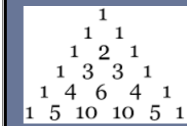
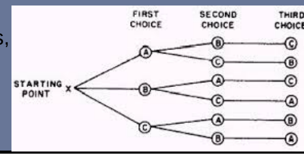


Unit 5: Permutations, Combinations, and the Binomial Theorem



5.1 Permutations and The Fundamental Counting Principle

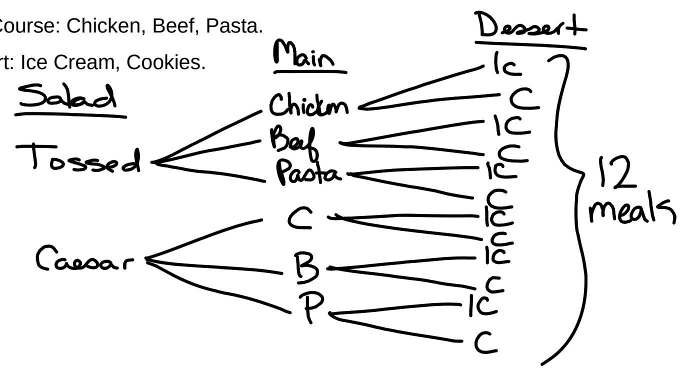
A **permutation** is an arrangement of items in a set. In a permutation, order matters.

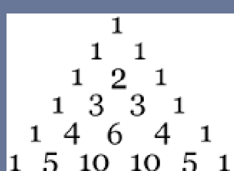
Ex.) Use a tree diagram to determine how many different meals could you have with the following choices:

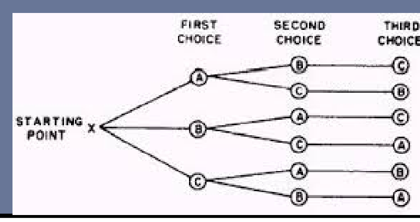
2 Salads: Tossed, Caesar.

3 Main Course: Chicken, Beef, Pasta.

2 Dessert: Ice Cream, Cookies.



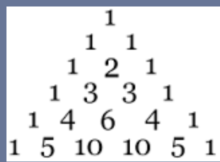


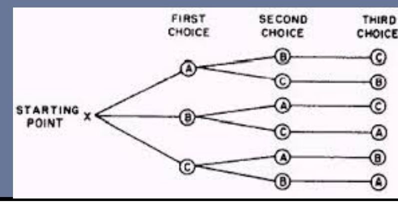


Tree diagrams work well for small sets, but can be time consuming for larger sets. The **Fundamental Counting Principle** allows us to do these calculations for larger sets more quickly.

With the Fundamental Counting Principle, we create blank spaces that will be filled with the number of options we have for that space. Use the Fundamental Counting Principle for the previous example:

$$\underset{\text{salad}}{\underline{2}} \cdot \underset{\text{main}}{\underline{3}} \cdot \underset{\text{dessert}}{\underline{2}} = \boxed{12 \text{ meals}}$$



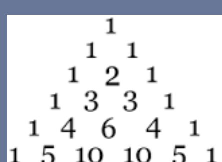


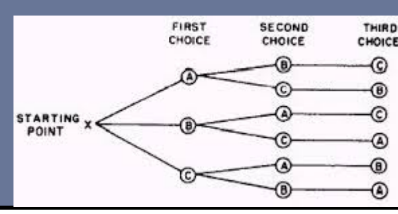
Ex.) You have 6 pairs of pants, 12 shirts, and 3 pairs of shoes. How many outfits could you make?

$$\underset{\text{pants}}{\underline{6}} \cdot \underset{\text{shirt}}{\underline{12}} \cdot \underset{\text{shoes}}{\underline{3}} = \boxed{216 \text{ outfits}}$$

Ex.) The final score in a hockey game was 5-4. How many different scoring combinations are there for the end of the second period?

$$\underset{\text{Home}}{\underline{6}} \cdot \underset{\text{Away}}{\underline{5}} = \boxed{30}$$





Restrictions: fill in the restriction first, then work with the remaining items

Ex.) How many 2-digit, odd numbers can be made from 1, 2, 4, 5, 7, 8, 9, if:

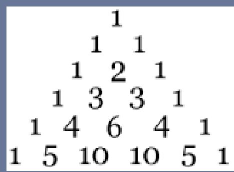
7 #'s

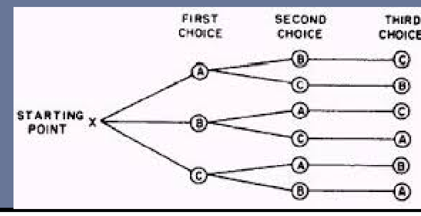
a) Repeats are allowed.

$$\underline{7} \cdot \underset{\text{odd}}{\underline{4}} = \boxed{28}$$

b) Repeats are not allowed.

$$\underline{6} \cdot \underset{\text{odd}}{\underline{4}} = \boxed{24}$$



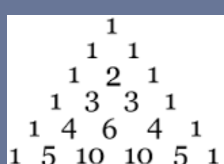


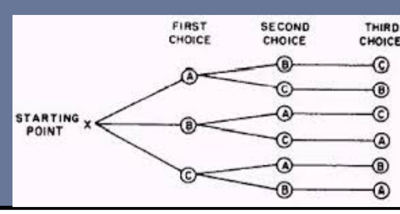
Ex.) How many 3-digit, odd numbers (with no repeats) can be made from: 0, 1, 4, 5, 6, 8, 9.

$$\underset{\text{not } 0}{\underline{5}} \cdot \underline{5} \cdot \underset{\text{odd}}{\underline{3}} = \boxed{75}$$

Ex.) How many 3-digit, even numbers (with repeats allowed) can be made from: 2, 3, 4, 6, 7, 8.

$$\underline{6} \cdot \underline{6} \cdot \underset{\text{even}}{\underline{4}} = \boxed{144}$$





Ex.) How many License Plates (3 digits and 3 letters) can be made with:

a) no restrictions

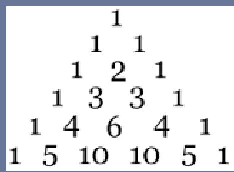
$$\underset{L}{\underline{26}} \cdot \underset{L}{\underline{26}} \cdot \underset{L}{\underline{26}} \cdot \underset{\#}{\underline{10}} \cdot \underset{\#}{\underline{10}} \cdot \underset{\#}{\underline{10}} = \boxed{17\ 576\ 000}$$

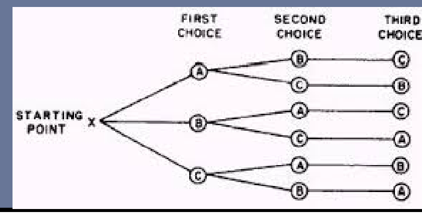
b) no *I* or *O*

$$\underline{24} \cdot \underline{24} \cdot \underline{24} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = \boxed{13\ 824\ 000}$$

c) first letter is your initial and last number is odd

$$\underset{\text{initial}}{\underline{1}} \cdot \underline{26} \cdot \underline{26} \cdot \underline{10} \cdot \underline{10} \cdot \underset{\text{odd}}{\underline{5}} = \boxed{338\ 000}$$





Ex.) *Create 'words' from the letters* questions: no repeats, like pulling letters from a hat.

arrangement of letters

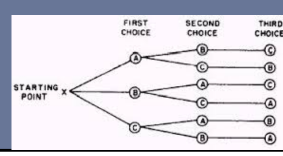
LACOMBE 7 Vowels: a e i o u, y?

a) total number of arrangements of all letters

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{5040}$$

b) 4-letter word arrangements

$$7 \cdot 6 \cdot 5 \cdot 4 = \boxed{840}$$



c) 4-letter word that ends in E

$$6 \cdot 5 \cdot 4 \cdot \underset{E}{1} = \boxed{120}$$

d) 4-letter word that contains E

$$\underset{E}{1} \cdot 6 \cdot 5 \cdot 4 = 120$$

$+$

or $$6 \cdot \underset{E}{1} \cdot 5 \cdot 4 = 120$$

$+$

or $$6 \cdot 5 \cdot \underset{E}{1} \cdot 4 = 120$$

$+$

or $$6 \cdot 5 \cdot 4 \cdot \underset{E}{1} = 120$$

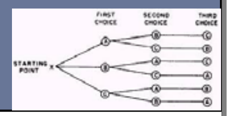
$\boxed{480}$

*

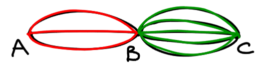
or $\rightarrow$ +

and $\rightarrow$ ×

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1



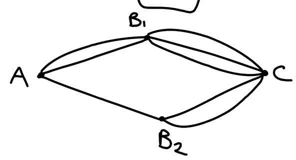
Ex.) Pathways:



$A \to B$ and $B \to C$

3×5

$\boxed{15}$



Top or Bottom

$A \to B_1, B_1 \to C$ $A \to B_2$ $B_2 \to C$

$2 \times 3 + 1 \times 2$

$6 + 2$

$\boxed{8}$