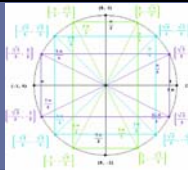


### Unit 4: Trigonometry



4.5 Solving Quadratic Trig Equations

Factor using the substitution method:

Ex.) Solve the following for  $0 \leq \theta \leq 360^\circ$  and  $0 \leq \theta \leq 2\pi$ .

a)  $\cos^2 \theta - \cos \theta - 2 = 0$        $x = \cos \theta$

$$x^2 - x - 2 = 0$$

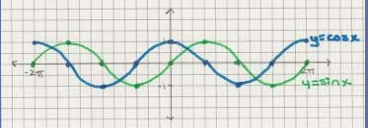
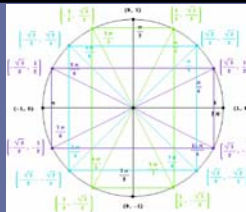
$$(x-2)(x+1) = 0$$

$$x-2 = 0$$

$$x = 2 \quad x = -1$$

$\cos \theta = 2$      $\cos \theta = -1$   
 $\theta_{\text{ref}} = \text{error}$      $\theta_{\text{ref}} = 180^\circ$   
 undefined

$\theta = 180^\circ \quad \theta = \pi$

b)  $\tan^2 \theta - 5 \tan \theta + 4 = 0$        $a = \tan \theta$

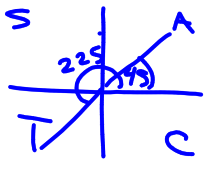
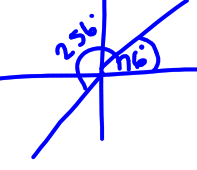
$$a^2 - 5a + 4 = 0$$

$$(a-1)(a-4) = 0$$

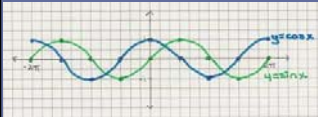
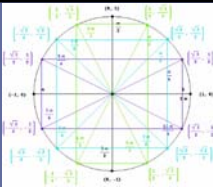
$$a = 1 \quad a = 4$$

$$\tan \theta = 1 \quad \tan \theta = 4$$

$$\theta_{\text{ref}} = 45^\circ \quad \theta_{\text{ref}} = 76^\circ$$

$45^\circ, 76^\circ, 225^\circ, 256^\circ$   
 $\frac{\pi}{4}, \frac{19\pi}{45}, \frac{5\pi}{4}, \frac{64\pi}{45}$

c)  $\csc^2 x - 1 = 0$   $a = \csc x$

$$a^2 - 1 = 0 \quad \text{or} \quad a^2 - 1 = 0$$

$$(a+1)(a-1) = 0 \quad \sqrt{a^2} = \sqrt{1}$$

$$a = \pm 1 \quad a = \pm 1$$

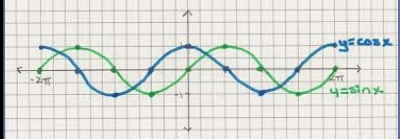
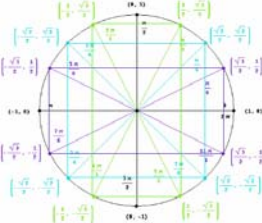
$$a = 1 \quad a = -1$$

$$\csc x = 1 \quad \csc x = -1$$

$$\frac{1}{\sin x} = 1 \quad \frac{1}{\sin x} = -1$$

$$\sin x = 1 \quad \sin x = -1$$

$x = 90^\circ$	$x = 270^\circ$
$x = \frac{\pi}{2}$	$x = \frac{3\pi}{2}$

d)  $\cos^2 x - \cos x = 0$   $w = \cos x$

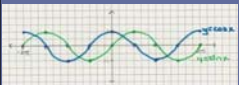
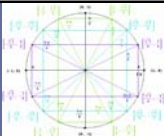
$$w^2 - w = 0$$

$$w(w-1) = 0$$

\*  $w = 0 \quad w = 1$

$$\cos x = 0 \quad \cos x = 1$$

$x = 90^\circ, 270^\circ, 0, 360^\circ$
$\frac{\pi}{2}, \frac{3\pi}{2}, 0, 2\pi$

e)  $2 \sin^2 \theta - 1 = \sin \theta$   $x = \sin \theta$

$2x^2 - 1 = x$

$-2 \quad 2x^2 - x - 1 = 0$

$\begin{matrix} 2 & & -1 \\ -1 & & \end{matrix} \quad 2x^2 - 2x \mid +1(x-1) = 0$

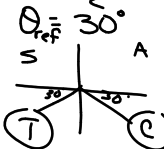
$2x(x-1) \mid +1(x-1)$

$(x-1)(2x+1) = 0$

$x = 1 \quad x = -\frac{1}{2}$

$\sin \theta = 1 \quad \sin \theta = -\frac{1}{2}$

$\theta = 90^\circ$



$\theta = 210^\circ, 330^\circ, 90^\circ$   
 $\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$

Pg. 212 # 7, 9, 13. ( $0 \leq \theta \leq 2\pi$ ).