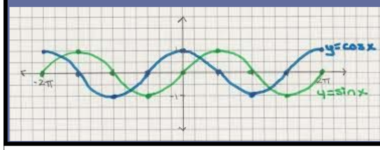
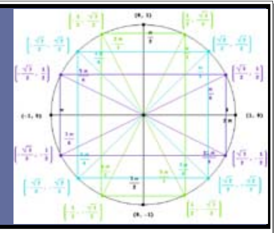


Unit 4: Trigonometry



4.9 Trig Identities

When working with Trig Identities, you can prove them three ways:

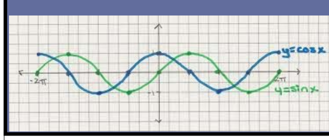
1) Graphically: LS = RS ($y_1 = y_2$) and see if the graphs overlap.

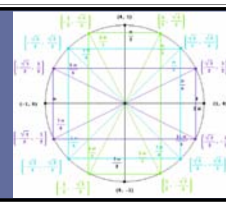
2) Numerically: Substitute an angle into the equations and check to see if LS = RS.

3) Algebraically: Simplify the identities using the formula sheet.

   * This will be presented in a "Two-Column Proof"/"T-Form Proof" arrangement. **You never cross the equal sign.**

   * Most problems can be solved by changing to sin and cosine.





Common Misconception:

$\sin x^2$ vs. $\underbrace{\sin^2 x}$

$\sin(x^2)$ $(\sin x)^2$

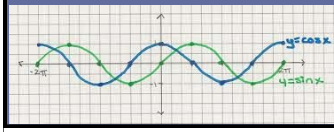
Helpful Understandings:

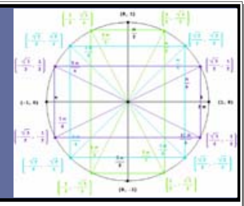
$\tan x = \frac{\sin x}{\cos x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$ $\sec^2 x = \frac{1}{\cos^2 x}$ $\cot^2 x = \frac{\cos^2 x}{\sin^2 x}$

$\csc x = \frac{1}{\sin x}$ $\cot^2 x = \frac{1}{\tan^2 x}$

$\csc^2 x = \frac{1}{\sin^2 x}$





Ex.) Prove the following identities numerically: sub in number.

a) sin θ cot θ = cos θ, when θ = 30°

LS	RS
$\sin 30^\circ \cdot \cot 30^\circ$	$\cos 30^\circ$
$0.5 \cdot \frac{1}{\tan 30^\circ}$	$0.866\ldots$
$0.5 \cdot 1.73\ldots$	
0.866	

calculator mode needs to change

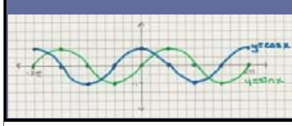
LS=RS

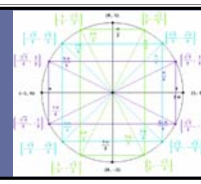
b) tan² x = sec² x - 1, when x = π/7

LS	RS
$\tan^2 x$	$\tan^2 x$
$\tan^2(\frac{\pi}{7})$	$(\tan(\pi/7))^2$
$0.2319\ldots$	$0.2319\ldots$

from formula sheet: $1+\tan^2 x = \sec^2 x$

LS=RS





Ex.) Prove the following identities algebraically: replace from formula sheet and make LS=RS.

a) $\frac{\sin x}{\tan x} = \cos x$, tan x ≠ 0

LS	RS
$\frac{\sin x}{\frac{\sin x}{\cos x}}$	$\cos x$
$\cancel{\sin x} \cdot \frac{\cos x}{\cancel{\sin x}}$	
$\cos x$	

*fraction within a fraction ⇒ multiply by reciprocal

$\sin x \div \frac{\sin x}{\cos x}$

LS=RS. ※ QED.

b) $\frac{1}{\cos x} - \frac{\cos x}{1} = \sin x \tan x$

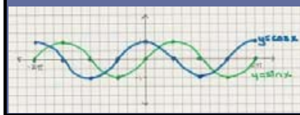
* common denominator.

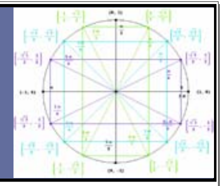
LS	RS
$\frac{1}{\cos x} - \frac{\cos x}{1} \cdot \frac{\cos x}{\cos x}$	$\sin x \cdot \frac{\sin x}{\cos x}$
$\frac{1-\cos^2 x}{\cos x}$	$\frac{\sin^2 x}{\cos x}$
$\frac{\sin^2 x}{\cos x}$	

* $\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1-\cos^2 x$

LS=RS ※





c) $\dfrac{\sec^2 x}{\sec^2 x - 1} = \csc^2 x$

$\dfrac{\left(\frac{1}{\cos^2 x}\right)}{\left(\frac{\sin^2 x}{\cos^2 x}\right)}$ | $\dfrac{1}{\sin^2 x}$

$\dfrac{1}{\cancel{\cos^2 x}} \cdot \dfrac{\cancel{\cos^2 x}}{\sin^2 x}$

$\dfrac{1}{\sin^2 x}$

$1 + \tan^2 x = \sec^2 x$

LS= RS ##

d) $\sin^3 x + \cos^2 x \cdot \sin x = \dfrac{1}{\csc x}$

$\sin x \underbrace{(\sin^2 x + \cos^2 x)}$ | $\dfrac{1}{\left(\frac{1}{\sin x}\right)}$

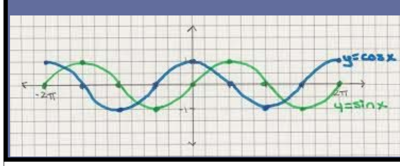
$\sin x \cdot 1$ | $1 \div \dfrac{1}{\sin x}$

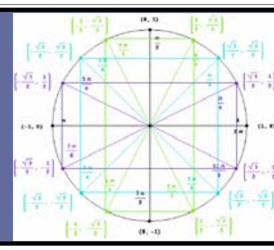
$\sin x$ | $1 \cdot \dfrac{\sin x}{1}$

$\sin x$

* factor GCF : sinx

LS=RS ###





e) $(\tan x - 1)^2 = \sec^2 x - 2\tan x$

$\tan^2 x - 2\tan x + 1$ |

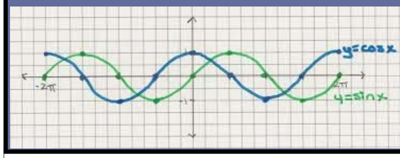
$\sec^2 x - 2\tan x$

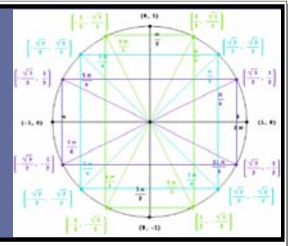
LS=RS ##

~~$(x-1)^2$~~
~~$x^2 - 1$~~

$(x-1)^2$

$(x-1)(x-1)$





f) (sin x + cos x)^2 + (sin x - cos x)^2 = 2

*FOIL

$\sin^2 x + \cancel{2\sin x\cos x} + \cos^2 x + \sin^2 x - \cancel{2\sin x\cos x} + \cos^2 x$ | 2

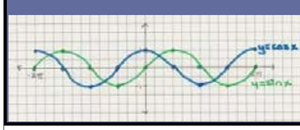
$2\sin^2 x + 2\cos^2 x$

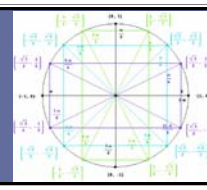
$2(\sin^2 x + \cos^2 x)$

$2 \cdot 1$

2

LS=RS





*Trick for fractions with binomial denominators...multiply by the conjugate.

g) $\dfrac{1 + \cos\theta}{\sin\theta} = \dfrac{\sin\theta}{(1 - \cos\theta)} \cdot \dfrac{(1+\cos\theta)}{(1+\cos\theta)}$

$\dfrac{\sin\theta + \sin\theta\cos\theta}{1 - \cos^2\theta}$

$\dfrac{\sin\theta + \sin\theta\cos\theta}{\sin^2\theta}$

$\dfrac{\cancel{\sin\theta}(1+\cos\theta)}{\sin^{\cancel{2}}\theta}$

$\dfrac{1+\cos\theta}{\sin\theta}$

$(x+1)\ (x-1)$

$x^2 + x - x - 1$

$x^2 - 1$

$\sin^2\theta + \cos^2\theta = 1$