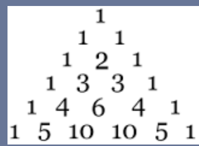
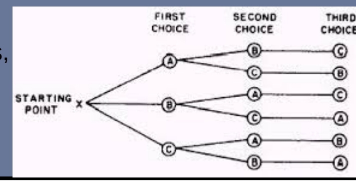


Unit 5: Permutations, Combinations, and the Binomial Theorem



5.3 Permuations and Combinations

ORDER MATTERS: Permutations

Order DOESN"T Matter: Combinations

$0! = 1$

$n! = n(n-1)(n-2)\ldots$

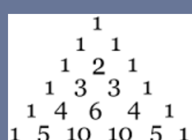
$n \in \mathbb{N}$

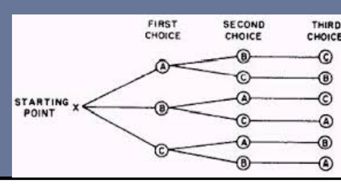
Ex.) Simplify the following Expressions:

a) $\dfrac{n!}{(n-3)!} = \dfrac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = \boxed{n(n-1)(n-2)}$

b) $\dfrac{(n+1)!}{(n-1)!} = \dfrac{(n+1)(n+1-1)\cancel{(n-1)!}}{\cancel{(n-1)!}} = \boxed{n(n+1)}$

$= n^2 + n$





Ex.) Solve for *n.*

a) $\dfrac{(n+1)!}{(n-1)!} = 6$

$\dfrac{(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}} = 6$

$n^2 + n = 6$

$n^2 + n - 6 = 0$

$(n+3)(n-2) = 0$

$\cancel{n=-3}$ $n = 2$

$n \in \mathbb{N}$

b) $\dfrac{(n+2)!}{n!} = \dfrac{12\cancel{n!}}{\cancel{n!}}$

$\dfrac{(n+2)(n+1)\cancel{(n)!}}{\cancel{n!}} = 12$

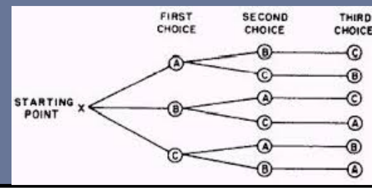
$n^2 + 3n + 2 = 12$

$n^2 + 3n - 10 = 0$

$(n+5)(n-2) = 0$

$\cancel{n=-5}$ $n = 2$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1



<u>Permutations</u>: arrangements, order of objects

$$\boxed{{}_nP_r = \frac{n!}{(n-r)!}}$$

total # objects

# of objects you are picking

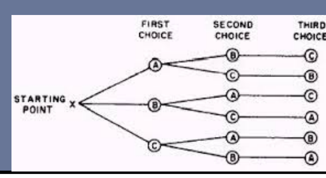
Ex.) A class of 32 students are voting for a president, vice president, and secretary. How many possible arrangements are there?

order matters

$${}_{32}P_3 = \frac{32!}{(32-3)!} = \frac{32!}{29!} = \frac{32 \cdot 31 \cdot 30 \cdot \cancel{29!}}{\cancel{29!}}$$

$$= \boxed{29\,760}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1



Ex.) There are 10 different book on a shelf. Four are chosen to be arranged for a display. How many possible arrangements are there?

$${}_{10}P_4 = \boxed{5040}$$

Ex.) For a play there are 4 male roles and 3 females roles. If there are 6 actors and 8 actresses to pick from, how many casts are available?

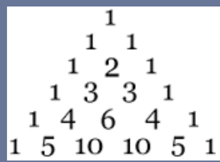
Actors & Actresses

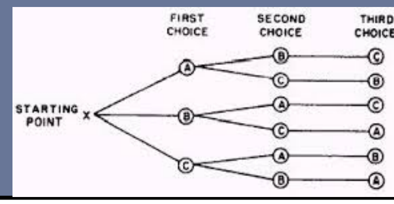
$${}_6P_4 \times {}_8P_3$$

$$360 \times 336$$

$$\boxed{120\,960}$$

and - mulitply
or - add





Combinations: order does NOT matter

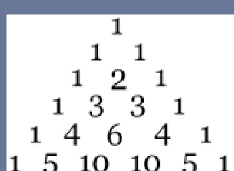
$$\boxed{{}_nC_r = \frac{n!}{(n-r)!r!} \qquad {}_nC_r = \binom{n}{r}}$$

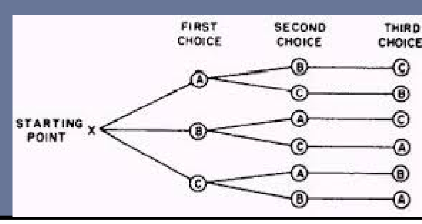
total # of objects

objects chosen

Ex.) Lotto 6-49 gets its name from the fact that of 49 numbers, 6 are chosen. How many combinations are possible in Lotto 6-49?

$${}_{49}C_6 = 13\ 983\ 816$$





Ex.) There are 8 people available to sit on a subcommittee of 3 people. How many ways are there is the subcommittee has:

a) no titles

$${}_8C_3 = 56$$

b) titles

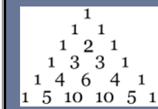
$${}_8P_3 = 336$$

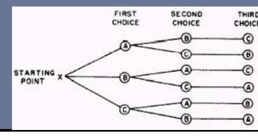
$${}_nC_r = \frac{n!}{(n-r)!r!}$$

$$\frac{8!}{(8-3)!3!} = \frac{8!}{5!3!}$$

$${}_8C_5 = \frac{8!}{(8-5)!5!}$$

$$= \frac{8!}{3!5!}$$





Ex.) A group of 15 people(9 females, 6 males) will form a subcommittee with 7 people. How many combinations are there with

a) exactly 3 females 3F and 4m

$${}_9C_3 \cdot {}_6C_4 = 84 \cdot 15 = \boxed{1260}$$

b) 7 females

$${}_9C_7 = \boxed{36}$$

c) at least four females

4F and 3m, OR 5F and 2m, OR 6F and 1m or 7F

$$= ({}_9C_4)({}_6C_3) + ({}_9C_5)({}_6C_2) + ({}_9C_6)({}_6C_1) + {}_9C_7$$

$$= \binom{9}{4}\cdot\binom{6}{3} + \binom{9}{5}\binom{6}{2} + \binom{9}{6}\binom{6}{1} + \binom{9}{7}$$

$$= 2520 + 1890 + 504 + 36$$

$$= \boxed{4950}$$

*Perms: Pg. 524 #2, 5, 6, 7a, 24.

*Combs: Pg. 534 #1, 4, 6b, 8, 9, 11, 14,17.