

4.3 Trigonometric Ratios

Recall: Good ol' SOH CAH TOA. This gave us the three primary trigonometric ratios. However, they all have companions:

COsecant secant Cotangent

Companions to the primary trig ratios are reciprocal trig ratios:

$$\text{COsecant: } \text{csc } \theta = \frac{1}{\sin \theta} \quad \text{csc } \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\text{Secant: } \text{sec } \theta = \frac{1}{\cos \theta} \quad \text{sec } \theta = \frac{\text{hyp}}{\text{adj}}$$

$$\text{Cotangent: } \text{cot } \theta = \frac{1}{\tan \theta} \quad \text{cot } \theta = \frac{\text{adj}}{\text{opp}}$$

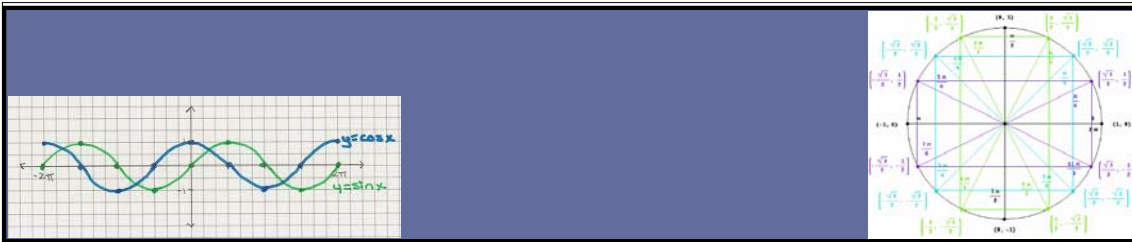


Here's what we get on the formula sheet:

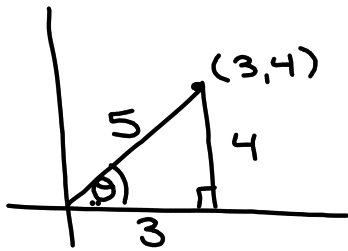
$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$



Ex.) Determine the 6 trig ratios for the point P (3 , 4) on the terminal arm of an angle in standard position.



$$\sin \theta = \frac{4}{5}$$

$$\csc \theta = \frac{5}{4}$$

$$\cos \theta = \frac{3}{5}$$

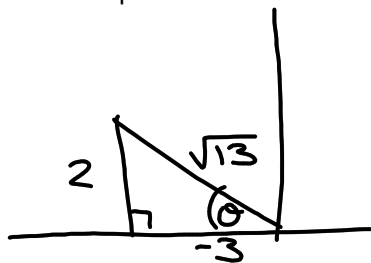
$$\sec \theta = \frac{5}{3}$$

$$\tan \theta = \frac{4}{3}$$

$$\cot \theta = \frac{3}{4}$$



Ex.) Determine the 6 trig ratios for the point P (-3 , 2) on the terminal arm of an angle in standard position.



$$\sin \theta = \frac{2}{\sqrt{13}}$$

$$\csc \theta = \frac{\sqrt{13}}{2}$$

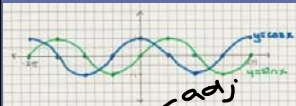
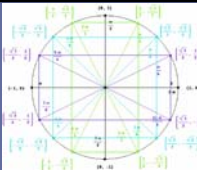
$$\cos \theta = \frac{-3}{\sqrt{13}}$$

$$\sec \theta = \frac{\sqrt{13}}{-3}$$

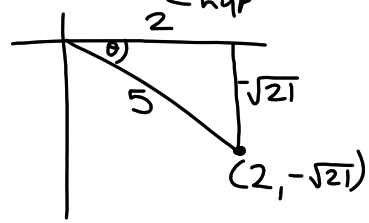
$$\tan \theta = \frac{-2}{3}$$

$$\cot \theta = \frac{-3}{2}$$

| | |
|-------|-----|
| Csc | A |
| Sin | |
| <hr/> | |
| Tan | Cos |
| Cot | Sec |

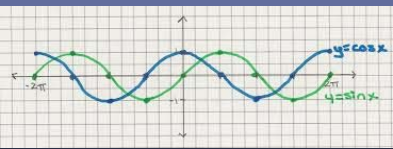
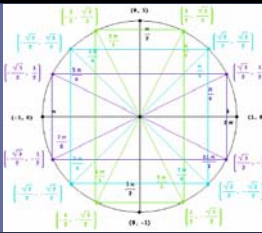
Ex.) If $\cos \theta = 2/5$ in quadrant IV, determine the other 5 trigonometric ratios.



$\sin \theta = -\frac{\sqrt{21}}{5}$
 $\tan \theta = -\sqrt{21}/2$
 $* \csc \theta = 5/\sqrt{21}$
 $\sec \theta = 5/2$
 $* \cot \theta = 2/\sqrt{21}$

Rationalizing the Denominator:

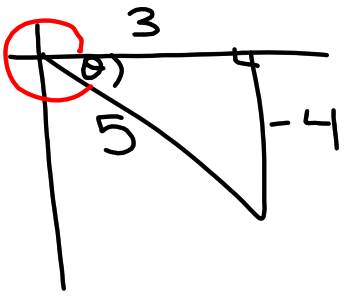
$$\csc \theta = \frac{-5}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{-5\sqrt{21}}{21}$$

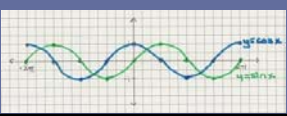
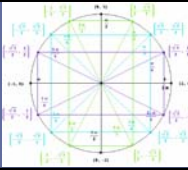
$$\cot \theta = \frac{-2}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{-2\sqrt{21}}{21}$$



Ex.) If $\cos \theta = 3/5$ and $\tan \theta < 0$, determine the reference angle and the angle in standard position.

neg.

$\theta_{ref} = \cos^{-1}(3/5) = \boxed{53^\circ}$
 $\theta = 360^\circ - 53 = \boxed{307^\circ}$
principle angle

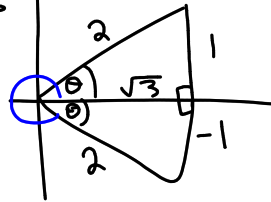


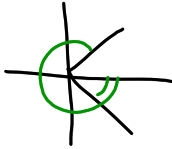
Ex.) Given $\sec \theta = 2/\sqrt{3}$, determine θ : $-2\pi < \theta < 2\pi$.

$\frac{1}{\cos \theta}$ hyp $\swarrow \nearrow$ adj $\theta_{ref} = 30^\circ$

$\frac{+}{-} \left| \begin{matrix} A \\ C \end{matrix} \right.$



$0 < \theta < 360^\circ$
 $30^\circ, 330^\circ$



$-360^\circ < \theta < 0$
 $-30^\circ, -330^\circ$

$\frac{-2\pi < \theta < 2\pi}{-\frac{11\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}}$

Pg. 201 # 1, 3, 8, 9, 10, 12.