

Math 30-1

Name: _____

Relation and Functions: Composite Functions

$$(f \circ g)(x) = f(\underline{g(x)})$$

1. Given $f(x) = x - 3$ $g(x) = x^2$, determine the following

a. $(f \circ g)(x)$

$$f(g(x))$$

$$f(x^2)$$

$$\boxed{x^2 - 3}$$

b. $(g \circ f)(x)$

$$g(f(x))$$

$$g(x-3)$$

$$(x-3)^2$$

$$(x-3)(x-3)$$

$$\boxed{x^2 - 6x + 9}$$

work from
the inside
out

2. Given $f(x) = x - 3$ $g(x) = x^2$, determine the following

a. $(f \circ g)(5)$

$$f(g(5))$$

$$f(25)$$

$$25 - 3 = \boxed{22}$$

b. $(g \circ f)(5)$

$$g(f(5))$$

$$g(2)$$

$$2^2 = \boxed{4}$$

3. Does order matter when performing a composition?

Yes!

4. Given $f(x) = x^2 - 3$ and $g(x) = \frac{1}{2}x$, determine the following

a. $(f \circ g)(x)$

$$f(g(x)) \\ f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^2 - 3 = \boxed{\frac{1}{4}x^2 - 3}$$

b. $(g \circ f)(x)$

$$g(f(x)) \\ = g(x^2 - 3) = \frac{1}{2}(x^2 - 3) = \boxed{\frac{1}{2}x^2 - \frac{3}{2}}$$

c. $(f \circ g)(5)$

$$f(g(5)) \\ = f\left(\frac{1}{2} \cdot 5\right) \\ = f\left(\frac{5}{2}\right) = \left(\frac{5}{2}\right)^2 - 3 = \frac{25}{4} - \frac{12}{4} = \boxed{\frac{13}{4}}$$

d. $(g \circ f)(5)$

$$g(f(5)) \\ = g(5^2 - 3) = g(22) = \frac{1}{2} \cdot 22 = \boxed{11}$$

e. $(g \circ g)(5)$

$$g(g(5)) \\ = g\left(\frac{5}{2}\right) = \frac{1}{2} \cdot \frac{5}{2} = \boxed{\frac{5}{4}}$$

f. $(f \circ f)(5)$

$$f(f(5)) \\ f(22) = 22^2 - 3 = \boxed{481}$$

5. Given the following functions, determine the following.

$$f(x) = \sqrt{x}$$

$$g(x) = \frac{1}{x}$$

$$m(x) = |x|$$

$$n(x) = x + 2$$

a. $h(x) = (m \circ n)(x)$
 $m(n(x))$
 $m(x+2) = |x+2|$

b. $h(3) = (g \circ f)(3)$
 $= g(\sqrt{3}) = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
 Rationalized Denominator

c. $h(x) = g(n(f(x)))$
 $= g(n(\sqrt{x}))$
 $= g(\sqrt{x} + 2) = \frac{1}{\sqrt{x} + 2} \cdot \frac{(\sqrt{x} - 2)}{(\sqrt{x} - 2)} = \frac{\sqrt{x} - 2}{x - 4}$

d. $h(9) = (g \circ n \circ f)(9)$
 $= g(n(f(9)))$
 $= g(n(3))$
 $= g(3+2) = g(5) = \frac{1}{5}$
 Restriction: $x \geq 0$

e. $h(x) = (f \circ n)(x)$
 $f(n(x))$
 $= f(x+2)$
 $= \sqrt{x+2}$
 Restriction: $x+2 \geq 0$
 $x \geq -2$

Pg. 507 #1, 2, 5, 6, 8.