Unit 1: Kinematics Review

The Kinematics Equations

1. $\vec{v}_i = +69\ m/s$, $\vec{v}_f = +6.1\ m/s$, $\vec{d} = +750\ m$, $\vec{a} = ?$

$$\vec{a} = \frac{\vec{v}_f^2 - \vec{v}_i^2}{2\vec{d}} = \frac{6.1^2 - 69^2}{(2 \cdot 750)} = \boxed{-3.1\ m/s^2}$$

2. $\vec{v}_i = +33\ m/s$, $\vec{v}_f = 0\ m/s$, $\vec{a} = -11\ m/s^2$, $\vec{d} = ?$

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\vec{d}$$
$$0^2 = 33^2 + 2(-11)\vec{d}$$
$$-1089 = -22\vec{d}$$
$$\boxed{\vec{d} = 50\ m\ [E]}$$

3. $\vec{v}_i = +35.0\ m/s$, $t = 2.00\ s$, $\vec{a} = -9.81\ m/s^2$, $\vec{v}_f = ?$

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$
$$= 35.0 + (-9.81)(2.00)$$
$$= \boxed{15.4\ m/s\ [up]}$$

4. $\vec{v}_i = +15\ m/s$, $\vec{v}_f = 0\ m/s$ (top of flight, half the time), $\vec{a} = -9.81\ m/s^2$, $t = ?$

$$t = \frac{\vec{v}_f - \vec{v}_i}{\vec{a}} = \frac{0 - 15}{-9.81} = 1.52\ldots$$

↑ times by 2 for total time

$$t_{total} = 2t = \boxed{3.1\ s}$$

5. $\vec{v}_i = +5.0\ m/s$, $\vec{v}_f = 0\ m/s$, $\vec{a} = -9.81\ m/s^2$, $\vec{d} = ?$

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\vec{d}$$
$$0^2 = 5.0^2 + 2(-9.81)\vec{d}$$
$$-25 = -19.62\vec{d}$$
$$\boxed{\vec{d} = 1.3\ m\ [up]}$$

Hilbert

6. $\vec{v}_f = -72.0\ m/s$
$\vec{a} = -9.81\ m/s^2$
$t = 23\ s$
$\vec{v}_i = ?$

$\vec{v}_f = \vec{v}_i + \vec{a}t$
$-72.0 = \vec{v}_i + (-9.81)(23)$

$\boxed{\vec{v}_i = 154\ m/s\ [up]}$

7. $\vec{a} = -9.81\ m/s^2$
$\vec{d} = 110\ m$
$\vec{v}_f = 0\ m/s$
$\vec{v}_i = ?$

$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\vec{d}$
$0^2 = \vec{v}_i^2 + 2(-9.81)(110)$
$\sqrt{2158.2} = \sqrt{\vec{v}_i^2}$

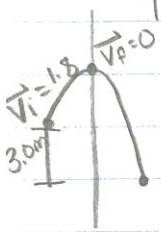
$\boxed{\vec{v}_i = 46\ m/s\ [up]}$

8. $\vec{v}_i = ?$
$\vec{v}_f = 0$
$t = 4\ s$ (half)
$\vec{a} = -9.81\ m/s^2$

$\vec{v}_f = \vec{v}_i + \vec{a}t$
$0 = \vec{v}_i + (-9.81)(4)$

$\boxed{\vec{v}_i = 39.2\ m/s}$

9. $\vec{v}_i = +1.8\ m/s$
$\vec{a} = -9.81\ m/s^2$
$\vec{v}_f = 0\ m/s$
$\vec{d} = ?$



(b) $\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\vec{d}$
$0^2 = 1.8^2 + 2(-9.81)\vec{d}$
$-3.24 = -19.62\vec{d}$
$\vec{d} = 0.17\ m$

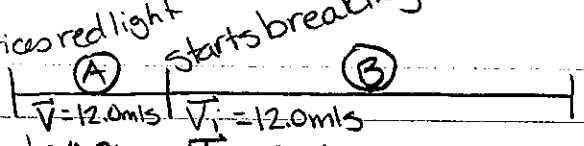
$\vec{d}_{up} = 0.17\ m + 3.0\ m = \boxed{3.2\ m\ [up]}$

(a) $\vec{v}_i = 0\ m/s$
$\vec{a} = -9.81\ m/s^2$
$\vec{d} = 3.2\ m$
$\vec{v}_f = ?$

$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\vec{d}$
$\vec{v}_f^2 = 0^2 + 2(-9.81)(-3.2)$
$\sqrt{\vec{v}_f^2} = \sqrt{62.784}$

$\boxed{\vec{v}_f = 7.9\ m/s\ [down]}$

10.



Ⓐ Uniform Motion: $\vec{d} = \vec{v}t = (12.0)(0.510) = \underline{6.12\ m}$

Ⓑ Decceleration: $\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\vec{d}$

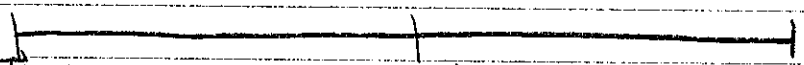
$0^2 = 12.0^2 + 2(-6.20)\vec{d}$

$-144 = -12.4\vec{d}$

$\vec{d} = 11.6\ldots m$

$\vec{d}_{total} = 6.12 + 11.6 = \boxed{17.7\ m}$

11.



$\vec{v}_i = 0$ m/s
$\vec{d} = 402$ m
$\vec{a} = +17.0$ m/s²
$\vec{v}_f = ?$

$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\vec{d}$

$v_f = \sqrt{0^2 + 2(17)(402)}$

$\vec{v}_f = 116.910\ldots$ m/s

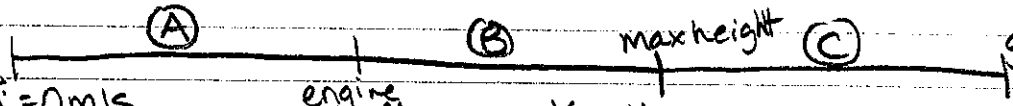
$\vec{a} = -6.10$ m/s²
$\vec{d} = 3.50 \times 10^2$ m
$\vec{v}_{f_1} = \vec{v}_i = 116.910\ldots$ m/s
$\vec{v}_{f_2} = ?$

$\vec{v}_f = \sqrt{\vec{v}_i^2 + 2\vec{a}\vec{d}}$

$\vec{v}_f = \sqrt{116.9^2 + 2(-6.10)(3.50\times10^2)}$

$\boxed{\vec{v}_f = 96.9\ \text{m/s [forward]}}$

12.



$\vec{v}_i = 0$ m/s
$\vec{a} = +20.0$ m/s²
$\vec{d} = 415$ m
$t = ?$
$\vec{v}_f = ?$

$\vec{v}_f = \vec{v}_i = 128.8$ m/s
$\vec{a} = -9.81$ m/s²
$t = ?$
$\vec{d} = ?$

$\vec{v}_f = 0$
$\vec{v}_i = 0$
$\vec{a} = -9.81$
$\vec{d} = 1261$
$t = ?$

$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$

$-1261 = 0t + \frac{1}{2}(-9.81)(t^2)$

$-1261 = -4.905t^2$

Ⓐ $\vec{v}_f = \sqrt{0^2 + 2(20.0)(415)}$

$\vec{v}_f = 128.8$ m/s

$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)t$

$415 = \left(\frac{128.8+0}{2}\right)t$

$t_A = 6.44$ s

$t = \frac{\vec{v}_f - \vec{v}_i}{\vec{a}}$

$= \frac{0 - 128.8}{-9.81}$

$t_B = 13.13$ s

$\vec{d} = \left(\frac{128.8+0}{2}\right)(13.13)$

$\vec{d}_B = 846$ m

$t_c = 16.03$ s

$t_{total} = t_A + t_B + t_c$

$t_{total} = \boxed{35.6\ s}$

13. $\vec{v}_i = 6.0\text{ m/s}$
$\vec{v}_f = 0$
$\vec{a} = -9.81\text{ m/s}^2$
$\vec{d} = ?$

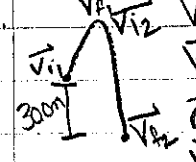
$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$

$t = \frac{0 - 6.0}{-9.81} = 0.611\ldots\text{ s}$

$t_{total} = 0.4 + 0.611\ldots\text{ s}$

$\vec{d} = \cancel{\vec{v}_f^{\,0} t} - \frac{1}{2}\vec{a}t^2$
$= -(\frac{1}{2})(-9.81)(1.011\ldots)^2$

$\boxed{\vec{d} = 5.0\text{ m [down]}}$

14. 

$\vec{v}_{i_1} = -20.0\text{ m/s}$
$\vec{v}_{f_1} = 0\text{ m/s}$
$\vec{a} = -9.81\text{ m/s}^2$
$\vec{v}_{i_2} = 0\text{ m/s}$
$\vec{v}_{f_2} = ?$
$t = ?$

a) $\vec{v}_{f_1}^{\,2} = \vec{v}_{i_1}^{\,2} + 2\vec{a}\vec{d}$
$0^2 = 20.0^2 + 2(-9.81)\vec{d}$
$\vec{d} = 20.387\ldots\text{ m}$
$+30\text{ m}$

$\vec{d}_{total} = 50.387\ldots\text{ m}$

$\vec{v}_{f_2}^{\,2} = \vec{v}_{i_2}^{\,2} + 2\vec{a}\vec{d}$
$\vec{v}_{f_2} = \sqrt{0^2 + 2(-9.81)(-50.387\ldots)}$

$\boxed{\vec{v}_{f_2} = 31.4\text{ m/s [down]}}$

b) $t = t_{up} + t_{down}$

$t_{up} = \frac{\vec{v}_f - \vec{v}_i}{\vec{a}} = \frac{0 - 20.0}{-9.81} = 2.038\ldots\text{ s}$

$t_{down} = \frac{\vec{v}_f - \vec{v}_i}{\vec{a}} = \frac{31.4 - 0}{-9.81} = 3.205\ldots\text{ s}$

$\boxed{t_{total} = 5.24\text{ s}}$

15. $\vec{v}_f = 0 m/s$
$\vec{a} = -9.81 m/s^2$
$t = \frac{5.30 s}{2} = 2.65 s$
$\vec{d} = ?$

$\vec{d} = \vec{v}_f t - \frac{1}{2}\vec{a}t^2$
$= 0 - (\frac{1}{2})(-9.81)(2.65)^2$

$\boxed{\vec{d} = 34.4 m}$

16. $\vec{v}_i = +5.0 m/s$
$\vec{v}_f = 0 m/s$
$t = 1.5 s \ (\frac{3.0 s}{2})$
$\vec{a} = ?$

$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t} = \frac{0 - 5.0}{1.5} = \boxed{-3.3 m/s^2}$

17. 2nd second: $1s - 2s$
Find $\vec{d}_2 - \vec{d}_1$.

$\vec{v}_i = +35.0 m/s$
$\vec{a} = -9.81 m/s^2$
$t = 2$

$\Delta d_2 = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$
$= (35.0)(2) + (\frac{1}{2})(-9.81)(2)^2$
$= 50.38 m$

$\Delta d_1 = (35.0)(1) + (\frac{1}{2})(-9.81)(1)^2$
$= 30.095 m$

a) $\vec{d}_2 - \vec{d}_1 = 50.38 - 30.095 = \boxed{20.3 m \text{ [up]}}$

b) 5th second: $4s - 5s$ Find $\vec{d}_5 - \vec{d}_4$

$\vec{d}_5 = (35.0)(5) + (\frac{1}{2})(-9.81)(5)^2$
$= 52.375 m$

$\vec{d}_4 = (35.0)(4) + (\frac{1}{2})(-9.81)(4)^2$
$= 61.52 m$

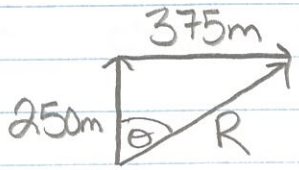
$\vec{d}_5 - \vec{d}_4 = 52.375 - 61.52 = -9.15 m$

$= \boxed{9.15 m \text{ [down]}}$

Vectors/Relative Motion

1. a) $400m + 275m + 150m + 650m = 1475m = \boxed{1.48 \times 10^3 m}$

b) $\vec{d}_x = -275 + {}^{+}650 = {}^{+}375\,m$
$d_y = +400 + -150 = +250m$



$R = \sqrt{250^2 + 375^2} = 451m$

$\tan\theta = \frac{375}{250} \quad \theta = 56°$

$\boxed{451m\ [56°\ E\ of\ N]}$

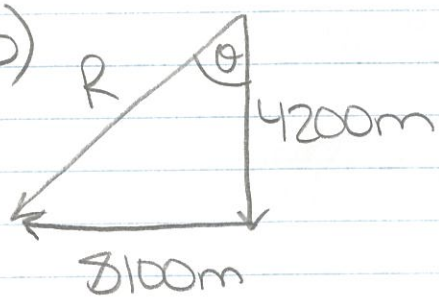
or $[34°\ N\ of\ E]$

c) $v = \frac{d}{t} = \frac{1475}{15} = \boxed{98.3\ m/min}$

2. a) $d_y = \frac{35m}{s} \times 120s = 4200m$ a) $v = \frac{d}{t} = \frac{12300m}{7min} = \boxed{1757\ m/min}$

$d_x = \frac{27m}{s} \times 300s = 8100m$

b)

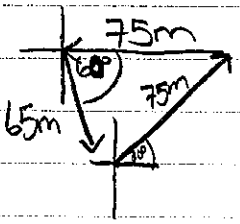


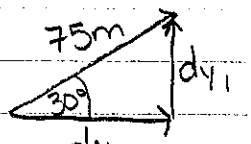
$R = \sqrt{4200^2 + 8100^2} = 9124m$

$\tan\theta = \frac{8100}{4200} \quad \theta = 63°$

$\boxed{9.1km\ [63°\ W\ of\ S]}$

3.

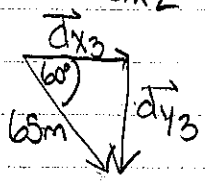




dx_1

$\leftarrow 75m$

dx_2



$\sin 30° = \frac{dy_1}{75}$ $dy_1 = +37.5m$

$\cos 30° = \frac{dx_1}{75}$ $dx_1 = +64.95m$

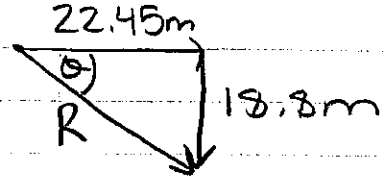
$dx_2 = -75m$

$\sin 60° = \frac{dy_3}{65}$ $dy_3 = -56.29m$

$\cos 60° = \frac{dx_3}{65}$ $dx_3 = +32.5m$

$\vec{dx}_{total} = +22.45$

$\vec{dy}_{total} = -18.8m$

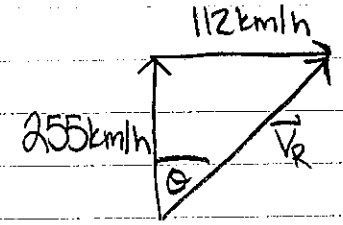


$R = \sqrt{18.8^2 + 22.45^2} = 29.3m$

$\tan\theta = \frac{18.8}{22.45}$ $\theta = 40°$

$\vec{d} = 29.3m\ [40°\ S\ of\ E]$

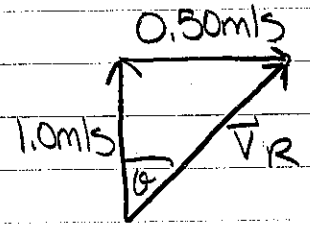
b) $v = \frac{d}{t} = \frac{(75+75+65)}{3min} = 71.7m/min$

4. 

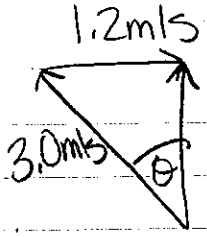
$\vec{V}_R = \sqrt{255^2 + 112^2} = 279\ km/h$

$\tan\theta = \frac{112}{255}$ $\theta = 24°$

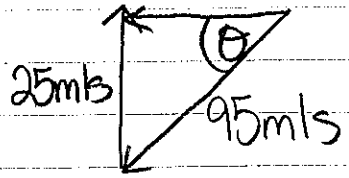
$\vec{V}_R =$ **279km/h [24° E of N]**

5. 

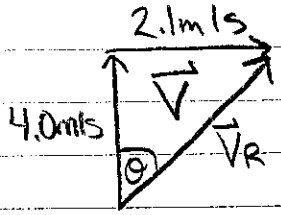
$\tan\theta = \frac{0.50}{1.0}$ $\theta =$ **27° E of N**

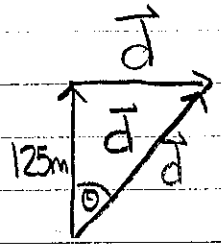
6. 

$\sin\theta = \frac{1.2}{3.0}$ $\theta =$ **24° W of N**

7. 

$\sin\theta = \frac{25}{95}$ $\theta =$ **15° S of W**

8. 



a) $\vec{V}_R = \sqrt{4.0^2 + 2.1^2}$
$= 4.5\ m/s$

$\tan\theta = \frac{2.1}{4.0}$ $\theta = 28°$

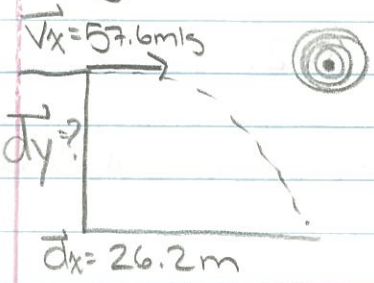
4.5m/s [28° E of N]

b) $t = \frac{\vec{d}}{\vec{V}}$
$= \frac{141}{4.5}$
$=$ **31s**

$\cos 28° = \frac{125}{\vec{d}}$
$\vec{d} = 141\ m$

c) $\tan 28° = \frac{\vec{d}}{125}$ **$\vec{d} = 66m$**

Projectile Motion

1. 

$\vec{d}_x = \vec{v}_x t$

$26.2 = 57.6t$

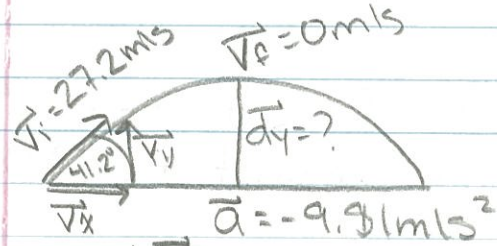
$t = 0.45$ s

$\vec{d}_y = \cancel{\vec{v}_{iy} t}^{\,0} + \frac{1}{2}\vec{a}_y t^2$

$= (\frac{1}{2})(-9.81)(0.45)^2$

$\vec{d}_y = -1.01$ m

$\vec{d}_y = 1.01$ m [down]

2. 

$\sin 41.2° = \frac{\vec{v}_{iy}}{27.2}$

$\vec{v}_{iy} = 17.9$ m/s

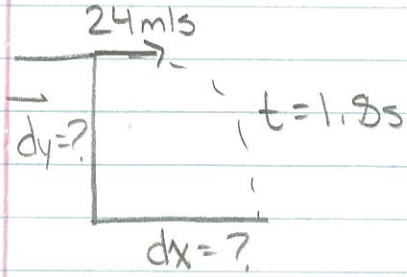
$\vec{v}_{fy} = 0$ m/s

$\vec{d}_y = ?$

$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{d}$

$0^2 = 17.9^2 + 2(-9.81)(\vec{d})$

$\vec{d}_y = 16.4$ m

3. 

$\vec{d}_x = \vec{v}_x t$

$= (24)(1.8)$

$= 43$ m

$\vec{d}_y = \cancel{\vec{v}_{iy} t}^{\,0} + \frac{1}{2} a t^2$

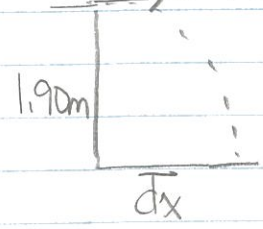
$\vec{d}_y = (\frac{1}{2})(-9.81)(1.8)^2$

$= -16$ m

$= 16$ m [down]

Hilroy

4. Pedro:



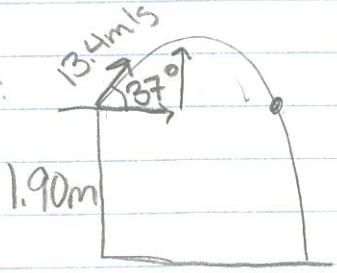
$\vec{d}_y = \frac{1}{2}\vec{a}t^2$

$-1.90 = (\frac{1}{2})(-9.81)t^2$

$t = 0.62$

$\vec{d}_x = \vec{v}_x t = (26.5)(0.62) = \boxed{16.5\text{m}}$

Juan:



$\Delta\vec{d}_y = \vec{v}_{iy}t + (\frac{1}{2})\vec{a}t^2$

$-1.90 = (8.06)t + (\frac{1}{2})(-9.81)t^2$

$0 = -4.905t^2 + 8.06t + 1.90$

(solve quadratic) graphing calc.

$t = 1.85\text{s}$

standard of excellence

$\sin 37° = \frac{\vec{v}_{iy}}{13.4}$

$\vec{v}_{iy} = 8.06\text{m/s}$

$\cos 37° = \frac{\vec{v}_x}{13.4}$

$\vec{v}_x = 10.7\text{m/s}$

$\vec{d}_x = \vec{v}_x t = (10.7)(1.85)$

$= \boxed{19.8\text{m}}$

Juan's goes farther by 3.3m.