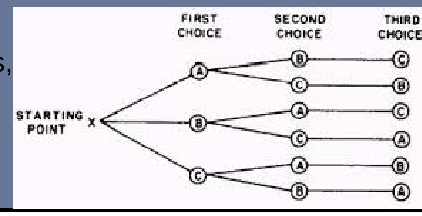
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Unit 5: Permutations, Combinations, and the Binomial Theorem



5.4 Binomial Expansion

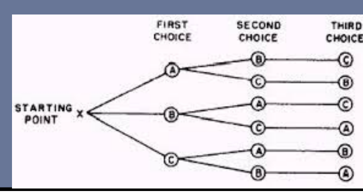
Ex.) Expand $(x+3)^3$ $= (x+3)(x+3)(x+3)$

$= (x^2+6x+9)(x+3)$

$= x^3+3x^2+6x^2+18x+9x+27$

$= \boxed{x^3+9x^2+27x+27}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1



In the expansion of $(x+y)^n$, written in descending powers of x, the general term is $t_{k+1} = {}_nC_k x^{n-k} y^k$.

Now expand $(x+3)^3$ using the binomial theorem above:

$t_1 = {}_3C_0 (x)^{3-0}(3)^0 = (1)(x^3)(1) = x^3$

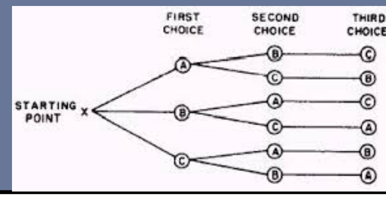
$t_2 = {}_3C_1 (x)^{3-1}(3)^1 = 3(x^2)(3) = 9x^2$

$t_3 = {}_3C_2 (x)^{3-2}(3)^2 = 3(x)(9) = 27x$

$t_4 = {}_3C_3 (x)^{3-3}(3)^3 = 1(1)(27) = 27$

$\boxed{x^3+9x^2+27x+27}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1



Connection between # of terms and the exponent of a binomial:

	Exponent	Terms
$(x+2)^1$	1	2
$(x+2)^2$	2	3
$(x+3)^3$	3	4

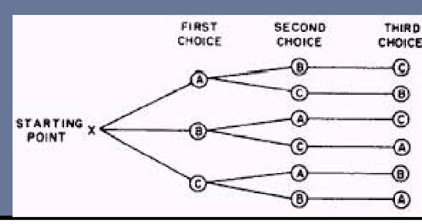
Ex.) $(3a-4b)^{3x-6}$, 22 terms

$3x-6=21$

$3x=27$

$x=9$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1



Specific Term in a Binomial Expansion:

$(x+y)^n$ $\quad t_{k+1} = {}_nC_k\,(x)^{n-k}(y)^k$

first term in binomial

second term in binomial

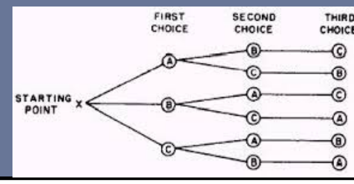
$t_6 = t_{5+1}$

$t_{30} = t_{29+1}$

The 'k' value is always one less than the term #.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$t_{k+1} = {}_nC_k (x)^{n-k}(y)^k$



Ex.) Determine the seventh term in the expansion of $(x + 4)^{11}$.

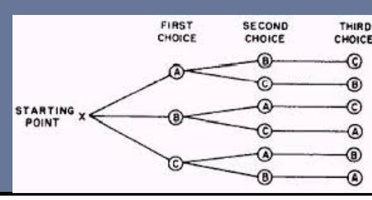
$$t_7 = t_{6+1} = {}_{11}C_6 (x)^{11-6}(4)^6$$
$$= (462)(x^5)(4096)$$
$$= \boxed{1\,892\,352x^5}$$

$n = 11$
$k = 6$
$x = x$
$y = 4$

Ex.) Determine the ninth term in the expansion of $(3 - 2x)^{14}$.

$$t_9 = t_{8+1} = {}_{14}C_8 (3)^{14-8}(-2x)^8$$
$$= (3003)(729)(256x^8)$$
$$= \boxed{560431872x^8}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1



Ex.) Determine the fourth term in the expansion of $(3x^2 - 4y)^7$.

$$t_4 = t_{3+1} = {}_7C_3 (3x^2)^{7-3}(-4y)^3$$
$$= (35)(81x^8)(-64y^3)$$
$$= \boxed{-181\,440x^8y^3}$$

$(x^2)^4$

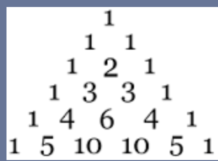
Ex.) Determine the middle term in the expansion of $(3x - 5)^6$.

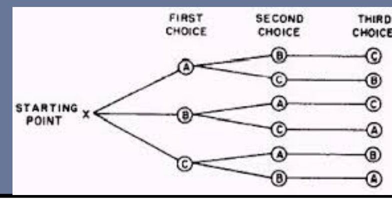
t_4

7 terms

_ _ _ ⊖ _ _ _

$$t_4 = t_{3+1} = {}_6C_3 (3x)^{6-3}(-5)^3$$
$$= 20(27x^3)(-125)$$
$$= \boxed{-67\,500x^3}$$





Ex.) Determine the numerical coefficient of the term with x^4 in the expansion of $(4 - 3x)^6$.

${}_nC_k (x)^{n-k}(y)^k$

$= {}_6C_4 (4)^{6-4}(-3x)^4$

$= 15(16)(81x^4)$

$= 19440x^4$

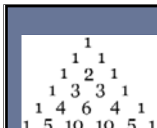
$\boxed{19440}$

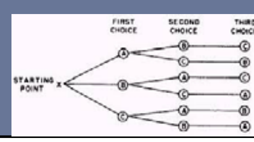
Ex.) Determine the numerical coefficient of the term with x^7 in the expansion: $(3x + 5)^{10}$.

${}_{10}C_3 (3x)^{10-3}(5)^3$

$= (120)(2187x^7)(125)$

$= \boxed{32805000x^7}$





Ex.) In the expansion of $(x - b)^{12}$, a term is $(309375/16)x^8$. The value of '*b*' is ______.

${}_{12}C_4 (x)^{12-4}(-b)^4$

$= 495\cancel{x^8}(b^4) = \frac{309375}{16}\cancel{x^8}$

$\frac{495b^4}{495} = \frac{\left(\frac{309375}{16}\right)}{495}$

$\sqrt[4]{b^4} = \sqrt[4]{39.0625}$

$\boxed{b = 5/2}$

$\sqrt[4]{x}$ $x^{(1/4)}$

Ex.) Determine the $\underbrace{\text{constant term}}_{x^0}$ in the expansion of $(x^2 + (1/x))^6$.

$(x^2 + x^{-1})^6$

${}_6C_k (x^2)^{6-k}(x^{-1})^k = x^0$

$2(6-k)+-k = 0$

$12-2k-k = 0$

$12-3k = 0$

$12 = 3k$

$k = 4$

${}_6C_4 (x^2)^{6-4}(x^{-1})^4$

$= 15x^4x^{-4}$

$= 15x^0 = \boxed{15}$