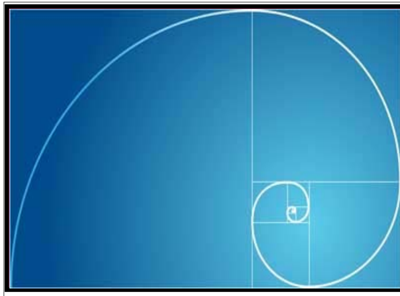


Unit 6: Sequences and Series

6.4 Geometric Series and Infinite Series

$$S_n = \frac{t_1\left(r^n - 1\right)}{r - 1} \qquad S_n = \frac{rt_n - t_1}{r - 1}$$

$r \neq 1$

Ex.) Determine the sum of the first 11 terms of each geometric series:

$n = 11$

a) 4 + 12 + 36 + ...

$t_1 = 4$

$r = 3$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_{11} = \frac{4(3^{11} - 1)}{3 - 1} = \boxed{354292}$$

b) $t_1 = 5$, $r = -1/2$

$n = 11$

$$S_{11} = \frac{5\left(\left(-\tfrac{1}{2}\right)^{11} - 1\right)}{\left(-\tfrac{1}{2} - 1\right)} = \frac{\left(\frac{-10245}{2048}\right)}{\left(-\tfrac{3}{2}\right)}$$

$$= \boxed{\frac{3415}{1024}}$$

Ex.) Determine the sum of each geometric series:

a) $\frac{1}{27}+\frac{1}{9}+\frac{1}{3}+\ ...+729$

$r=(1/9)\div(1/27)$

$r=3$

$t_1=1/27$

$t_n=729$

$S_n=\dfrac{rt_n-t_1}{r-1}$

$S_n=\dfrac{3(729)-(1/27)}{2}=\boxed{1093.5}$

b) 4 - 16 + 64 - ... - 65 536

$r=-4$

$t_1=4$

$t_n=-65536$

$S_n=\dfrac{(-4\cdot -65536)-4}{-4-1}$

$=\boxed{-52428}$

Infinite Geometric Series - a geometric series the never ends (ie. continues indefinitely)

ex. 1 + 1/3 + 1/9 + 1/27 + ...

$1+1/3=1.333333\ldots$

$1+1/3+1/9=1.44444\ldots$

$1+1/3+1/9+1/27=1.481481\ldots$

$1+1/3+1/9+1/27+1/81=1.49382716\ldots$

$\vdots$

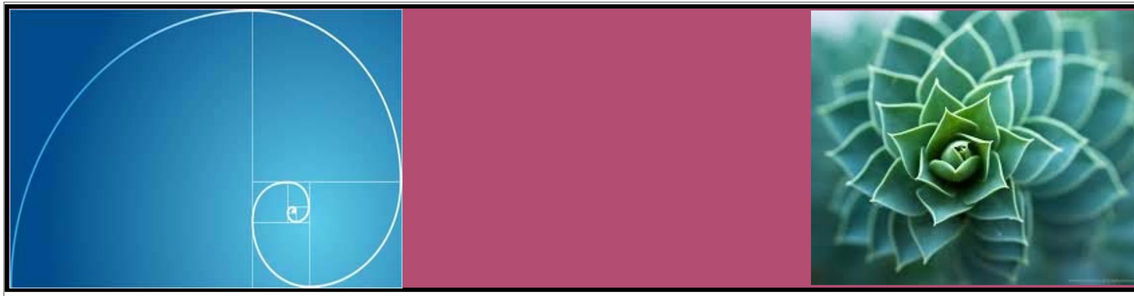
Convergent Series 1.5

$S_\infty=\dfrac{t_1}{1-r}=\dfrac{1}{1-(1/3)}=\dfrac{1}{\frac{3}{3}-\frac{1}{3}}\quad\dfrac{1}{\frac{2}{3}}$

$\frac{1}{1}\div\frac{2}{3}$

$\frac{1}{1}\times\frac{3}{2}=\frac{3}{2}$

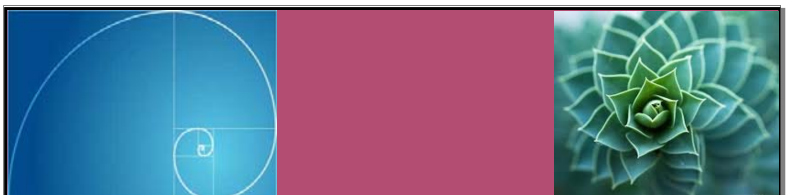
$=1.5$



Convergent Series

* terms get smaller and smaller
* sum reaches a limit that we say is the sum of the series

$$\boxed{S_\infty = \frac{t_1}{1-r}}$$



Divergent Series

* terms get larger and larger
* cannot determine its sum

Ex.) 1 + 3 + 9 + 27 + ...

$1+3=4$

$1+3+9=13$

$1+3+9+27=40$

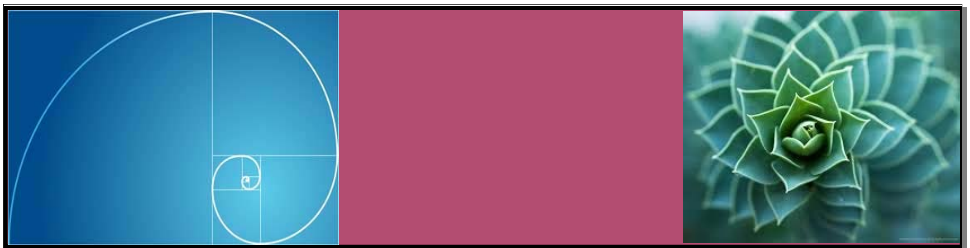
$1+3+9+27+81=121$

$\vdots$

∞

$\therefore$ Divergent Series

no sum.



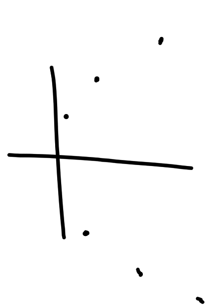
Ex.) Decide whether each infinite geometric series is convergent of divergent. State the sum of the series, if it exists.

a) 1 - 1/3 + 1/9 - ...

$r = -1/3$ $S_\infty = \frac{t_1}{1-r} = \frac{1}{1-(-1/3)} = \boxed{3/4}$

$t_1 = 1$

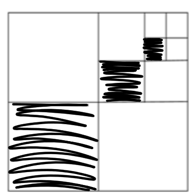
b) 2 - 4 + 8 - ...



$r = -2$ Divergent

$t_1 = 2$ ∴ no sum

Ex.) In the diagram below, assume that each shaded square represents 1/4 of the area of the larger square bodering two of its adjacent sides and that the shading continues indefinitelv in the indicated manner.



a) Write the series of terms that would represent this situation.

$1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$

b) How much of the total area of the largest square is shaded?

Convergent $\Rightarrow S_\infty = \frac{t_1}{1-r} = \frac{1}{1-(\frac{1}{4})} = \boxed{4/3}$

Pg. 53 #1, 2, 3ac, 4ac, 5, 9, 10.

Pg. 63 # 1-2, 4-10.