

Math 30 Pure
Binomial Expansions

Name: Key

1. Find the value of n if the expansion of

a) $(2x+3)^n$ has 18 terms

$$n = 17$$

b) $(3x-5)^{4n-3}$ has 26 terms

$$\begin{aligned} 4n-3 &= 25 \\ 4n &= 28 \end{aligned} \quad \boxed{n=7}$$

2. Find the indicated term of each expansion.

a) the fifth term of $(a-b)^5$

$$\begin{aligned} t_{4+1} &= {}_5C_4 (a)^{5-4} (-b)^4 \\ &= \boxed{5ab^4} \end{aligned}$$

b) the second term of $(x-2)^6$

$$\begin{aligned} t_{1+1} &= {}_6C_1 (x)^{6-1} (-2)^1 \\ &= 6x^5(-2) \\ &= \boxed{-12x^5} \end{aligned}$$

c) the fourth term of $(a^2-2a)^7$

$$\begin{aligned} t_{3+1} &= {}_7C_3 (a^2)^{7-3} (-2a)^3 \\ &= 35a^8(-8a^3) \\ &= \boxed{-280a^{11}} \end{aligned}$$

d) the middle term of $\left(2-\frac{x}{2}\right)^6$

$$\begin{aligned} t_{3+1} &= {}_6C_3 (2)^{6-3} \left(-\frac{x}{2}\right)^3 \\ &= 20(8)\left(-\frac{x^3}{8}\right) = \boxed{-20x^3} \end{aligned}$$

3. The term that contains b^3 in the expansion of $(5-2b)^{12}$ is

$$\begin{aligned} {}_{12}C_k (5)^{12-k} (-2b)^k &= {}_{12}C_3 (5)^{12-3} (-2b)^3 \\ k &= 3 \\ &= 220(1953125)(-8b^3) \\ &= \boxed{-3437500000b^3} \end{aligned}$$

4. The term that contains x^6 in the expansion of $(2x^2-9)^8$ is

$$\begin{aligned} {}_8C_k (2x^2)^{8-k} (-9)^k \\ k &= 5 \\ &= {}_8C_5 (2x^2)^{8-5} (-9)^5 \\ &= 56(8x^6)(-59049) = \boxed{-26453952x^6} \end{aligned}$$

5. Determine the value of m if one term in the expansion of $(x+m)^{11}$ is $-4455x^8$.

$${}^{11}C_k (x)^{11-k} (m)^k$$

$$k=3$$

$${}^{11}C_3 (x)^{11-3} (m)^3 = -4455x^8$$

$$\frac{165x^8 m^3}{165} = \frac{-4455x^8}{165}$$

$$m^3 = -27$$

$$\boxed{m = -3}$$

5. Find the constant term in the expansion of $\left(x - \frac{1}{x^3}\right)^{12}$.

$$x^0$$

$$\left(x - x^{-3}\right)^{12}$$

$${}^{12}C_k (x)^{12-k} (x^{-3})^k$$

$$12 - k - 3k = 0$$

$$12 - 4k = 0$$

$$12 = 4k$$

$$k = 3$$

$${}^{12}C_3 (x)^{12-3} (x^{-3})^3$$

$$= 220x^9 (x^{-9}) = \boxed{-220}$$