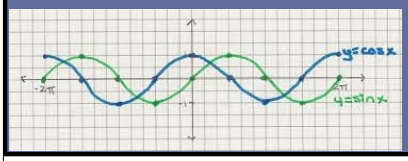
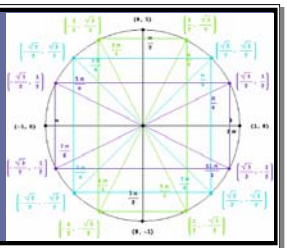
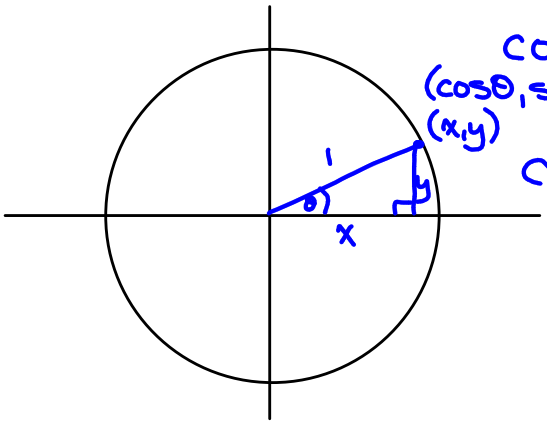
Unit 4: Trigonometry





4.2 The Unit Circle

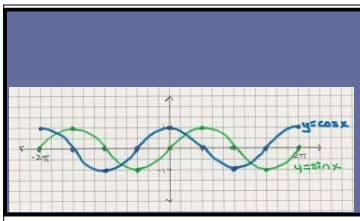
$x^2 + y^2 = 1$

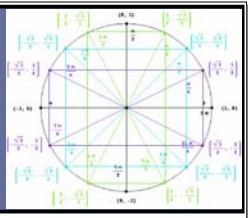


$\cos\theta = \frac{x}{1}$ $\quad (\cos\theta, \sin\theta)$ $\quad \sin\theta = \frac{y}{1}$

(x, y)

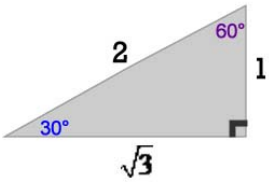
$\cos\theta = x$ $\quad \sin\theta = y$

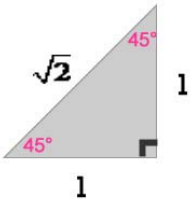




Special Triangles:

Evaluating Functions of a 30°, 45°, or 60° Angle

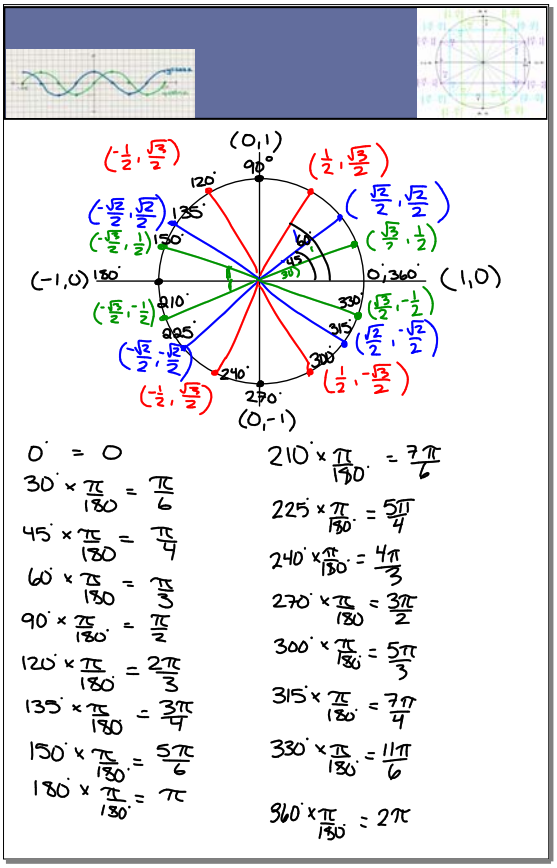




$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{\sqrt{2}}{2}$

$\sin 30° = \frac{1}{2}$	$\sin 60° = \frac{\sqrt{3}}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$
$\cos 30° = \frac{\sqrt{3}}{2}$	$\cos 60° = \frac{1}{2}$	$\cos 45° = \frac{\sqrt{2}}{2}$
$\tan 30° = \frac{\sqrt{3}}{3}$	$\tan 60° = \sqrt{3}$	$\tan 45° = 1$



$0^\circ = 0$

$30^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{6}$

$45^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{4}$

$60^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{3}$

$90^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{2}$

$120^\circ \times \frac{\pi}{180^\circ} = \frac{2\pi}{3}$

$135^\circ \times \frac{\pi}{180^\circ} = \frac{3\pi}{4}$

$150^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{6}$

$180^\circ \times \frac{\pi}{180^\circ} = \pi$

$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$

$225^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{4}$

$240^\circ \times \frac{\pi}{180^\circ} = \frac{4\pi}{3}$

$270^\circ \times \frac{\pi}{180^\circ} = \frac{3\pi}{2}$

$300^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{3}$

$315^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{4}$

$330^\circ \times \frac{\pi}{180^\circ} = \frac{11\pi}{6}$

$360^\circ \times \frac{\pi}{180^\circ} = 2\pi$

Using the Unit Circle:

a) $\tan 45^\circ = \frac{\sin 45^\circ}{\cos 45^\circ} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = \boxed{1}$

b) $\tan 240^\circ = \frac{\sin 240^\circ}{\cos 240^\circ} = \frac{-\sqrt{3}/2}{-1/2} = -\frac{\sqrt{3}}{2} \div \frac{-1}{2} = +\frac{\sqrt{3}}{2} \cdot \frac{2}{+1} = \boxed{\sqrt{3}}$

c) $\tan \pi = \frac{\sin \pi}{\cos \pi} = \frac{0}{-1} = \boxed{0}$

d) $\tan(\pi/2) = \frac{\sin(\pi/2)}{\cos(\pi/2)} = \frac{1}{0}$ undefined

Ex.) ~~Do~~ the following points exist on the Unit Circle?

a) $P(\frac{1}{2}, -\frac{\sqrt{3}}{2})$

$x^2 + y^2 = 1$

$(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2 \mid 1$

$\frac{1}{4} + \frac{3}{4}$

$\frac{4}{4}$

$1 \mid 1$

LS = RS

∴ the point exists

b) $P(\frac{1}{2}, \frac{3}{4})$

$x^2 + y^2 = 1$

$(\frac{1}{2})^2 + (\frac{3}{4})^2 \mid 1$

$\frac{1}{4} + \frac{9}{16}$

$\frac{4}{16} + \frac{9}{16}$

$\frac{13}{16} \neq 1$

LS ≠ RS

∴ P is not on the unit circle

Ex.) The point P(5/6 , y) is on the unit circle. What is/are the value(s) of 'y'?

$x^2 + y^2 = 1$

$(\frac{5}{6})^2 + y^2 = 1$

$\frac{25}{36} + y^2 = 1$

$y^2 = \frac{36}{36} - \frac{25}{36}$

$\sqrt{y^2} = \sqrt{\frac{11}{36}}$

$y = \pm \frac{\sqrt{11}}{6}$

QI: $y = \sqrt{11}/6$

QIV: $y = -\sqrt{11}/6$

Pg. 186 # 2, 3, 4, 5, 10.