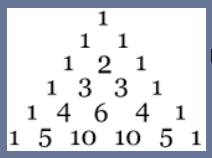
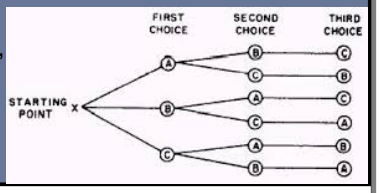


Unit 5: Permutations, Combinations, and the Binomial Theorem



5.2 Factorial Notation

$n! = n(n-1)(n-2)\ldots 3 \times 2 \times 1,$
where $n \in N$ and $0! = 1$

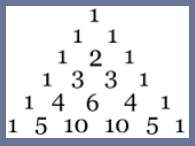
Calc. Math PRB 4:!

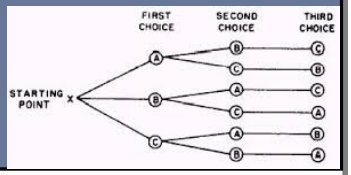
Ex.)

a) 5! = $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

b) 3! = 6

c) 8! = 40320





Ex.) Determine the number of arrangements of the letters in the word ORANGES:

a) with no restrictions

$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 7! = 5040$

b) starting with the letter R

R $1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6! = 720$

c) with all the vowels together

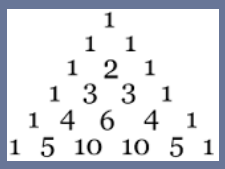
$3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 3!4!5$

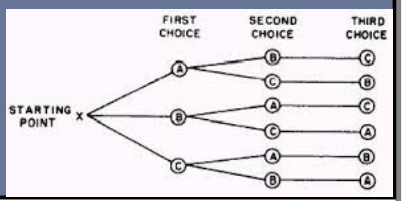
Vowels

$= 3!5!$

x5

$= 720$





Ex.) Determine the number of arrangements of the letters in the word BRAINS:

a) with no restrictions

$6! = 720$

b) with all the vowels together

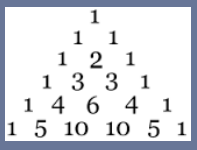
$2 \cdot 1 \quad 4 \cdot 3 \cdot 2 \cdot 1 = 2!5! = 240$

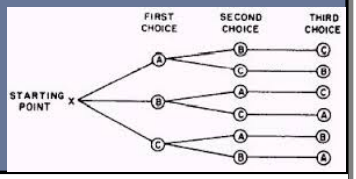
vowels $\times 5$

c) with vowels not together

$720 - 240 = 480$

total − vowels together = vowels NOT together





Repetitions: - identical objects, repeating letters

- divide by the repeats

$\frac{n!}{a!b!c!}$ — n! = total # letters; a!b!c! = repeats

Ex.) KISSING

a) no restrictions

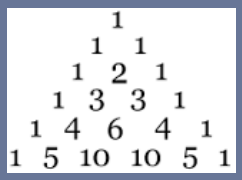
$\frac{7!}{2!2!1!} = \frac{7!}{4} = 1260$

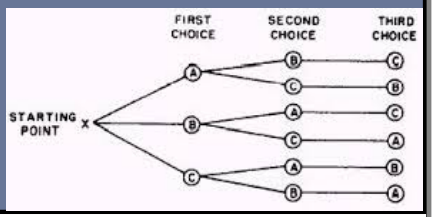
I · S · S

b) SS are first

$\frac{2 \cdot 1}{S\ S} \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2! 2!} = \frac{2!5!}{2!2!} = \frac{5!}{2} = 60$

$5 \cdot 4 \cdot 3 = 60$





Ex.) You have 15 lollipops and an equal amount of red, green, yellow. How many possible arrangements are there?

5 each

total / repeats

$\frac{15!}{(5!5!5!)} = 756\,756$

$15!/(5!5!5!)$