Name Key

Trigonometry2 Assignment #3

1. Verify the following identities for the given value of the variable:

a) $\cot x = \dfrac{\cos x}{\sin x}$ for $x = \dfrac{\pi}{3}$

$$\frac{1}{\tan(\pi/3)} \qquad \frac{\cos(\pi/3)}{\sin(\pi/3)}$$

$$\frac{1}{\frac{\sqrt{3}}{2}\cdot\frac{2}{1}} \qquad \frac{1/2}{\sqrt{3}/2}$$

$$\frac{1}{\sqrt{3}} \qquad \frac{1}{2}\cdot\frac{2}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} \checkmark$$

b) $\sin^2 x + \cos^2 x = 1$ for $x = \dfrac{\pi}{4}$

$$\left[\sin\left(\tfrac{\pi}{4}\right)\right]^2 + \left[\cos\left(\tfrac{\pi}{4}\right)\right]^2 \qquad 1$$

$$\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2$$

$$\frac{2}{4} + \frac{2}{4}$$

$$\frac{4}{4}$$

$$1 \checkmark$$

2. Verify the identity $1 + \cot^2 x = \csc^2 x$ for the given values:

a) $x = \dfrac{\pi}{6}$

$$1 + \left(\frac{\cos \pi/6}{\sin \pi/6}\right)^2 = \left(\frac{1}{\sin \pi/6}\right)^2$$

$$1 + \left(\frac{\sqrt{3}/2}{1/2}\right)^2 = \left(\frac{1}{1/2}\right)^2$$

$$1 + \sqrt{3}^2 = 2^2$$

$$1 + 3 = 4$$

$$4 = 4$$

$\checkmark$

b) $x = \dfrac{4\pi}{3}$

$$1 + \left(\frac{\cos \frac{4\pi}{3}}{\sin \frac{4\pi}{3}}\right)^2 = \left(\frac{1}{\sin \frac{4\pi}{3}}\right)^2$$

$$1 + \left(\frac{(-1/2)}{(-\sqrt{3}/2)}\right)^2 = \left(\frac{1}{(-\sqrt{3}/2)}\right)^2$$

$$1 + \left(\frac{1}{\sqrt{3}}\right)^2 = \left(\frac{-2}{\sqrt{3}}\right)^2$$

$$1 + \frac{1}{3} = \frac{4}{3}$$

$$\frac{3}{3} + \frac{1}{3} = \frac{4}{3} \checkmark$$

3. Factor to write each in a simpler form.

a) $\sec x \sin^2 x - \sec x$

$$= \sec x(\sin^2 x - 1)$$

$$= \frac{1}{\cos x}(-\cos^2 x)$$

$$= \boxed{-\cos x}$$

b) $\sin^4 \theta - \cos^4 \theta$

$$a^4 - b^4$$

$$(a^2+b^2)(a^2-b^2)$$

$$(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$$

$$1\,(-1(-\sin^2\theta + \cos^2\theta))$$

$$-(\cos^2\theta - \sin^2\theta)$$

$$\boxed{-\cos 2\theta}$$

Answers can vary.

4. ~~In each of the following:~~

i) verify the possibility of an identity using a graphing calculator
ii) prove the identity using an algebraic approach
iii) state any restrictions.

a) $\frac{\tan\theta\cos\theta}{\sin\theta} = 1$ $\sin\theta \neq 0$ $\cos\theta \neq 0$

$\left(\frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{1}\right)$

$\sin\theta$

$\frac{\sin\theta}{\sin\theta}$

1

1 #

b) $\sec^2 x - \sin^2 x = \cos^2 x + \tan^2 x$ $\cos x \neq 0$

$\frac{1}{\cos^2 x} - \frac{\sin^2 x}{1}$

$\frac{1 - \sin^2 x\cos^2 x}{\cos^2 x}$

$\frac{1 - (1-\cos^2 x)\cos^2 x}{\cos^2 x}$

$\frac{1 - \cos^2 x + \cos^4 x}{\cos^2 x}$

$\frac{1-\cos^2 x}{\cos^2 x} + \frac{\cos^4 x}{\cos^2 x}$

$\cos^2 x + \frac{\sin^2 x}{\cos^2 x}$

$\cos^2 x + \frac{1-\cos^2 x}{\cos^2 x}$ #

OR $1+\tan^2\theta - (1-\cos^2\theta)$

$\cancel{1}+\tan^2\theta - \cancel{1}+\cos^2\theta$

$\tan^2 x + \cos^2 x$

5. Prove the following identities using an algebraic approach.

a) $(1-\cos^2 x)(\csc x) = \sin x$

$\frac{\sin^2 x}{1} \cdot \frac{1}{\sin x}$

$\sin x$ #

b) $(\sin x + \cos x)^2 = 1 + 2\sin x\cos x$

$\sin^2 x + 2\sin x\cos x + \cos^2 x$

$1 + 2\sin x\cos x$ #

c) $\frac{1-\cos x}{\sin x} = \frac{\tan x - \sin x}{\tan x \sin x}$.

$\frac{1}{\sin x} - \frac{\cos x}{\sin x}$

$\frac{\tan x}{\tan x\sin x} - \frac{\sin x}{\tan x\sin x}$

$\frac{1}{\sin x} - \frac{1}{\tan x}$

$\frac{1}{\sin x} - \frac{\cos x}{\sin x}$ #

d) $\frac{2}{1-\sin x}(1-\sin x) + \frac{2}{1+\sin x}(1+\sin x) = 4\sec^2 x$

$\frac{2 - 2\sin x + 2 + 2\sin x}{1-\sin^2 x}$

$\frac{4}{\cos^2 x}$

$\frac{4}{\cos^2 x}$ #

e) $\frac{1+\cos x}{\tan x + \sin x} = \cot x$

$\frac{1+\cos x}{\frac{\sin x}{\cos x} + \frac{\sin x\cos x}{\cos x}}$

$\frac{1+\cos x}{\frac{\sin x(1+\cos x)}{\cos x}}$

$(1+\cancel{\cos x}) \cdot \frac{\cos x}{\sin x}$

$\frac{\cos x}{\sin x}$ #

f) $\sec x - \cos x = \frac{\sin x}{\cot x}$

$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$

$\frac{1-\cos^2 x}{\cos x}$

$\frac{\sin^2 x}{\cos x}$

$\sin x \div \frac{\cos x}{\sin x}$

$\frac{\sin^2 x}{\cos x}$ #

In questions #7 - #11 assume the appropriate restrictions.

7. $\dfrac{\cos x}{1-\sin^2 x}$ is equal to $= \dfrac{\cos x}{\cos^2 x}$ $= \dfrac{1}{\cos x}$ $= \sec x$

A. $\sec x$

B. $\csc x$

C. $\sin x$

D. $\tan x$

8. $\dfrac{\tan^2 x + 1}{\sec x}$ is equal to $= \dfrac{\sec^2 x}{\sec x}$ $= \sec x$

A. $\sec x$

B. $\csc x$

C. $\sin x$

D. $\tan x$

9. $\dfrac{\csc x}{\cot x}$ is equal to $= \dfrac{1}{\sin x} \div \dfrac{\cos x}{\sin x}$ $= \dfrac{1}{\cos x}$ $= \sec x$

A. $\cos x$

B. $\sin x$

C. $\sec x$

D. $\tan x$

10. The expression $\dfrac{\tan A \cos^2 A}{\sec A}$, expressed in terms of $\sin A$ is

$= \dfrac{\dfrac{\sin A}{\cos A} \cdot \cos^2 A}{\dfrac{1}{\cos A}}$

$= \sin A \cos A \div \dfrac{1}{\cos A}$

$= \sin A \cos^2 A$

$= \sin A(1-\sin^2 A)$

$= \sin A - \sin^3 A$

A. $\dfrac{\sin A}{1-\sin^2 A}$

B. $\dfrac{1-\sin^2 A}{\sin A}$

C. $\sin^2 A$

D. $\sin A - \sin^3 A$

11. $\sec x - \cos x$ is equal to $= \dfrac{1}{\cos x} - \dfrac{\cos^2 x}{\cos x}$ $= \dfrac{1-\cos^2 x}{\cos x}$ $= \dfrac{\sin^2 x}{\cos x}$ $= \sin x \tan x$

A. $\dfrac{1-\cos x}{\cos x}$

B. $\sin^2 x$

C. $\dfrac{1-2\cos x}{\cos x}$

D. $\sin x \tan x$

12. Which of the following in NOT an identity?

A. $\cos^2 x + \sin^2 x = 1$

B. $\sin x + \cos x = 1$

C. $\sec^2 x - \tan^2 x = 1$

D. $\tan x \cot x = 1$

13. If $\tan x \neq 0$, $\cos x \neq 0$, $\cot x \neq 0$ then

$$\frac{1}{\tan x \cos x \cot x}$$ is equal to

$$= \frac{1}{\frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} \cdot \frac{\cos x}{\sin x}}$$

$$= \frac{1}{\cos x}$$

A. $\frac{1}{\sin x}$

B. $\sin x$

C. $\frac{1}{\cos x}$

D. $\cos x$

14. If $\sin x \neq 0$, $\cos x \neq 0$ then

$$\frac{\tan x \cos x}{3 \sec x \cot x}$$ is equal to

$$\frac{\frac{\sin x}{\cos x} \cdot \frac{\cos x}{1}}{3 \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x}} = \frac{\sin x}{3/\sin x}$$

$$= \sin x \div \frac{3}{\sin x} = \frac{\sin^2 x}{3}$$

A. $\frac{1}{3}$

B. 3

C. $\frac{1}{3}\sin^2 x$

D. $\frac{1}{3}\csc^2 x$

15. When verifying the identity $\cot^2 x + 1 = \csc^2 x$ for $x = \frac{\pi}{7}$, the value on each side of the identity, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right)

5	.	3	

$$\cot^2 x + 1 = \csc^2 x$$

$\frac{\cos^2(\pi/7)}{\sin^2(\pi/7)} + 1$	$\frac{1}{\sin^2(\pi/7)}$
5.3	5.3

NAME_____

TRIGONOMETERY 2 ASSIGNMENT # 4

Prove the following identities.

1. $\cos x = \sin x \cot x$

$\cos x$	$\frac{\cancel{\sin x}\cos x}{\cancel{\sin x}}$
	$\cos x$
	#

2. $\frac{\tan\theta\cos\theta}{\sin\theta} = 1$

$\frac{\sin\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta}$	1
$\sin\theta$	
$\frac{\sin\theta}{\sin\theta}$	
1	#

3. $\sec\theta - \cos\theta = \sin\theta\tan\theta$

$\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}$	$\sin\theta\frac{\sin\theta}{\cos\theta}$
$\frac{\sin^2\theta}{\cos\theta}$	$\frac{\sin^2\theta}{\cos\theta}$
	#

4. $\sec A - \cos A = \sin A\tan A$

$\frac{1}{\cos A} - \frac{\cos^2 A}{\cos A}$	$\sin A\frac{\sin A}{\cos A}$
$\frac{\sin^2 A}{\cos A}$	$\frac{\sin^2 A}{\cos A}$
	#

5. $(\sin r + \cos r)^2 = 1 + 2\sin r\cos r$

$\sin^2 r + 2\sin r\cos r + \cos^2 r$	$1 + 2\sin r\cos r$
$1 + 2\sin r\cos r$	
	#

6. $\frac{\cos\theta - \sin\theta}{\cos\theta} = 1 - \tan\theta$

$\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$	$1 - \tan\theta$
$1 - \tan\theta$	#

7. $\sec^2\theta - \sin^2\theta = \cos^2\theta + \tan^2\theta$

$1 + \tan^2\theta - \sin^2\theta$

$\cos^2\theta + \tan^2\theta$

#

8. $\dfrac{\cot x - 1}{\tan x - 1} = -\cot x$

$\left(\dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\sin x}\right) \div \left(\dfrac{\sin x}{\cos x} - \dfrac{\cos x}{\cos x}\right)$

$-1\dfrac{\cancel{(-\cos x + \sin x)}}{\sin x} \cdot \dfrac{\cos x}{\cancel{(\sin x - \cos x)}}$

$-\dfrac{\cos x}{\sin x}$

$-\cot x$

#

9. $\dfrac{1-\cos A}{\sin A} = \dfrac{\sin A}{1+\cos A}$

$\dfrac{\sin A(1-\cos A)}{(1+\cos A)(1-\cos A)}$

$\dfrac{\sin A(1-\cos A)}{1-\cos^2 A}$

$\dfrac{\cancel{\sin A}(1-\cos A)}{\sin^{\cancel{2}} A}$

$\dfrac{1-\cos A}{\sin A}$ #

10. $(1-\cos^2\alpha)(\csc\alpha) = \sin\alpha$

$\sin^{\cancel{2}}\alpha \cdot \dfrac{1}{\cancel{\sin\alpha}}$

$\sin\alpha$

#

11. $\tan\theta\cos\theta = \sin\theta$

$\dfrac{\sin\theta\cancel{\cos\theta}}{\cancel{\cos\theta}}$

$\sin\theta$

#

12. $\sin\theta\cot\theta = \cos\theta$

$\cancel{\sin\theta} \cdot \dfrac{\cos\theta}{\cancel{\sin\theta}}$

$\cos\theta$

#

13. $\csc\theta(1+\sin\theta) = 1+\csc\theta$

$\csc\theta + \sin\theta\csc\theta$

$\csc\theta + \sin\theta\left(\dfrac{1}{\cancel{\sin\theta}}\right)$

$\csc\theta + 1$

$1 + \csc\theta$ #

14. $\cos\theta(\sec\theta - 1) = 1 - \cos\theta$

$\cos\theta\sec\theta - \cos\theta$

$\cancel{\cos\theta}\left(\dfrac{1}{\cancel{\cos\theta}}\right) - \cos\theta$

$1 - \cos\theta$

#

15. $\sin\theta\tan\theta + \sec\theta = \dfrac{\sin^2\theta + 1}{\cos\theta}$

$\dfrac{\sin\theta \cdot \sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta}$

$\dfrac{\sin^2\theta}{\cos\theta} + \dfrac{1}{\cos\theta}$

$\dfrac{\sin^2\theta + 1}{\cos\theta}$ #

16. $\dfrac{1+\sin\theta}{1-\sin\theta} = \dfrac{\csc\theta + 1}{\csc\theta - 1}$

$\dfrac{1}{\sin\theta} + \dfrac{\sin\theta}{\sin\theta}$

$\dfrac{1}{\sin\theta} - \dfrac{\sin\theta}{\sin\theta}$

$\dfrac{1+\sin\theta}{\cancel{\sin\theta}} \cdot \dfrac{\cancel{\sin\theta}}{1-\sin\theta}$

$\dfrac{1+\sin\theta}{1-\sin\theta}$ #

17. $\dfrac{\sin\theta + \tan\theta}{\cos\theta + 1} = \tan\theta$

$\dfrac{\sin\theta\cos\theta}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$ | $\dfrac{\sin\theta}{\cos\theta}$

$\cos\theta + 1$

$\dfrac{\sin\theta(\cancel{\cos\theta+1})}{\cos\theta} \div (\cancel{\cos\theta+1})$

$\dfrac{\sin\theta}{\cos\theta}$ #

18. $\sin^2\theta\cot^2\theta = 1 - \sin^2\theta$

$\dfrac{\cancel{\sin^2\theta} \cdot \cos^2\theta}{\cancel{\sin^2\theta}}$ | $\cos^2\theta$

$\cos^2\theta$

#

19. $\sin^2\theta = \dfrac{\tan^2\theta}{1+\tan^2\theta}$

$\dfrac{\tan^2\theta}{\sec^2\theta}$

$\dfrac{\sin^2\theta}{\cos^2\theta} \div \dfrac{1}{\cos^2\theta}$

$\sin^2\theta$

#