5.8 Probability Using Counting Methods

Ari, Ben, Cam and Dan have booked a flight to Florida. The airplane has four seats in each row with an aisle down the middle. The boys have booked all four seats in the same row. We want to know: what is the probability that Cam and Dan will sit beside each other on the same side of the plane?

1. Develop a sample space of the number of ways the boys can sit together. There should be 6 ways. Can you get that number using methods from chapter 2?

$_4C_2 = 6$

2. How many of those ways has Cam and Dan sitting together?

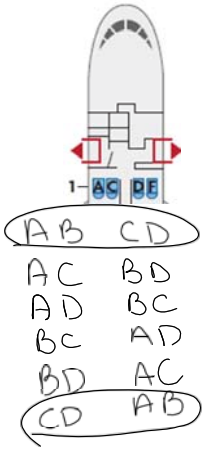
2

3. What is the probability Cam and Dan will sit together?

$\frac{2}{6} = 33\% = \frac{1}{3}$

4. What are the odds Cam and Dan will sit together?

2 : 4 1 : 2



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Ex.) Ten students have volunteered to put on a talent show at a retirement home, but only three are allowed to go. Victoria really really REALLY wants to go since her grandparents live in the home. Unfortunately for Victoria, each student's name will be put in a hat and three names will be drawn at random. Victoria wants to know what her chances are of her name being drawn.

A. Is this a fair way of determining who gets to perform at the retirement home? In other words, is Victoria just as likely to be drawn as any other name?

Yes, each person has the same prob. of being chosen.

B. Does the order in which the names are drawn matter? What hint does that provide in helping you choose which function to use on your calculator?

Order doesn't matter ⇒ Combination

C. How many ways can the three names be drawn from the hat?

$_{10}C_3 = 120$

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D. To determine Victoria's probability of being drawn, ALWAYS assume success. To successfully draw Victoria's name, we use ${}_1C_1$. How many ways can the others be drawn?

${}_1C_1$ and ${}_9C_2$

$1 \times 36 = 36$

E. What is the probability of Victoria's name being drawn from the hat?

$P(\text{Success}) = \frac{(\text{Number of successes})}{(\text{Number of all outcomes})}$

$P(\text{Victoria chosen}) = \frac{(\text{number of ways to draw Victoria}) \times (\text{number of ways to draw other two names})}{(\text{number of ways to draw the three names})} = \frac{36}{120} = 30\%$

F. Suppose only eight students volunteered for the talent show and three students are randomly picked to go. What are Victoria's chances now?

$P(\text{Victoria goes}) = \frac{{}_1C_1 \times {}_7C_2}{{}_8C_3} = 38\%$

G. Because only eight students volunteered, the teacher decided only two students would be selected to go. What are Victoria's chances now?

$= \frac{{}_1C_1 \times {}_7C_1}{{}_8C_2} = 25\%$

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Ex.) Jamaal, Ethan and Alberto are competing with seven other boys to be on the schools cross country team. Only the fastest three runners in a trial race will represent the school. What are the chances (probability) Jamaal, Ethan and Alberto will represent their school?

$\frac{3C_3}{10C_3} = \frac{1}{120} = 0.0083 \times 100 = 0.83\%$

Ex.) Suppose Zachary is also trying out for the team, so there are now 11 runners. What is the probability that three of these four boys will represent their school?

$\frac{4C_3}{11C_3} = \frac{4}{165} = 2.4\%$

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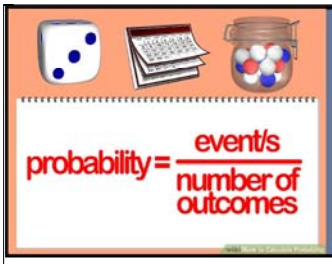

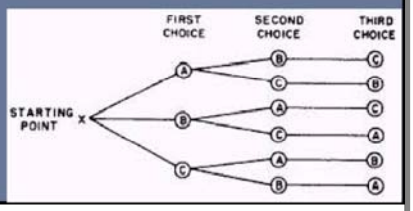

Ex.) Determine the probability that in a family with three children, at least one is a boy.

1 boy or 2 boys or 3 boys

bgg bbg bbb

gbg bgb

ggb gbb

3 + 3 + 1 = 7

$P(\text{at least one boy}) = \frac{7}{8} = 88\%$

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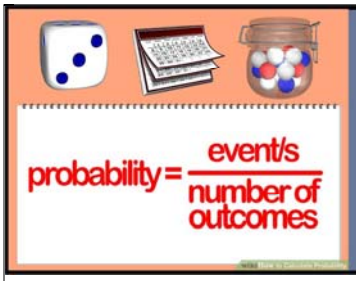

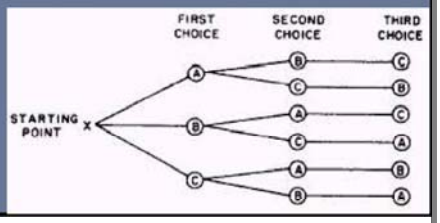

Ex.) The Scrabble® tiles for SASKATCHEWAN are placed in a cloth bag and drawn one at a time. What is the probability you will draw the name SASKATCHEWAN in that order?

$P(\text{Sask}) = \frac{1}{\left(\frac{12!}{(2!3!)}\right)} = \frac{1}{39\,916\,800}$

Ex.) How are the probabilities different if the tiles spelled SASKATOON?

$\frac{1}{\left(\frac{9!}{2!2!2!}\right)} = \frac{1}{45\,360}$

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Ex.) There are 18 exercise bikes in 3 rows of 6 bikes each. Allison, Brett, Carol, Doug, Erica and Franco each call the gym to reserve a bike. What is the probability they will all be in the same row with Allison on one end and Franco on the other? Two possible solutions are show below. Explain each one.

$$P(F) = \frac{3({}_2P_2)({}_4P_4)({}_{12}P_{12})}{({}_{18}P_{18})}$$

3 = 3 rows

${}_2P_2$ = Alison + Franco in specific spots

${}_4P_4$ = other 4 people in remaining spots of that row

${}_{12}P_{12}$ = fill remaing 12 bikes

${}_{18}P_{18}$ = total # of way 18 people fill 18 bikes

$$P(F) = \frac{3\times 2\times({}_4P_4)}{({}_{18}P_6)}$$

3 = 3 rows

2 = Alison & Franco

${}_4P_4$ = other 4 people in remaining bike of that row

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${}_{18}P_6$ = one row of bikes

$P = \frac{1}{92\,820}$

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