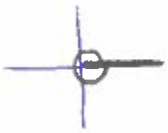
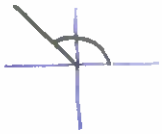
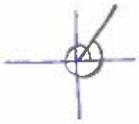
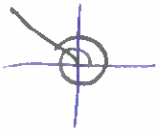
Math 30-1
Assignment 1: 4.1-4.3

$\frac{\pi}{180^\circ}$ $\frac{180^\circ}{\pi}$

Name: Key.

Answers vary

1. Angles in Degrees and Radians

Angle (in degrees and radians)	Sketch	Principle Angle	Reference Angle	Co-terminal Angles (one positive and one negative)
0° 0		0°, 0	0°	-360°, 360°
270° $\frac{3\pi}{2}$		270°, $\frac{3\pi}{2}$	90°	-90°, 630°
$\frac{\pi}{2}$ 90°		90°, $\pi/2$	$\frac{\pi}{2}$	$-\frac{3\pi}{2}$, $\frac{5\pi}{2}$
2π 360°		360°, 2π	0	0, 4π
$\frac{2\pi}{3}$ 120°		120°, $\frac{2\pi}{3}$	$\frac{\pi}{3}$	$-\frac{4\pi}{3}$, $\frac{8\pi}{3}$
$\frac{5\pi}{3}$ 300°		300°, $\frac{5\pi}{3}$	$\frac{\pi}{3}$	$-\frac{\pi}{3}$, $\frac{11\pi}{3}$
420° $\frac{7\pi}{3}$		60°, $\pi/3$	60°	60°, 780°, -300°
$-\frac{2\pi}{3}$ -120°		240°, $\frac{4\pi}{3}$	$\frac{\pi}{3}$	$-\frac{8\pi}{3}$, $\frac{4\pi}{3}$
210° $\frac{7\pi}{6}$		210°, $\frac{7\pi}{6}$	30°	-150°, 570°
$\frac{11\pi}{4}$ 495°		135°, $\frac{3\pi}{4}$	$\frac{\pi}{4}$	$-\frac{5\pi}{4}$, $\frac{3\pi}{4}$

Check with calculator

Using the unit circle worksheet:

2. Find the exact value of:

a) $\cos 120^0 = -\frac{1}{2}$

$\frac{2\pi}{3}$

b) $\tan 300^0 = -\sqrt{3}$

$\frac{\sin 300^\circ}{\cos 300^\circ} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1}$

c) $\sin 135^0 = \frac{\sqrt{2}}{2}$

d) $\sin(-30)^0 = -\frac{1}{2}$

e) $\cos^2 225^0 = \frac{1}{2}$

$= (\cos 225^\circ)^2$

$= \left(-\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4}$

f) $\tan 480^0 = -\sqrt{3}$

$\frac{\sqrt{3}}{2} \cdot \frac{2}{-1} = -\sqrt{3}$

g) $\sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

h) $\tan \frac{7\pi}{6} = \frac{\sqrt{3}}{3}$

$-\frac{1}{2} \cdot \frac{2}{-\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

i) $\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$

-120°

j) $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

k) $\tan^2 \frac{2\pi}{3} = 3$

$= \left(\frac{\sqrt{3}}{2} \cdot \frac{2}{-1}\right)^2 = \frac{3}{1}$

l) $\cos\left(-\frac{5\pi}{3}\right) = \frac{1}{2}$

-300°

m) $\sec 300^0 = 2$

$\frac{1}{\cos 300^\circ} = \frac{1}{1/2}$

n) $\cot \frac{5\pi}{6} = -\sqrt{3}$

$\frac{\cos \frac{5\pi}{6}}{\sin \frac{5\pi}{6}} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1}$

o) $\csc\left(-\frac{5\pi}{3}\right) = \frac{2}{\sqrt{3}}$

$\frac{1}{\sin -\frac{5\pi}{3}} = \frac{1}{\sqrt{3}/2}$

p) $\cot 930^0 = \sqrt{3}$

$\frac{\cos 210^\circ}{\sin 210^\circ} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{-1} = \sqrt{3}$

q) $\sec \frac{3\pi}{2}$ undefined

$= \frac{1}{\cos \frac{3\pi}{2}} = \frac{1}{0}$

r) $\csc 5\pi$ undefined

$\frac{1}{\sin \pi} = \frac{1}{0}$

s) $\sin 90^0 = 1$

t) $\csc\left(\frac{\pi}{6}\right) = 2$

$\frac{1}{\sin \pi/6} = \frac{1}{1/2}$

u) $\cot 540^0$ undefined

$\frac{\cos 180^\circ}{\sin 180^\circ} = \frac{-1}{0}$

v) $\cot\left(\frac{7\pi}{4}\right) = -1$

$= \frac{\cos \frac{7\pi}{4}}{\sin \frac{7\pi}{4}} = \frac{\sqrt{2}}{2} \cdot \frac{2}{-\sqrt{2}}$

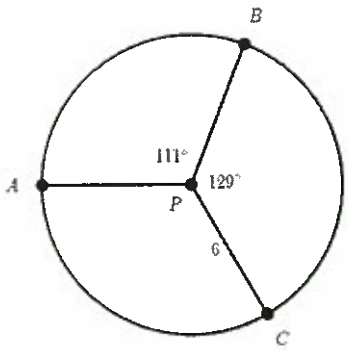
w) $\sec 150^0 = -\frac{2}{\sqrt{3}}$

$\frac{1}{\cos 150^\circ} = \frac{1}{-\sqrt{3}/2}$

x) $\csc(-330^0) = 2$

$\frac{1}{\sin -330^\circ} = \frac{1}{1/2}$

3. In the figure below, the radius of circle P is 6 units.



What is the length of $\overset{\frown}{ABC}$

$\theta = 111° + 129° = 240° \times \frac{\pi}{180°} = \frac{4\pi}{3}$

$\theta = \frac{a}{r}$

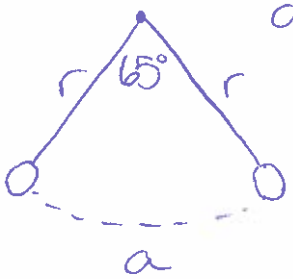
$a = \theta \cdot r = \left(\frac{4\pi}{3}\right)\left(\frac{6}{1}\right) = \frac{24\pi}{3} = \boxed{8\pi \text{ units}}$

≈ 25.1 units

4. A pendulum swings back and forth creating an angle of 65°. What is the length of the pendulum string, to the nearest hundredth, if the bob traveled a distance of 4.5 in?

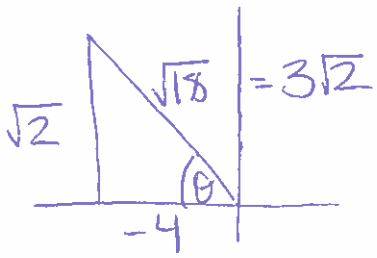
a $\quad r$

$\theta = 65° \times \frac{\pi}{180°} = \frac{13\pi}{36}$



$\theta = \frac{a}{r}$ $\quad r = \frac{a}{\theta} = \frac{4.5}{\left(\frac{13\pi}{36}\right)} = \boxed{3.97 \text{ in}}$

5. Given the point $(-4, \sqrt{2})$ on the terminal arm, determine the exact value of the six trigonometric ratios of θ. Rationalize any denominators if necessary.



$\sin\theta = \frac{\sqrt{2}}{3\sqrt{2}} = \frac{1}{3}$ $\quad \csc\theta = 3$

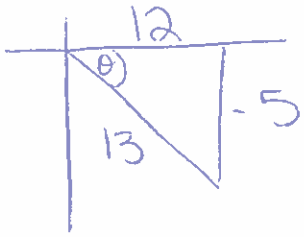
$\cos\theta = \frac{-4}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-4\sqrt{2}}{6} = \frac{-2\sqrt{2}}{3}$ $\quad \sec\theta = \frac{3\sqrt{2}}{-4}$

$\tan\theta = \frac{\sqrt{2}}{-4}$ $\quad \cot\theta = \frac{-4}{\sqrt{2}} = \frac{-4\sqrt{2}}{2}$

$= -2\sqrt{2}$

6. Find the exact values of the other five trigonometric ratios for an angle, θ, in standard position, given the following:

a) $\sin\theta = \frac{-5}{13}$, $\frac{3\pi}{2} < \theta < 2\pi$ Q IV



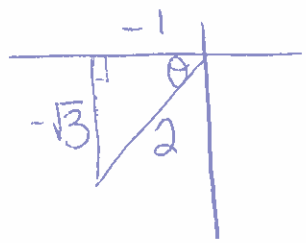
$\csc\theta = -\frac{13}{5}$

$\cos\theta = \frac{12}{13}$ $\sec\theta = \frac{13}{12}$

$\tan\theta = -\frac{5}{12}$ $\cot\theta = -\frac{12}{5}$

b) $\sec\theta = \frac{-2}{1}$, $\pi < \theta < 2\pi$ Q III or IV

$\cos\theta = -\frac{1}{2}$



$\sin\theta = -\frac{\sqrt{3}}{2}$ $\csc\theta = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\cos\theta = -\frac{1}{2}$ $\sec\theta = -2$

$\tan\theta = \sqrt{3}$ $\cot\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

c) $\sin\theta = \frac{2}{3}$, $\tan\theta < 0$ Q II

$\sin\theta = \frac{2}{3}$ $\csc\theta = \frac{3}{2}$

$\cos\theta = -\frac{\sqrt{5}}{3}$ $\sec\theta = -\frac{3}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$

$\tan\theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$ $\cot\theta = -\frac{\sqrt{5}}{2}$

7. Prove or disprove: Does the point $(-3/5, 4/5)$ lie on the unit circle?

$x^2 + y^2 = 1$	
$(-3/5)^2 + (4/5)^2$	1
$\frac{9}{25} + \frac{16}{25}$	
$\frac{25}{25}$	
1	1

LS = RS

$\therefore (-3/5, 4/5)$ exists on the Unit Circle.