5.4 Combinations

Combinations: order does NOT matter

$${}_nC_r = \frac{n!}{(n-r)!r!} \qquad {}_nC_r = \binom{n}{r}$$

total # objects

chosen

Ex.) Lotto 6-49 gets its name from the fact that of 49 numbers, 6 are chosen. How many combinations are possible in Lotto 6-49?

$${}_{49}C_6 = \binom{49}{6} = \frac{49!}{(49-6)!6!} = 13\,983\,816$$

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Ex.) There are 8 people available to sit on a subcommittee of 3 people. How many ways are there is the subcommittee has:

a) no titles

$${}_8C_3 = \frac{8!}{(8-3)!3!} = \frac{8!}{5!3!} = \frac{8\cdot7\cdot6\cdot5!}{5!3!} = 56$$

b) titles (order matters; vice different from secretary...)

$${}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8\cdot7\cdot6\cdot5!}{5!} = 336$$

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Ex.) A group of 15 people (9 females, 6 males) will form a subcommittee with 7 people. How many combinations are there with

a) exactly 3 females $3F \text{ and } 4M$

$${}_9C_3 \times {}_6C_4 = 84 \times 15 = \boxed{1260}$$

b) 7 females

$${}_9C_7 = \boxed{36}$$

c) at least four females

$$= \underbrace{\binom{9}{4}\cdot\binom{6}{3}}_{4F \text{ and } 3M} + \underbrace{\binom{9}{5}\cdot\binom{6}{2}}_{5F \text{ and } 2M} + \underbrace{\binom{9}{6}\cdot\binom{6}{1}}_{6F \text{ and } 1M} + \underbrace{\binom{9}{7}}_{7F}$$

$$= (126 \times 20) + (126 \times 15) + (84 \times 6) + 36$$

$$= 2520 + 1890 + 504 + 36$$

$$= \boxed{4950}$$

Pg. 118 # 1-4, 7, 8, 10, 11, 15.

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