\boxed{V = 2.3\,m/s}$$




Finding Period

Objects undergoing Uniform Circular Motion can replicate objects in SImple Harmonic Motion.

So recall from Unit 3 that:

$$\boxed{\vec{v} = \frac{2\pi r}{T}}$$

and that:

$$\boxed{\vec{v} = \vec{x}\sqrt{\frac{k}{m}}}$$

Therefore:

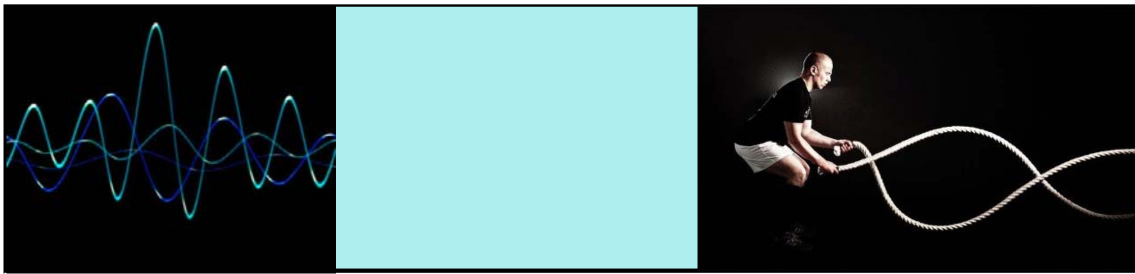
$$v = v$$

$$\frac{2\pi r}{T} = x\sqrt{\frac{k}{m}}$$

If we let x = r

$$\frac{2\pi \cancel{x}}{T} = \cancel{x}\sqrt{\frac{k}{m}} \rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$



Formula for Period of a Mass-Spring System

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Where:

T = period (s)

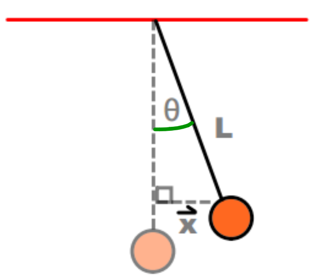
m = mass of oscillator (kg)

k = spring constant (N/m)

Note: the period of a mass-spring system does not depend on displacement (how far it is pulled back)

2. An Ideal Pendulum

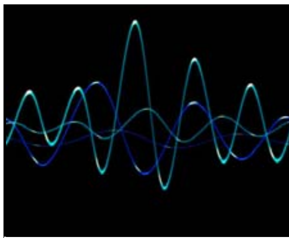
Since there is no spring constant to apply to a pendulum we cannot use the previous formula. However, we do pull a pendulum back through an angle, θ, so we will use this idea to create an appropriate equation:



where:
L = length of pendulum
$\vec{x}$ = displacement

We could write that:

$$\sin\theta = \frac{\vec{x}}{L}$$



Formula for Period of a Pendulum

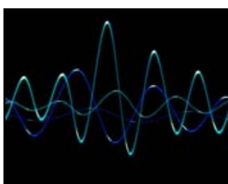
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Where:

- T - period (s)
- L = length of pendulum from end of string to centre of mass (m)
- g - acceleration due to gravity (m/s^2)

Note: period of a pendulum does not depend on mass or amplitude. Only length and gravitational field strength.

You could perform a simple experiment with pendulums (if you know length and period) to calculate gravitational field strengths on any planet without any other equipment.



Ex.) An astronaut lands on a planet and constructs a simple pendulum with length 5.5 m to determine the acceleration due to gravity. She measures the period of the pendulum to be 6.7 s. What is the gravitational field strength of the planet?

T

$$\frac{T}{2\pi} = \frac{\cancel{2\pi}\sqrt{\frac{L}{g}}}{\cancel{2\pi}}$$

$$\left(\frac{T}{2\pi}\right)^2 = \sqrt{\frac{L}{g}}^{\,2}$$

$$\frac{T^2}{4\pi^2} = \frac{L}{g}$$

$$\frac{g}{4\pi^2} = \frac{L}{T^2}$$

$$g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(5.5)}{6.7^2} = \boxed{4.8\ N/kg}$$

Simple Harmonic Motion in Real Life: <u>Resonance</u>

resonance - vibrating or oscillating system

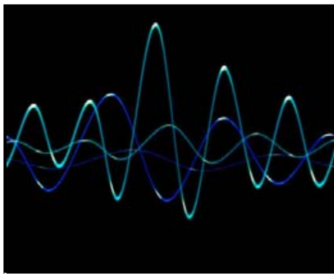
forced frequency - a force is added to an oscillator to keep it resonating

Eg. Swinging on a swing, analog clock (electrically charged quartz crystal provides force to keep gears in time)

Resonance Disasters:

- 1940 Tacoma Narrow's Bridge destroyed by the force of a ~~small~~ gale force wind.
- 1850 Angers, France bridge being marched upon by soldiers in unison.
- Mexican mid rise building during 1985 Earthquake.

https://www.youtube.com/watch?v=j-zczJXSxnw



Questions: Pg. 390-391 # 11-26.