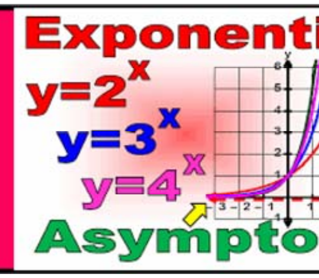
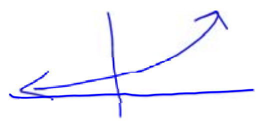
Unit 2

Exponents and Logarithms



4.3 Exponential Problem Solving



$y = a \cdot b^x$

Review:

	$y=(4)^x$	$y=2\left(\frac{1}{2}\right)^x$	$y=-\frac{1}{2}(3)^x$
* x-intercept	none	none	none
y-intercept	(0,1)	(0,2)	(0,-½)
* Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$	$x \in \mathbb{R}$
Range	$y>0$	$y>0$	$y<0$

What patterns do you see?

- no x-int
- domain always $x \in \mathbb{R}$
- y-int 'a'
- check range on calculator

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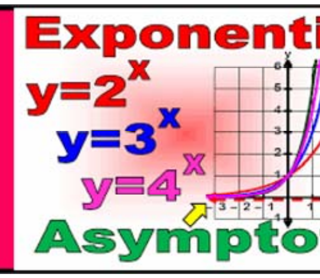

How do you know, from a table of values that a functions is exponential?

	x	$g(x)$
2^1	1	2
2^2	2	4
2^3	3	8
2^4	4	16
2^5	5	32

y doubles each time

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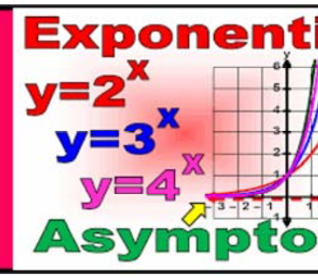

Types of word problems:

- Finance (compound interest)
- Earthquake Richter Scales
- Decibel scale
- pH scales
- growth and decay (populations, radioactive materials, etc.)

$\div 100 = 0.065$

$* \; 1.065 *$ add one

Ex.) The population of alligators in a swamp is increasing at a rate of 6.5 % per year.

a) Write an exponential function that relates the population P(t) and the time, in years, from now. The initial alligator population is 500.

$$P(t) = 500(1.065)^t$$

$$y = a \cdot b^x$$

initial amount

b) What will be the population in 9 years?

t

$$P(9) = 500(1.065)^9 = \boxed{881 \text{ alligators}}$$

c) In how many years will the population double?

P(t)

$$\frac{1000}{500} = \frac{\cancel{500}(1.065)^x}{\cancel{500}}$$

$$2 = 1.065^x$$

y_1 y_2

Intersection (11.0067, 2)

$$\boxed{t \approx 11 \text{ years}}$$

Feb 6-8:22 AM

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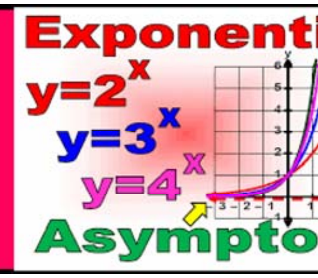

100% − 16% per annum = per year

Ex.) The value of a car is decreasing at an average rate of 16%/a. The initial value of the car was $ 24 500. Write an exponential function to model the value of the car, *v(t)*, after *t* years. When will the car be worth half its original value, to the nearest year?

$$v(t) = 24500(0.84)^t$$

Value remaining (84%)

$$\frac{12250}{24500} = \frac{\cancel{24500}(0.84)^t}{\cancel{24500}}$$

$$0.5 = 0.84^t$$

y_1 y_2

$(3.97, 0.5)$

$$\boxed{t = 4 \text{ years}}$$

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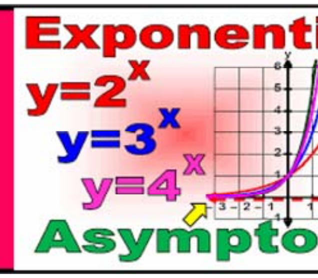

The intensity of an earthquake can be calculated using the formula

$$\frac{I}{I_o} = \frac{\cancel{I_o}(10)^M}{\cancel{I_o}}$$

where I represents the intensity of an earthquake, M is the magnitude of the earthquake on the Richter scale, and I_o represents the intensity of an earthquake with a magnitude of 0.

Ex.) Explain why an earthquake with a magnitude of 8.5 is about 40 times as intense as an earthquake with a magnitude of 6.9.

divide means comparison $\quad \frac{I}{I_0} = 10^M$

$$\frac{10^{8.5}}{10^{6.9}} \leftarrow \text{"compared to"}$$

Ex.) Early in the century, an earthquake in San Francisco registered 8.3 on the Richter scale. In the same year, another earthquake was recorded in South America that was four times stronger. What was the magnitude of the earthquake in South American?

$$\cancel{10^{8.3}} \cdot \frac{10^R}{\cancel{10^{8.3}}} = 4 \cdot 10^{8.3}$$

$$10^R = 798104926$$

$y_1 \qquad y_2$

$$\boxed{R = 8.9}$$

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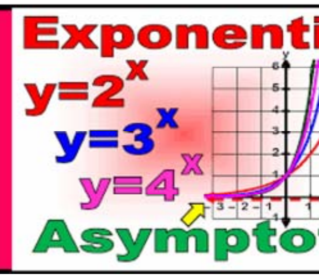

The half-life of carbon-14 is approximately 5 730 years. As a sample of carbon-14 decays, the [illegible]f carbon-14 remaining, P, at any time during the process can be modelled by the function

$$P = 100\left(\frac{1}{2}\right)^{\left(\frac{t}{5\,730}\right)}$$

wher[illegible]s the approximate age of the sample, [illegible]

Ex.) To the nearest year, determine the approximate age of carbon-14 when 33% of the original sample remains.

$$P = 100\left(\tfrac{1}{2}\right)^{(t/5730)}$$

$$\frac{33}{100} = \frac{\cancel{100}\left(\tfrac{1}{2}\right)^{(t/5730)}}{\cancel{100}}$$

$$0.33 = \left(\tfrac{1}{2}\right)^{(x/5730)}$$

Pg. 361 # 2, 4, 5, 7-11.

$$\boxed{t = 9165 \text{ years}}$$