5.5 Choosing FCP, Perms, or Combs

FCP: choosing one thing from each category

Perms: order matters

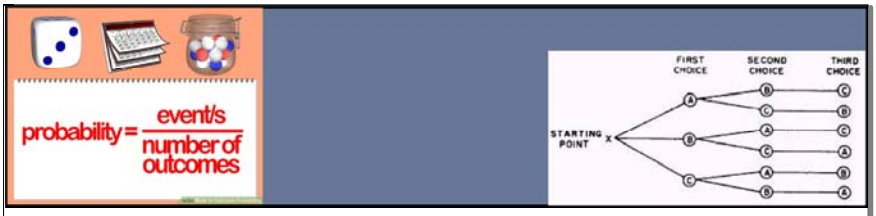
Combs: order doesn't matter

Ex.) There are five swimmers in a race: Aubrey, Betz, Cam, Deanna and Elena. How many different ways can the swimmers finish first, second and third?

Order matters ⇒ perm

$_5P_3 = 5 \cdot 4 \cdot 3 = 60$

May 9-12:57 PM



Ex.) Suppose the above race was a *heat*, where the top three advance no matter what order they finish in. How many different ways can the swimmers advance?

Order doesn't matter ⇒ comb

$_5C_3 = \binom{5}{3} = \frac{5!}{(5-3)!3!} = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3!}{2! 3!} = \frac{20}{2} = 10$

Ex.) A piano teacher and her students are having a group photograph taken. There are three boys and five girls. The photographer wants the boys to sit together and the girls to sit together for one of the poses. How many ways can the students and teacher sit in a row of nine chairs for this pose?

$\underbrace{3 \cdot 2 \cdot 1}_{\text{boys}} \cdot \underbrace{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}_{\text{girls}} \cdot 1 = 3!5!3!$

$= 4320$

3 · 2 · 1

$_3P_3 \cdot {}_5P_5 \cdot {}_3P_3$

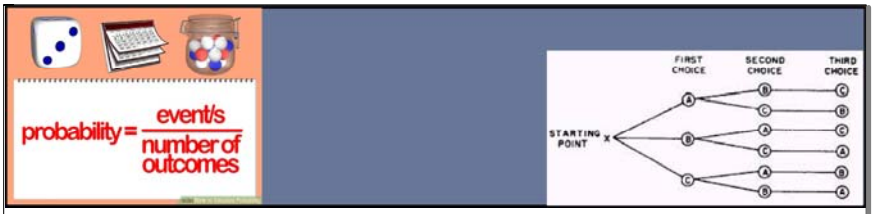
May 9-1:00 PM

Ex.) For another pose, the photographer wants the two tallest students, Jill and Sam, to sit at either end with Jill on the left and Sam on the right and the teacher to sit in the middle. How many different seating arrangements are there for this pose?

$1 \cdot 6 \cdot 5 \cdot 4 \cdot 1 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 6! = 720$

$= {}_6P_6$

May 9-1:02 PM



Ex.) How many different five-card hands that contain at most one black card can be dealt to one person from a standard deck of playing cards? There are two ways to solve this problem logically.

① zero black (all red) or 1 black (4 red)

${}_{26}C_5 + {}_{26}C_1 \times {}_{26}C_4$

$\binom{26}{5} + \binom{26}{1}\binom{26}{4} = 454\,480$

② total hands − (2B + 3B + 4B + 5B)

Pg. 127 # 9-14.

May 9-1:03 PM