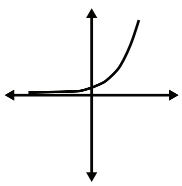
Mathematics 30-1

Student Workbook

Unit 3

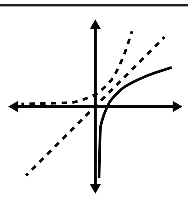


$y = b^x$

Lesson 1: Exponential Functions
Approximate Completion Time: 3 Days

$\log_B A = E$

Lesson 2: Laws of Logarithms
Approximate Completion Time: 4 Days



$y = \log_b x$

Lesson 3: Logarithmic Functions
Approximate Completion Time: 3 Days

UNIT THREE

Exponential and Logarithmic Functions

Mathematics 30-1

Student Workbook

Unit
3

Complete this workbook by watching the videos on **www.math30.ca**.
Work neatly and use proper mathematical form in your notes.

UNIT THREE
Exponential and Logarithmic Functions

$y = b^x$

Exponential and Logarithmic Functions
LESSON ONE - *Exponential Functions*
Lesson Notes

Example 1
Exponential Functions

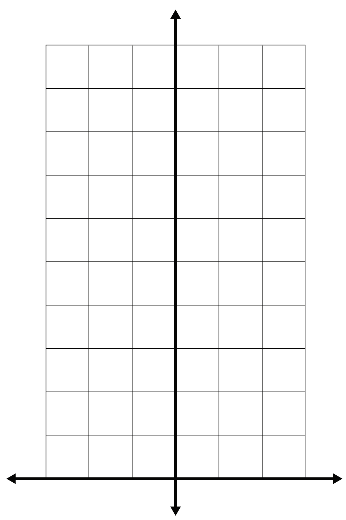
Graphing Exponential Functions

For each exponential function:

i) Complete the table of values and draw the graph.
ii) State the domain, range, intercepts, and the equation of the asymptote.

a) $y = 2^x$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



Domain:

Range:

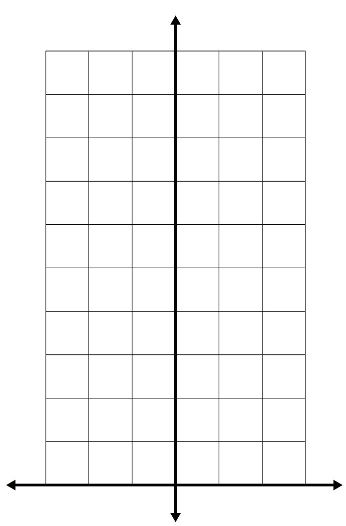
x-intercept:

y-intercept:

Asymptote:

b) $y = 3^x$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



Domain:

Range:

x-intercept:

y-intercept:

Asymptote:

Set-Builder Notation

A ***set*** is simply a collection of numbers, such as {1, 4, 5}. We use ***set-builder notation*** to outline the rules governing members of a set.

$\{x \mid x \in R, x \geq -1\}$

State the variable.

List conditions on the variable.

In words: "The variable is x, such that x can be any real number with the condition that x ≥ -1".

As a shortcut, set-builder notation can be reduced to just the most important condition.

$x \geq -1$

While this resource uses the shortcut for brevity, as set-builder notation is covered in previous courses, Math 30-1 students *are* expected to know how to read and write full set-builder notation.

Interval Notation

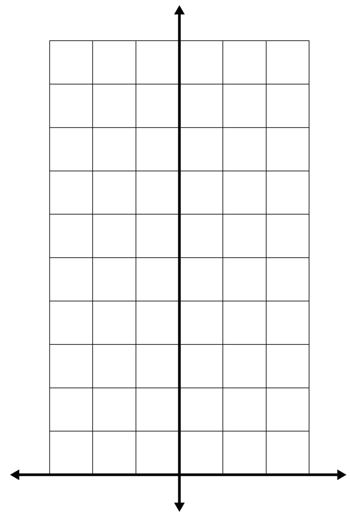
Math 30-1 students are expected to know that domain and range can be expressed using ***interval notation***.

() - Round Brackets: Exclude point from interval.
[] - Square Brackets: Include point in interval.
Infinity ∞ always gets a round bracket.

Examples: x ≥ -5 becomes [-5, ∞);
1 < x ≤ 4 becomes (1, 4];
x ε R becomes (-∞ , ∞);
-8 ≤ x < 2 or 5 ≤ x < 11 becomes [-8, 2) U [5, 11), where U means "or", or ***union of sets***;
x ε R, x ≠ 2 becomes (-∞ , 2) U (2, ∞);
-1 ≤ x ≤ 3, x ≠ 0 becomes [-1 , 0) U (0, 3].

c) $y = \left(\frac{1}{2}\right)^x$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



Domain:

Range:

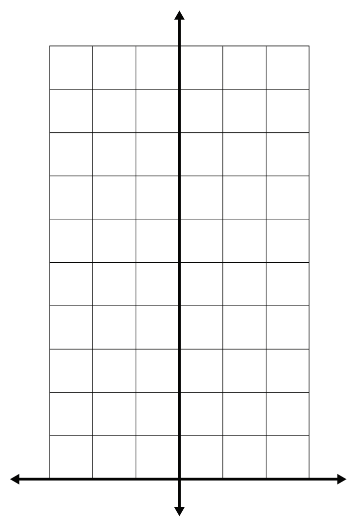
x-intercept:

y-intercept:

Asymptote:

d) $y = \left(\frac{1}{3}\right)^x$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



Domain:

Range:

x-intercept:

y-intercept:

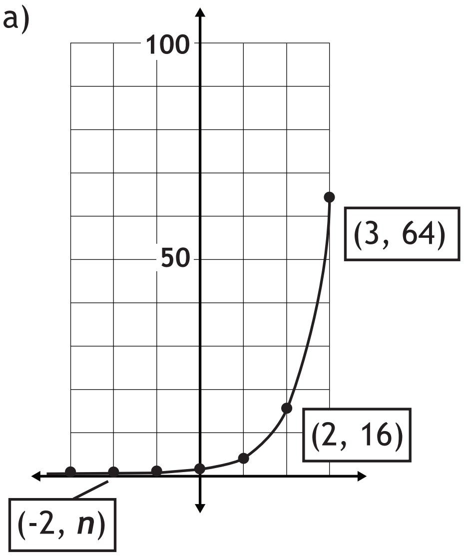
Asymptote:

e) Define *exponential function*. Are the functions $y = 0^x$ and $y = 1^x$ considered exponential functions? What about $y = (-1)^x$?

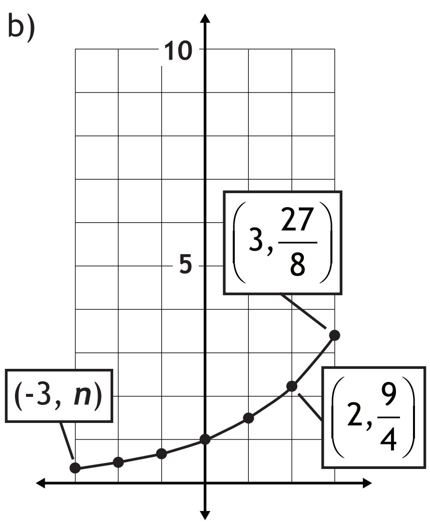
Example 2 Determine the exponential function corresponding to each graph, then use the function to find the unknown. All graphs in this example have the form $y = b^x$.

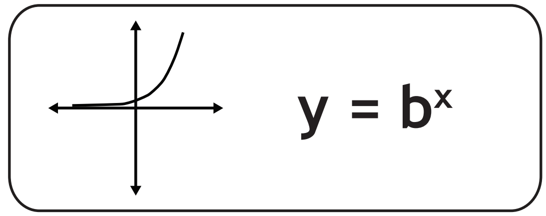
Exponential Function of a Graph. *($y = b^x$)*

a)

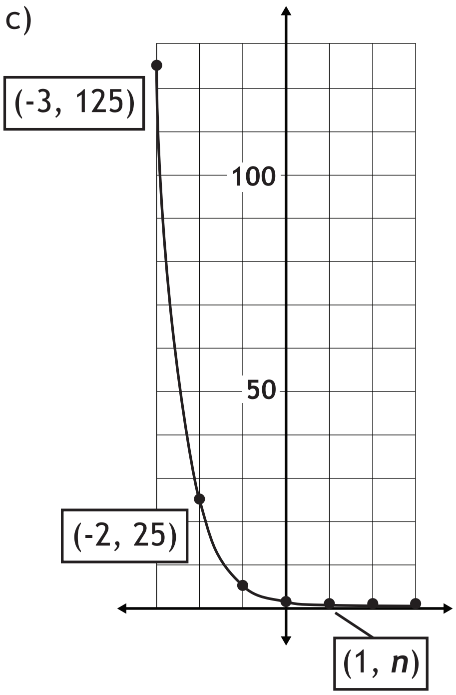


b)

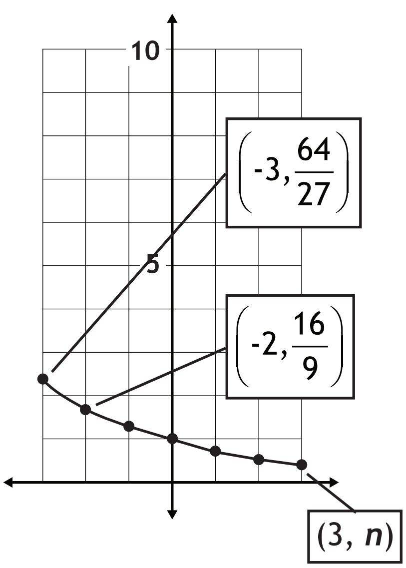



y = bˣ

c)


(-3, 125)
100
50
(-2, 25)
(1, n)

d)

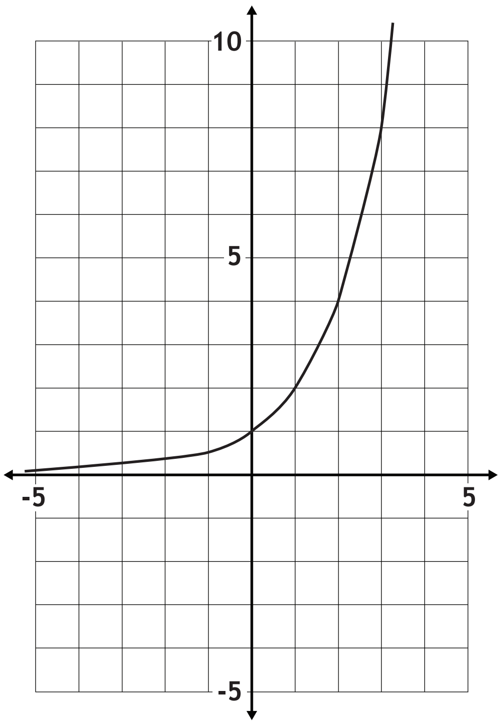

10
5
$\left(-3, \frac{64}{27}\right)$
$\left(-2, \frac{16}{9}\right)$
(3, n)

Example 3

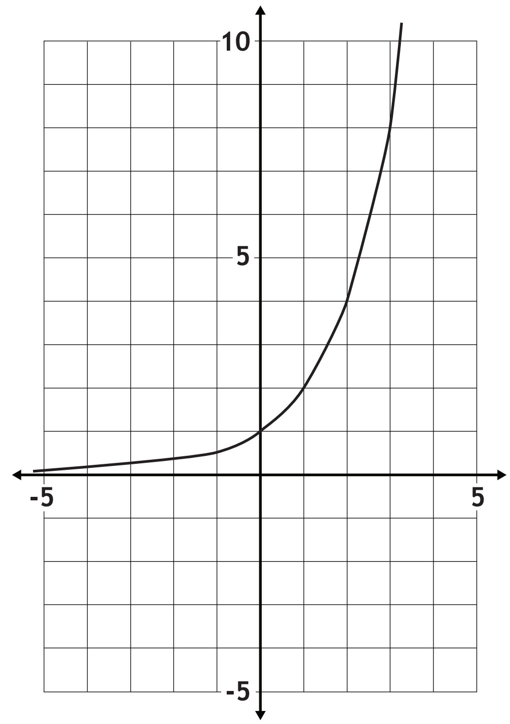
Draw the graph. The graph of $y = 2^x$ is provided as a convenience. State the domain, range, and equation of the asymptote.

Transformations

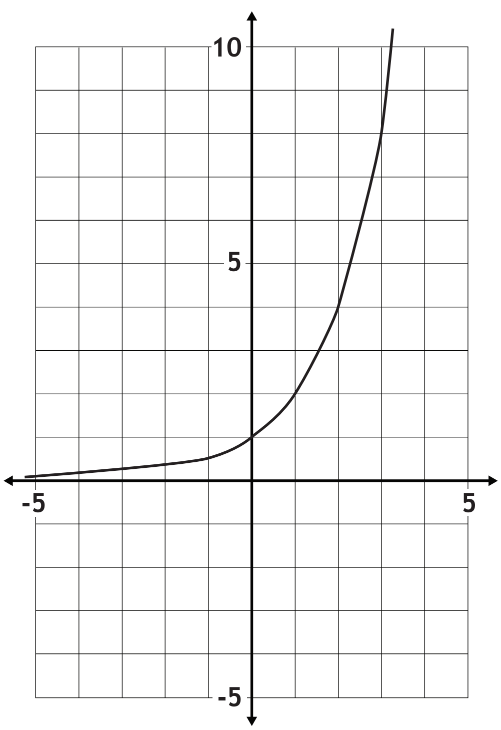
a) $y = 3(2)^x$



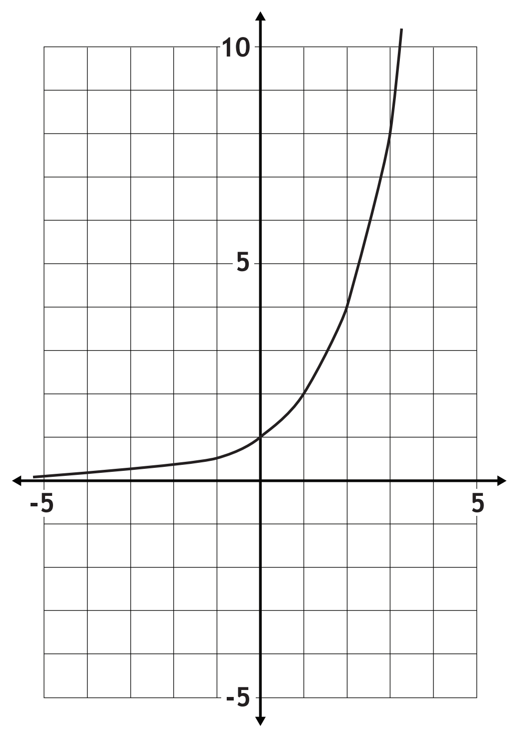
b) $y = 2^{\frac{x}{4}}$



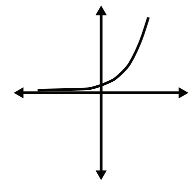
c) $y = 2^x + 3$



d) $y = 2^{x-1}$



Lesson Notes



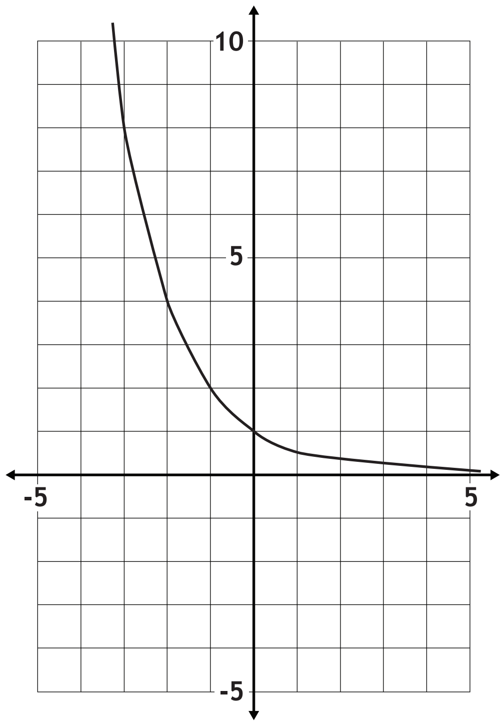
$y = b^x$

Example 4

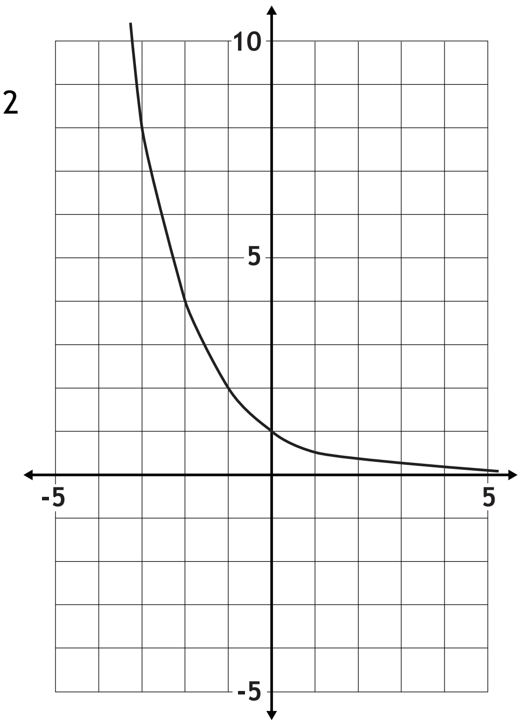
Draw the graph. The graph of $y = (1/2)^x$ is provided as a convenience. State the domain, range, and equation of the asymptote.

Transformations

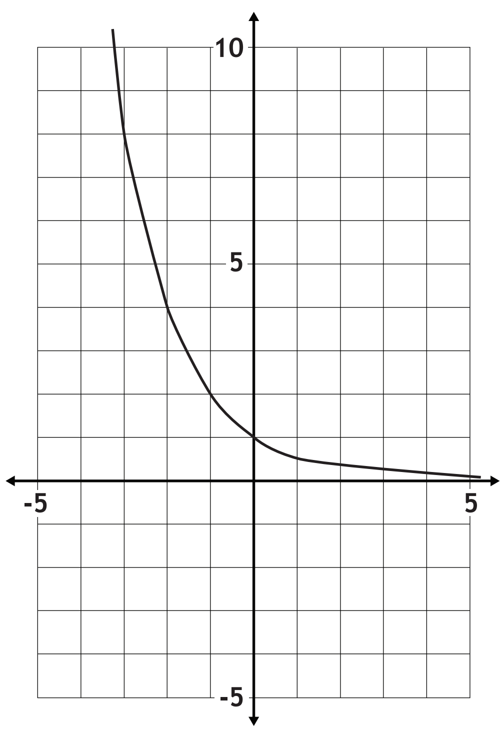
a) $y = 2\left(\frac{1}{2}\right)^{x} - 4$



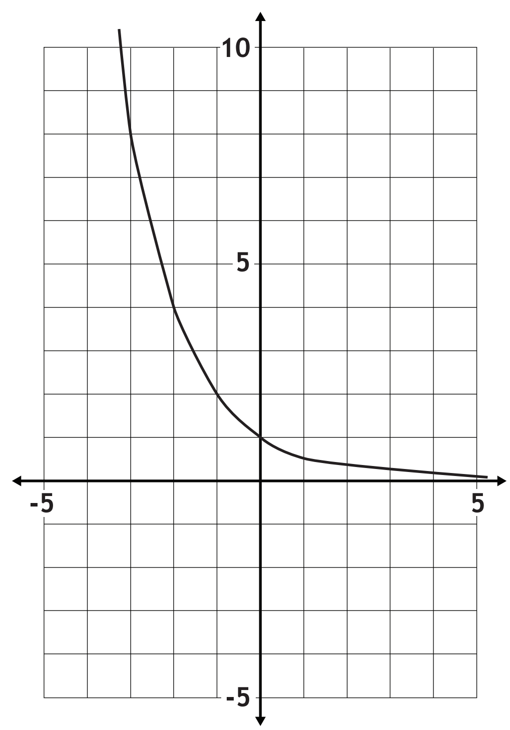
b) $y = \left(\frac{1}{2}\right)^{x+3} - 2$



c) $y = \left(\frac{1}{2}\right)^{\frac{1}{2}(x-1)}$



d) $y = \left(\frac{1}{2}\right)^{2x+6}$

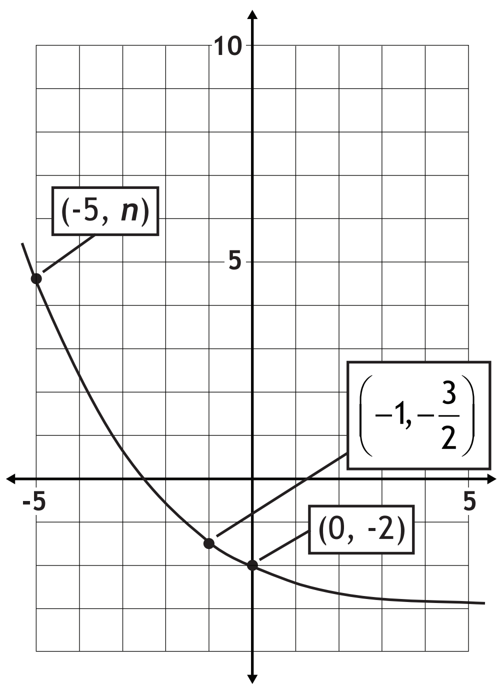


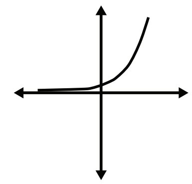
Example 5

Determine the exponential function corresponding to each graph, then use the function to find the unknown. Both graphs in this example have the form $y = ab^x + k$.

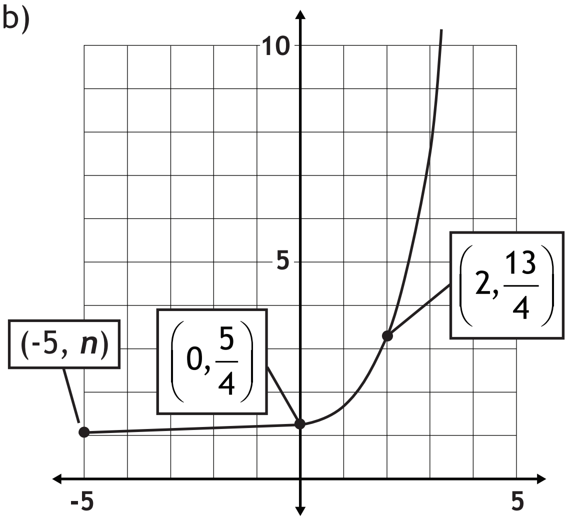
Exponential Function of a Graph. *($y = ab^x + k$)*

a)





b)



Example 6

Answer each of the following questions.

Assorted Questions

a) What is the y-intercept of $f(x) = ab^{x-4}$?

b) The point $\left(-1, \frac{5}{3}\right)$ exists on the graph of $y = a(5)^x$. What is the value of a?

c) If the graph of $y = \left(\frac{1}{3}\right)^x$ is stretched vertically so it passes through the point $\left(2, \frac{1}{12}\right)$, what is the equation of the transformed graph?

d) If the graph of $y = 2^x$ is vertically translated so it passes through the point (3, 5), what is the equation of the transformed graph?

e) If the graph of $y = 3^x$ is vertically stretched by a scale factor of 9, can this be written as a horizontal translation?

f) Show algebraically that each pair of graphs are identical.

i) $y = 25(5)^x$ and $y = 5^{x+2}$

ii) $y = \frac{1}{8}(2)^x$ and $y = 2^{x-3}$

iii) $y = 2^{-x}$ and $y = \left(\frac{1}{2}\right)^x$

iv) $y = \frac{64}{27}\left(\frac{3}{4}\right)^{-x}$ and $y = \left(\frac{4}{3}\right)^{x+3}$

v) $y = \frac{3}{4}\left(\frac{1}{3}\right)^x$ and $y = \frac{1}{4}\left(\frac{1}{3}\right)^{x-1}$

Example 7

Solving equations where x is in the base.

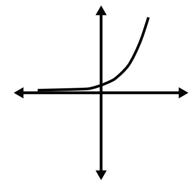
Raising Reciprocals

a) $x^3 = 8$

b) $x^{\frac{1}{4}} = 2$

c) $x^{-\frac{3}{5}} = 27$

d) $(16x)^{\frac{2}{3}} = 4$



$y = b^x$

Example 8 Solving equations where *x* is in the exponent.

Common Base

a) $2^{2x+1} = 8^{x-1}$

b) $2^{3x} = 32^{x-2}$

c) $8^{x-1} = 16^{x-2}$

d) $9^{\frac{x}{2}} = 27^{x-4}$

e) Determine *x* and *y*:

$$8^x = \frac{1}{64}$$
$$25^{x+y} = 125$$

f) Determine *m* and *n*:

$$27^{2m-n} = \frac{1}{9}$$
$$49^{3m-2n} = 7$$

Example 9

Solving equations where x is in the exponent.

Common Base
(fractional base)

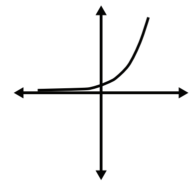
a) $\left(\frac{1}{6}\right)^{x} = 36$

b) $\left(\frac{125}{8}\right)^{x-2} = \left(\frac{25}{4}\right)^{2x-5}$

c) $\left(\frac{9}{4}\right)^{x-4} = \left(\frac{8}{27}\right)^{2x}$

d) $\left(\frac{16}{81}\right)^{6x} = \left(\frac{27}{8}\right)^{-10x+1}$

Lesson Notes



y = bˣ

Example 10 Solving equations where *x* is in the exponent.

Common Base
(fractional exponents)

a) $3^{\frac{2x}{3}} = 9^{x-4}$

b) $25^{\frac{10+x}{3}-2} = 125^{\frac{2x}{5}}$

c) $\left(\frac{1}{8}\right)^{\frac{x}{9}-6} = 4^{\frac{x}{2}-3}$

d) $\left(\frac{3}{4}\right)^{\frac{2}{3}(x+3)} = \left(\frac{64}{27}\right)^{\frac{x}{3}-9}$

Example 11

Solving equations where x is in the exponent.

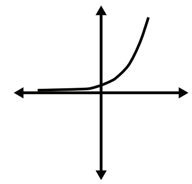
Common Base
(multiple powers)

a) $16^{3x} = \left(2^{5x+2}\right)\left(8^{2x}\right)$

b) $27^{x+1} = \left(3^{x-3}\right)\left(9^{x+3}\right)$

c) $125\left(\frac{4}{5}\right)^{2x+1} = 64$

d) $8^{x+1} = \frac{1}{64^{1-x}}$



$y = b^x$

Example 12 Solving equations where *x* is in the exponent.

Common Base
(radicals)

a) $3^x = 9\sqrt{3}$

b) $5^x = 125\sqrt{5}$

c) $64^{x-2} = \left(\sqrt[4]{4}\right)^{3x+3}$

d) $3^{4x} = \left(\sqrt[3]{9}\right)^{2x+4}$

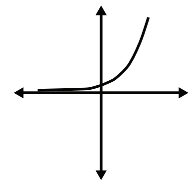
Example 13 Solving equations where *x* is in the exponent. Factoring

a) $4^{2x} - 6(4)^x + 8 = 0$

b) $2(2)^{-2x} - 9(2)^{-x} + 4 = 0$

c) $2^{x+3} + 2^{x+4} = 96$

d) $3^x - 3^{x-1} = 162$



$y = b^x$

Example 14

Solving equations where x is in the exponent.

No Common Base
(use technology)

a) $3^x = 7$

b) $\left(\frac{1}{2}\right)^x = -3$

c) $2(4)^{x-1} = 6$

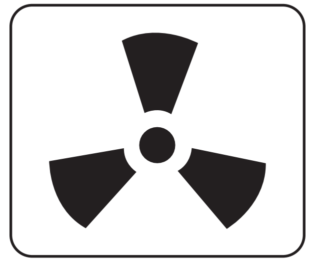
d) $12\left(\frac{1}{2}\right)^{x-1} = 3$

Example 15

A 90 mg sample of a radioactive isotope has a half-life of 5 years.

$$y = ab^{\frac{t}{p}}$$

a) Write a function, $m(t)$, that relates the mass of the sample, m, to the elapsed time, t.



b) What will be the mass of the sample in 6 months?

Logarithmic Solutions

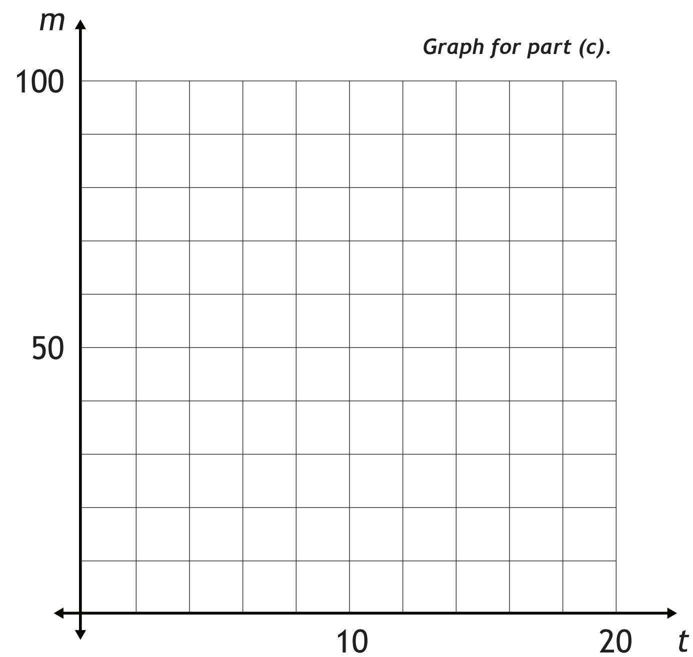
Some of these examples provide an excellent opportunity to use logarithms.

Logarithms are not a part of this lesson, but it is recommended that you return to these examples at the end of the unit and complete the logarithm portions.

c) Draw the graph for the first 20 years.

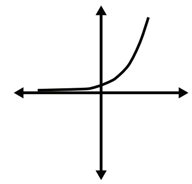
d) How long will it take for the sample to have a mass of 0.1 mg?

Solve Graphically | **Solve with Logarithms**



Graph for part (c).

Exponential and Logarithmic Functions
LESSON ONE- *Exponential Functions*
Lesson Notes



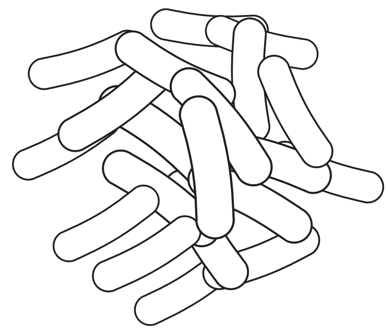
$y = b^x$

Example 16

A bacterial culture contains 800 bacteria initially and doubles every 90 minutes.

$y = ab^{\frac{t}{P}}$

a) Write a function, *B(t)*, that relates the quantity of bacteria, *B*, to the elapsed time, *t*.



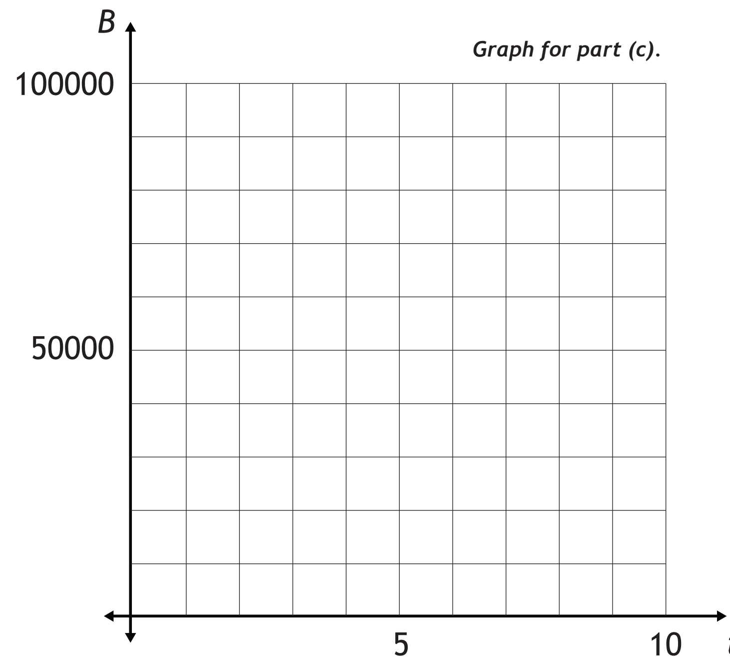
b) How many bacteria will exist in the culture after 8 hours?

c) Draw the graph for the first ten hours.

d) How long ago did the culture have 50 bacteria?

Solve Graphically

Solve with Logarithms

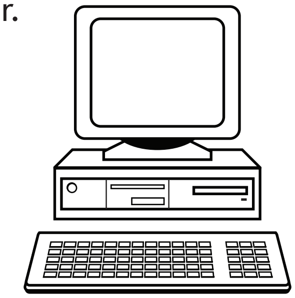


Graph for part (c).

Example 17

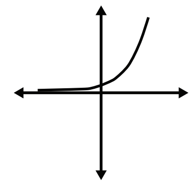
In 1990, a personal computer had a processor speed of 16 MHz. In 1999, a personal computer had a processor speed of 600 MHz. Based on these values, the speed of a processor increased at an average rate of 44% per year.

$$y = ab^{\frac{t}{P}}$$



a) Estimate the processor speed of a computer in 1994 (t = 4).
How does this compare with actual processor speeds (66 MHz) that year?

b) A computer that cost \$2500 in 1990 depreciated at a rate of 30% per year.
How much was the computer worth four years after it was purchased?



$y = b^x$

Example 18

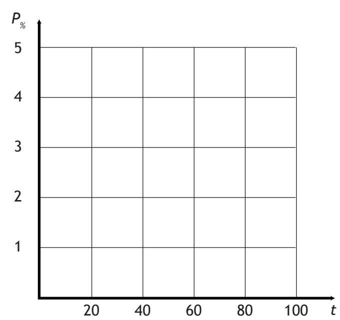
A city with a population of 800,000 is projected to grow at an annual rate of 1.3%.

$$y = ab^{\frac{t}{P}}$$

a) Estimate the population of the city in 5 years.

b) How many years will it take for the population to double?

Solve Graphically

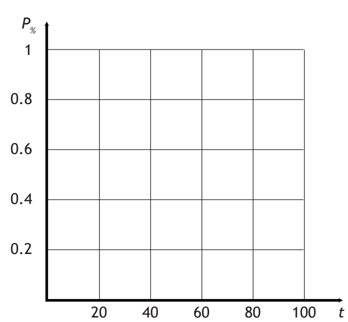


Solve with Logarithms

c) If projections are incorrect, and the city's population *decreases* at an annual rate of 0.9%, estimate how many people will leave the city in 3 years.

d) How many years will it take for the population to be reduced by half?

Solve Graphically



Solve with Logarithms

Example 19

$500 is placed in a savings account that compounds interest annually at a rate of 2.5%.

$y = ab^{\frac{t}{P}}$

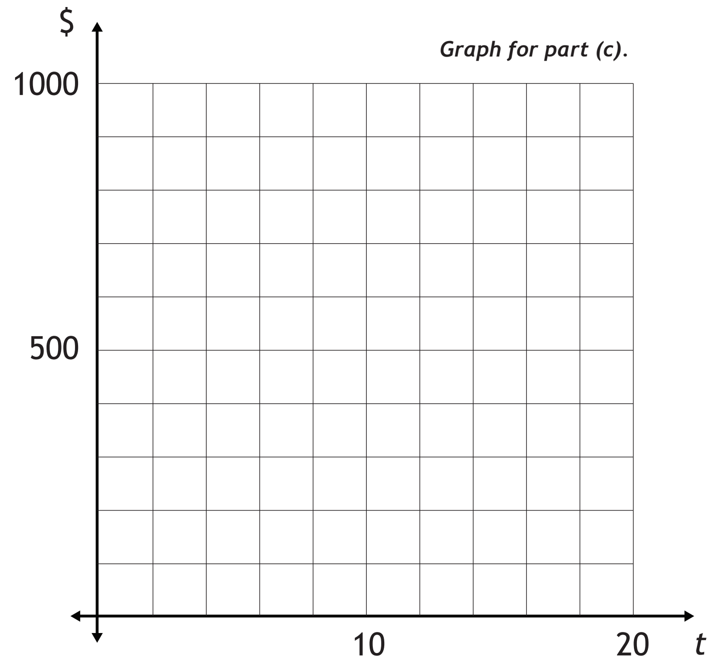
a) Write a function, $A(t)$, that relates the amount of the investment, A, with the elapsed time t.

b) How much will the investment be worth in 5 years? How much interest has been received?

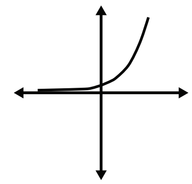
c) Draw the graph for the first 20 years.

d) How long does it take for the investment to double?

Solve Graphically | **Solve with Logarithms**



Graph for part (c).



$y = b^x$

e) Calculate the amount of the investment in 5 years if compounding occurs i) semi-annually, ii) monthly, and iii) daily. Summarize your results in the table.

Future amount of $500 invested for 5 years and compounded:

Annually	*Use answer from part b.*
Semi-Annually	
Monthly	
Daily	

Example 1 Introduction to Logarithms. | Logarithm Components |

a) Label the components of $\log_B A = E$ and $A = B^E$.

$$\log_B A = E$$

____

____

____

$$A = B^E$$

____

____

____

b) Evaluate each logarithm.

i) $\log_2 1 = \square$

$\log_2 2 = \square$

$\log_2 4 = \square$

$\log_2 8 = \square$

ii) $\log 1 = \square$

$\log 10 = \square$

$\log 100 = \square$

$\log 1000 = \square$

c) Which logarithm is bigger?

i) $\log_2 1$ or $\log_4 2$

ii) $\log_3\left(\frac{1}{9}\right)$ or $\log_9\left(\frac{1}{3}\right)$

Lesson Notes

$\log_B A = E$

Example 2 Order each set of logarithms from least to greatest.

Ordering Logarithms

a) $\log 10$, $\log_2 16$, $\log_9\left(\frac{1}{3}\right)$, $\log_{16}\left(\frac{1}{2}\right)$, $\log_5 1$

b) $\log_{\frac{1}{3}} 27$, $\log_{\frac{1}{4}} 8$, $\log_{\frac{1}{8}}\left(\frac{1}{2}\right)$, $\log_{\frac{1}{4}}\left(\frac{1}{2}\right)$, $\log_{\frac{1}{8}}\left(\frac{1}{8}\right)$

c) $\log_3 25$, $\log_6 7$, $\log_{\frac{1}{4}}\left(\frac{1}{15}\right)$, $\log_8 3$ (Estimate the order using benchmarks)

Example 3

Convert each equation from logarithmic to exponential form.

Express answers so y is isolated on the left side.

Logarithmic to Exponential Form
(The Seven Rule)

$\log_b y = x \rightarrow b^x = y$

a) $\log_2 y = x$

b) $2 = \log_4 y$

c) $a \log y = x$

d) $\log_3 (2y) = x$

e) $\frac{1}{2} = \log_x y$

f) $\log_2 (y - x) = 3$

g) $2 = \log_{x+1} (y + 1)$

h) $\log_3 (3y) = 2x$

Lesson Notes

$$\log_B A = E$$

Example 4

Convert each equation from exponential to logarithmic form.

Express answers with the logarithm on the left side.

Exponential to Logarithmic Form
(A Base is Always a Base)

$\boldsymbol{b}^x = y \rightarrow \log_{\boldsymbol{b}} \boxed{y} = \boxed{x}$

a) $y = x^2$

b) $10x^4 = y$

c) $y = \left(\frac{1}{3}\right)^x$

d) $\sqrt{x} = 3y$

e) $y = \sqrt[3]{\frac{x}{2}}$

f) $y = (x - 3)^2$

g) $y = \frac{k^x}{k}$

h) $10^{y-x} = a$

Example 5 Evaluate each logarithm using change of base.

Evaluating Logarithms
(Change of Base)

a) $\log_4 64$

b) $\log_{\frac{2}{3}} \frac{8}{27}$

General Rule

$$\log_b a = \frac{\log_c a}{\log_c b}$$

For Calculator
(base-10)

$$\log_b a = \frac{\log a}{\log b}$$

c) $\log_{\sqrt{2}} 2$

d) $\log 100$

In parts (e - h), condense each expression to a single logarithm.

e) $\dfrac{\log 5}{\log 25}$

f) $\dfrac{\log \sqrt{3}}{\log 3}$

g) $\dfrac{\log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{3}\right)}$

h) $\left(\log_a x\right)\left(\log_x b\right)$

Lesson Notes

$\log_B A = E$

Example 6

Expand each logarithm using the product law.

Expanding Logarithms
(Product Law)

$\log_b(M \times N) = \log_b M + \log_b N$

a) $\log(xy)$

b) $\log(x + y)$

c) $\log\left(3(x+1)\right)$

d) $\log(10x)$

In parts (e - h), condense each expression to a single logarithm.

e) $\log 3 + \log 4$

f) $\log\frac{2}{3} + \log\frac{3}{4}$

g) $\log x^2 + \log x^3$

h) $\log(x+1) + \log(x-2)$

$\log_B A = E$

Exponential and Logarithmic Functions
LESSON TWO - *Laws of Logarithms*
Lesson Notes

Example 7 Expand each logarithm using the quotient law.

Expanding Logarithms
(Quotient Law)

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

a) $\log\left(\frac{x}{y}\right)$

b) $\log(x - y)$

c) $\log\left(\frac{x+1}{100}\right)$

d) $\log_3\left(\frac{x}{3(x+1)}\right)$

In parts (e - h), condense each expression to a single logarithm.

e) $\log 12 - \log 4$

f) $\log\frac{1}{3} - \log 2$

g) $\log x^5 - \log x^2$

h) $\log 2 + \log x - \log(x + 3)$

$\log_B A = E$

Example 8 Expand each logarithm using the power law.

Expanding Logarithms
(Power Law)
$\log_b\left(M^n\right) = n\log_b M$

a) $\log x^2$

b) $\left(\log x\right)^2$

c) $\log x^3 + \log x^4$

d) $\log x^{a+1}$

In parts (e - h), condense each expression to a single logarithm.

e) $3\log x$

f) $2\log\left(x-1\right)$

g) $3\log\left(2x^2\right)$

h) $5\log x - 3\log x$

$\log_B A = E$

Exponential and Logarithmic Functions
LESSON TWO - *Laws of Logarithms*
Lesson Notes

Example 9 Expand each logarithm using the appropriate logarithm rule.

Expanding Logarithms
(Other Rules)

$\log_b x$ has the domain x > 0
$\log_b 1 = 0$
$\log_b b = 1$
$b^{\log_b x} = x$
$\log_b b^x = x$

a) $\log_2 0$

b) $\log(-3)$

c) $\log_2 1$

d) $\log_4 4$

e) $5^{\log_5 x}$

f) $\log_2 2^x$

g) $\log_5 25^k$

h) $\log_a \left(\sqrt{a}\right)^k$

$\log_B A = E$

Example 10 Use logarithm laws to answer each of the following questions.

Substitution Questions

a) If $10^k = 4$, then $10^{1 + 2k} =$

b) If $3^a = k$, then $\log_3 k^4 =$

c) If $\log_b 4 = k$, then $\log_b 16 =$

d) If $\log_2 a = h$, then $\log_4 a =$

e) If $\log_b h = 3$ and $\log_b k = 4$, then $\log_b\left(\frac{1}{hk}\right) =$

f) If $\log_h 4 = 2$ and $\log_8 k = 2$, then $\log_2(hk) =$

g) Write $\log x + 1$ as a single logarithm.

h) Write $3 + \log_2 x$ as a single logarithm.

Example 11 Solving Equations. Express answers using exact values.

Solving Exponential Equations
(No Common Base)

a) $3^x = 4$

b) $5^x = -2$

c) $2 \times 5^{x+2} = 7$

d) $\left(\frac{2}{5}\right)^{x-3} = \frac{1}{3}$

Lesson Notes

$\log_B A = E$

Example 12

Solving Equations. Express answers using exact values.

Solving Exponential Equations
(No Common Base)

a) $6^{5x} = 3^{2x-1}$

b) $2^{x+3} = 3^{2x-1}$

c) $\dfrac{4^{2x-1}}{3} = 5^x$

d) $2 \times 3^{x+3} = 6^{3x}$

Example 13 Solving Equations. Express answers using exact values.

Solving Logarithmic Equations
(One Solution)

a) $3\log x + 5 = 8$

b) $2\log_5 3 = \log_5 (x + 1)$

c) $\log_3 (x - 2) = \log_3 (3x + 2)$

d) $\log_3 x - \log_3 2 = \log_3 7$

$\log_B A = E$

Example 14 Solving Equations. Express answers using exact values.

Solving Logarithmic Equations
(Multiple Solutions)

a) $\log_2 x + \log_2 (x + 2) = 3$

b) $\log_2 (x - 1) + \log_2 (x - 2) - \log_2 3 = 2$

c) $\log x^2 + \log 3 = \log 2x$

d) $\log_4 (x^2 + 1) - \log_4 6 = \log_4 5$

Example 15

Solving Equations. Express answers using exact values.

Solving Logarithmic Equations
(Multiple Solutions)

a) $\log_{x-1} 25 = 2$

b) $2\log(x-3) = \log 4 + \log(6-x)$

c) $(\log x)^2 - 4\log x - 5 = 0$

d) $(\log x)^4 - 16 = 0$

$\log_B A = E$

Example 16 Assorted Questions. Express answers using exact values.

Assorted Mix I

a) Evaluate.

$\log_6 \sqrt[4]{6}$

b) Condense.

$\frac{1}{2}\log a - 3\log b - 2\log c$

c) Solve.

$3\log_2 x = 12$

d) Evaluate.

$\log_2\left(\log(10000)\right)$

e) Write as a logarithm.

$b^{\frac{5}{4}} = 2a$

f) Show that:

$\log_{\frac{1}{5}}\left(\frac{1}{x}\right) = \log_5 x$

g) If $\log_a 3 = x$ and $\log_a 4 = 12$,
then $\log_a 12^2 =$
(express answer in terms of x.)

h) Condense.

$2 + \frac{1}{3}\log_3 x$

Exponential and Logarithmic Functions
LESSON TWO - *Laws of Logarithms*
Lesson Notes

$\log_B A = E$

Example 17 Assorted Questions. Express answers using exact values.

Assorted Mix II

a) Evaluate.

$\log_3 9 + \log_3 9^2 + \log_3 9^3$

b) Evaluate.

$\log_3 9 + \left(\log_3 9\right)^2 + \left(\log_3 9\right)^3$

c) What is one-third of 3^{234}?

d) Solve.

$8 = \left(x + 1\right)^3$

e) Evaluate.

$\log_b\left(\dfrac{1}{b^{-100}}\right)$

f) Condense.

$\log_2 a + \log_4 b$

g) Solve.

$\log(x+2) + \log(x-1) = 1$

h) If xy = 8, then $5\log_2 x + 5\log_2 y =$

$$\log_B A = E$$

Example 18 Assorted Questions. Express answers using exact values.

Assorted Mix III

a) Evaluate.

$\log_2 \frac{1}{8}$

b) Solve.

$\log x - \log(x+5) = 1$

c) Condense.

$\log_4 8^x - \log_4 2^x$

d) Solve.

$(\log x)^2 = 2\log x$

e) Condense.

$$\left(\frac{1}{2}\right)^{\log_{\frac{1}{2}} a}\left(\frac{1}{2}\right)^{\log_{\frac{1}{2}} a}$$

f) Evaluate.

$$\log_9\left(\log_2 8\right)$$

g) Show that:

$$\log_{\frac{1}{2}} 81 = \log_2\left(\frac{1}{81}\right)$$

h) Condense.

$$\log_2\left(2x+1\right)+1$$

$\log_B A = E$

Example 19 Assorted Questions. Express answers using exact values.

Assorted Mix IV

a) Solve.

$\log_3(2x+1) - \log_3(x-1) = 1$

b) Condense.

$3(\log a + \log b)$

c) Solve.

$\log_{\sqrt{2}} x^4 + 4 = 12$

d) Condense.

$\log(a^2 + 2a + 1) - \log(a + 1)$

e) Evaluate.

$-\frac{1}{3}\log_2 64$

f) Solve.

$\log(2 - x) + \log(2 + x) = \log 3$

g) Evaluate.

$\frac{1}{4}\log_2 16 + \log_3 \sqrt{27}$

h) Condense.

$3\log_{16} x + \frac{1}{2}$

$\log_B A = E$

Example 20

Assorted Questions. Express answers using exact values.

Assorted Mix V

a) Solve.

$\log(x+2) = \log x + \log 2$

b) Solve.

$2^{3x-1} = 5^{2x+3}$

c) Evaluate.

$\log_3 9^{99} + \log_4 64 + \log_a 1 + \log_{\frac{1}{2}} 8 + \log_{\sqrt{a}} \sqrt{a}$

d) Condense.

$\log x - 4\log\sqrt{x}$

e) Solve.

$$\log_4(\log_3 x) = \frac{1}{2}$$

f) Solve.

$$2\log x + 3\log x = 8$$

g) Condense.

$$4\log a - \frac{1}{2}\log b + \log c$$

h) Solve.

$$\log_{2x}(4x + 8) = 2$$

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Exponential and Logarithmic Functions
LESSON THREE - *Logarithmic Functions*
Lesson Notes

Example 1

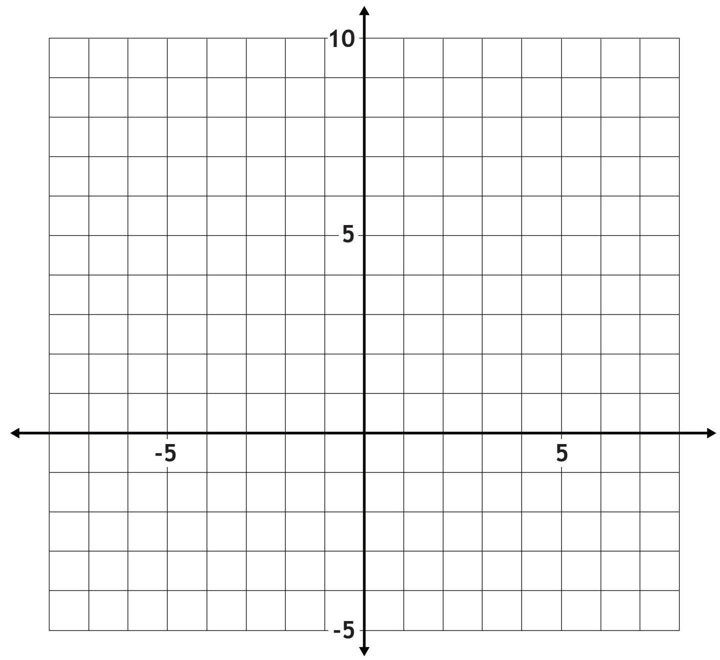
Logarithmic Functions

Graphing Logarithms

a) Draw the graph of $f(x) = 2^x$

b) Draw the inverse of $f(x)$.

c) Show algebraically that the inverse of $f(x) = 2^x$ is $f^{-1}(x) = \log_2 x$.



d) State the domain, range, intercepts, and asymptotes of both graphs.

	$y = 2^x$	$y = \log_2 x$
Domain		
Range		
x-intercept		
y-intercept		
Asymptote Equation		

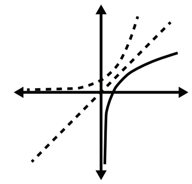
e) Use the graph to determine the value of:
i) $\log_2 0.5$, ii) $\log_2 1$, iii) $\log_2 2$, iv) $\log_2 7$

f) Are $y = \log_1 x$, $y = \log_0 x$, and $y = \log_{-2} x$ logarithmic functions?
What about $y = \log_{\frac{1}{10}} x$?

g) Define logarithmic function.

h) How can $y = \log_2 x$ be graphed in a calculator?

Exponential and Logarithmic Functions
LESSON THREE- *Logarithmic Functions*
Lesson Notes



$y = \log_b x$

Example 2 Draw each of the following graphs without technology. State the domain, range, and asymptote equation.

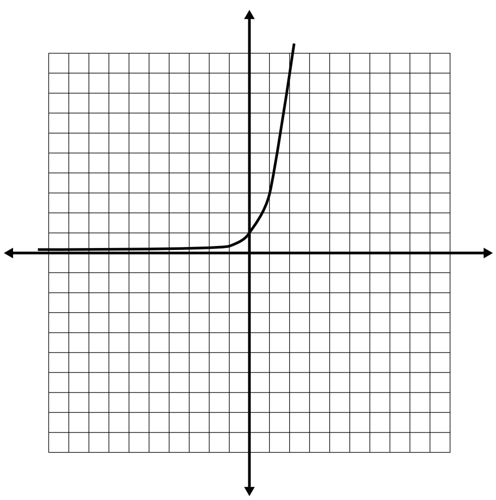
Graphing Logarithms

a) $y = \log_3 x$

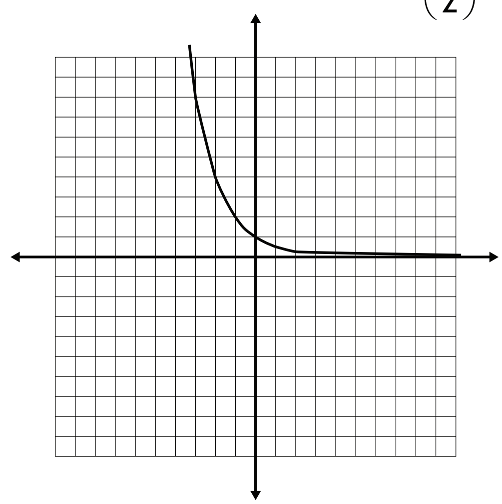
b) $y = \log_{\frac{1}{2}} x$

The exponential function corresponding to the base is provided as a convenience.

Given: $y = 3^x$



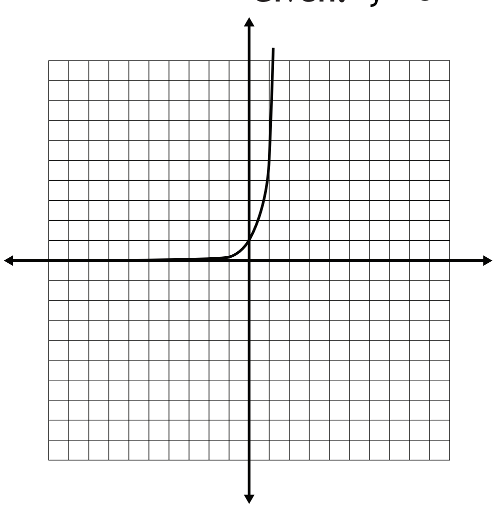
Given: $y = \left(\frac{1}{2}\right)^x$



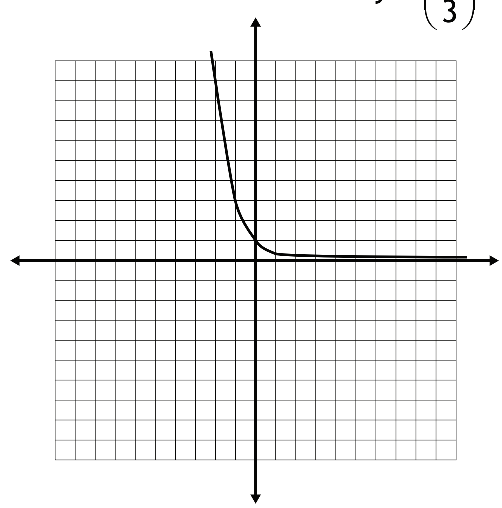
c) $y = \log_5 x$

d) $y = \log_{\frac{1}{3}} x$

Given: $y = 5^x$



Given: $y = \left(\frac{1}{3}\right)^x$



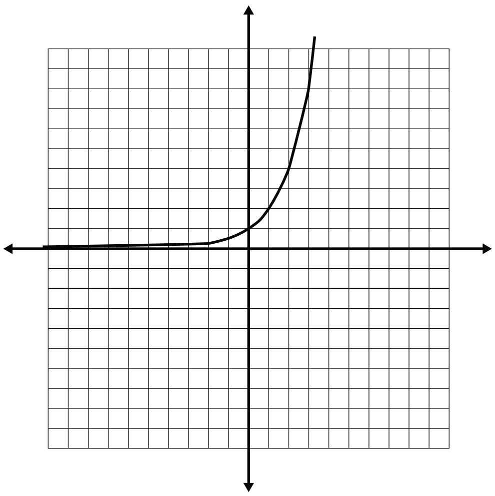
Example 3 Draw each of the following graphs without technology. State the domain, range, and asymptote equation.

Stretches and Reflections

The exponential function corresponding to the base is provided as a convenience.

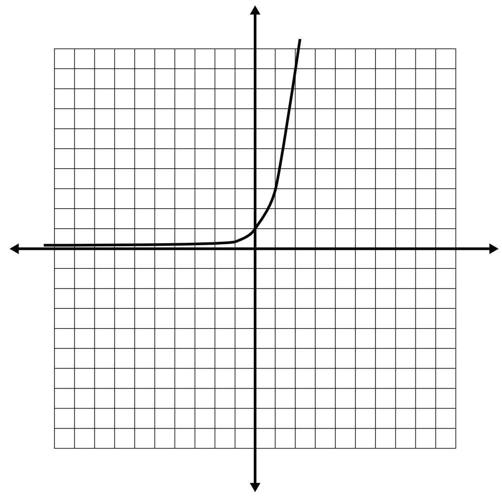
a) $y = 2\log_2 x$

Given: $y = 2^x$



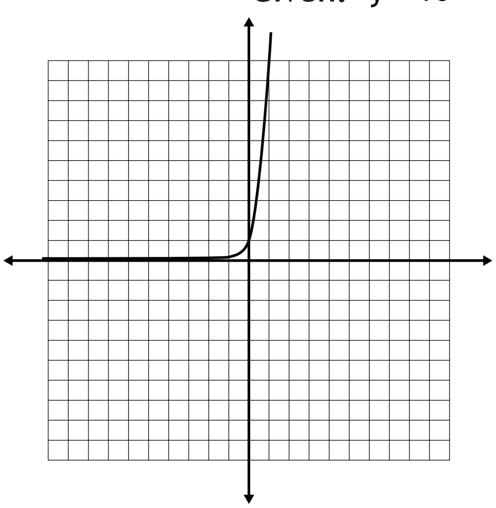
b) $y = -\frac{1}{3}\log_3 x$

Given: $y = 3^x$



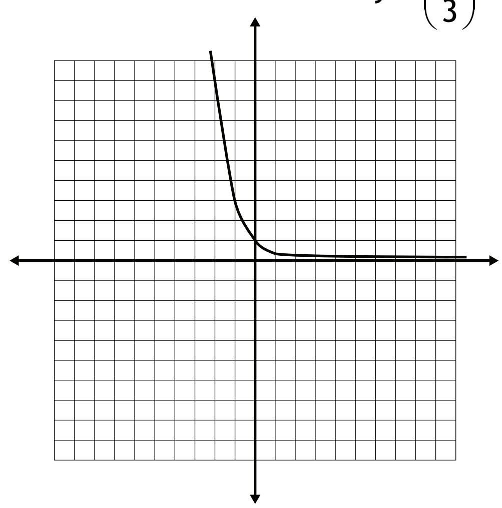
c) $y = \log(2x)$

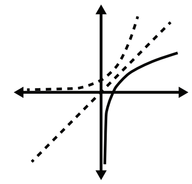
Given: $y = 10^x$



d) $y = \log_{\frac{1}{3}}\left(\frac{1}{2}x\right)$

Given: $y = \left(\frac{1}{3}\right)^x$





$y = \log_b x$

Example 4 Draw each of the following graphs without technology. State the domain, range, and asymptote equation.

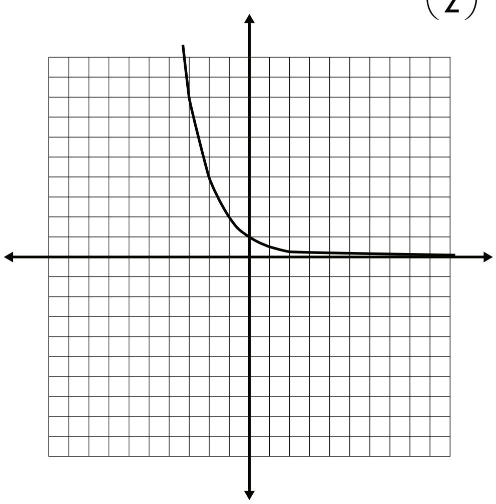
Translations

a) $y = \log_{\frac{1}{2}} x - 1$

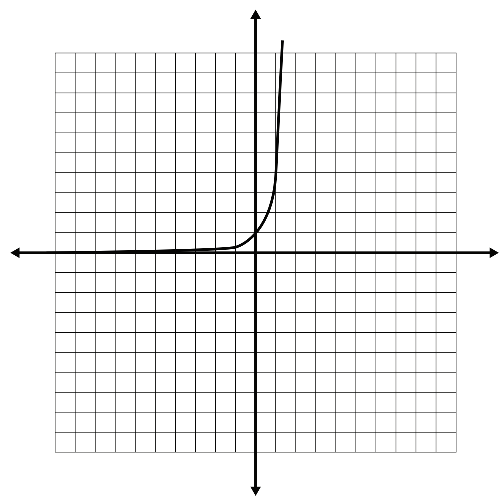
b) $y = \log_4 (x + 2)$

The exponential function corresponding to the base is provided as a convenience.

Given: $y = \left(\frac{1}{2}\right)^x$



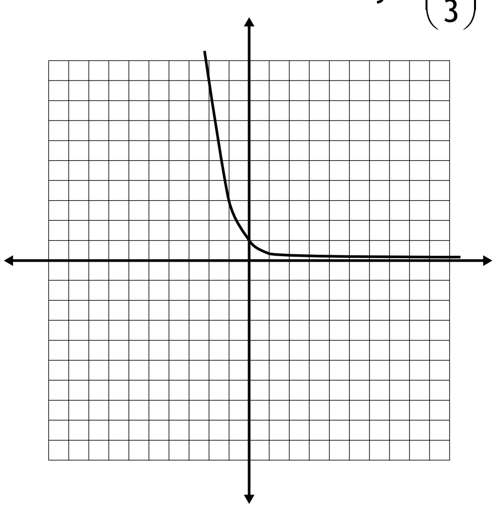
Given: $y = 4^x$



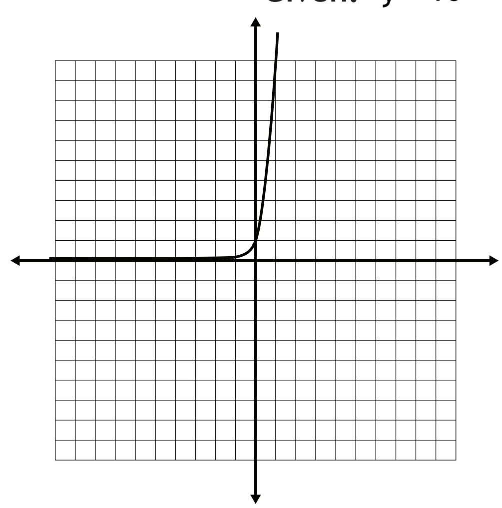
c) $y = \log_{\frac{1}{3}} (x - 3) - 1$

d) $y = \log(x + 4) + 2$

Given: $y = \left(\frac{1}{3}\right)^x$



Given: $y = 10^x$



Example 5

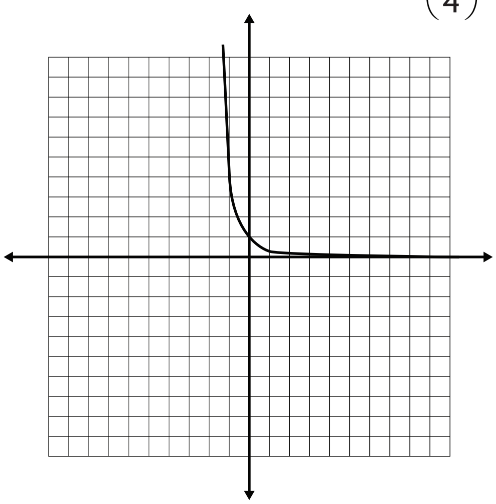
Draw each of the following graphs without technology. State the domain, range, and asymptote equation.

Combined Transformations

The exponential function corresponding to the base is provided as a convenience.

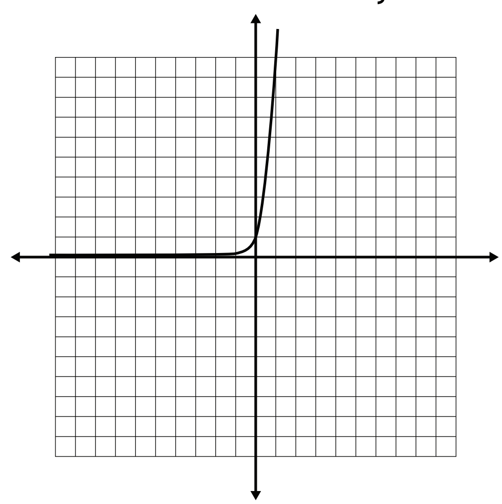
a) $y = \frac{1}{2}\log_{\frac{1}{4}}(x+3)$

Given: $y = \left(\frac{1}{4}\right)^x$



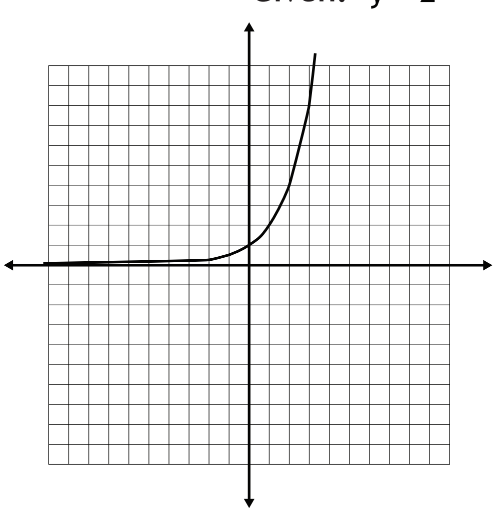
b) $y = -5\log x - 3$

Given: $y = 10^x$



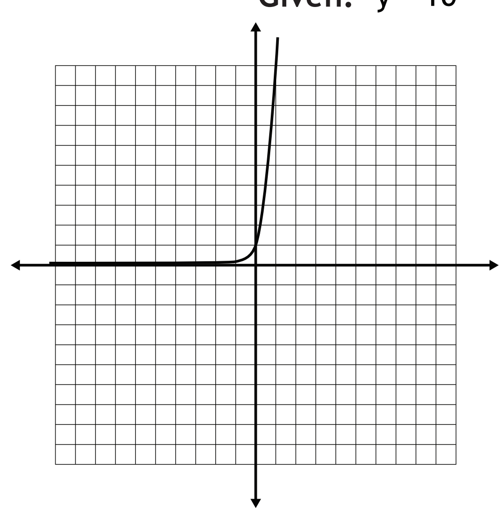
c) $y = 2\log_2(2x+6) - 1$

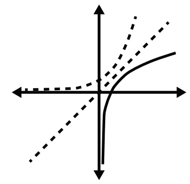
Given: $y = 2^x$



d) $y = 10\log(x+2) - 2$

Given: $y = 10^x$





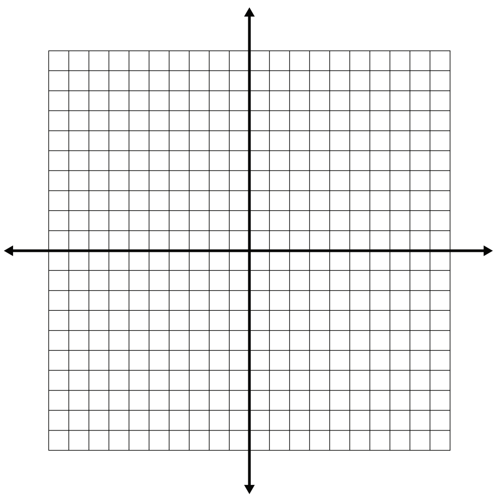
$y = \log_b x$

Example 6

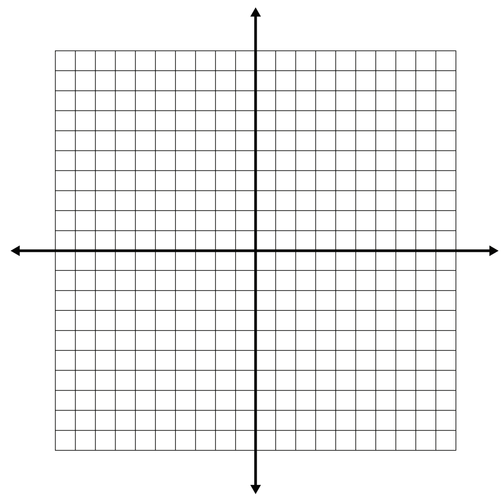
Draw each of the following graphs without technology. State the domain, range, and asymptote equation.

Other Logarithmic Functions

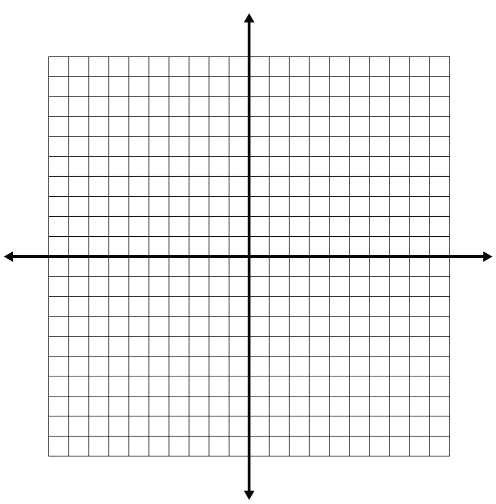
a) $y = \log_2 \sqrt{x}$



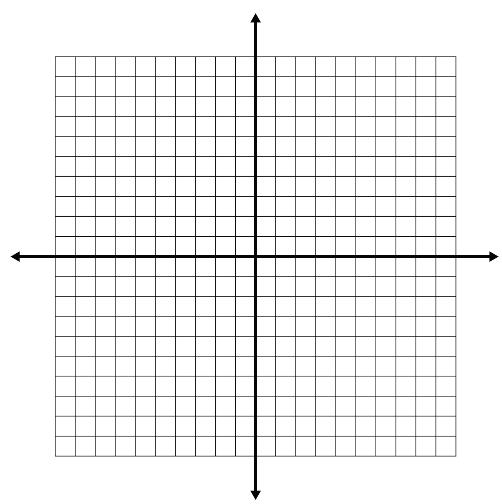
b) $y = \log(x-1)^5$



c) $y = \log_3(x^2 - 4) - \log_3(x-2)$



d) $y = \log_9 x + \log_3 x$



Lesson Notes

Example 7

Solve each equation by (i) finding a common base (if possible), (ii) using logarithms, and (iii) graphing.

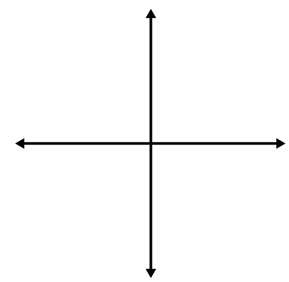
Exponential Equations
(solve multiple ways)

a) $8^{x-2} = 4^{x+1}$

i) Common Base

ii) Solve with Logarithms

iii) Solve Graphically

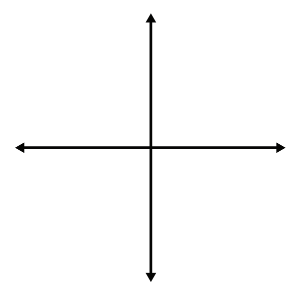


b) $5^{2x+1} = 3^{\frac{x}{2}}$

i) Common Base

ii) Solve with Logarithms

iii) Solve Graphically

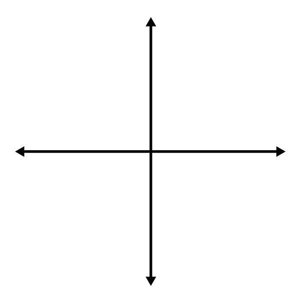


c) $5 = 2^{x-2} + 11$

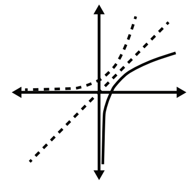
i) Common Base

ii) Solve with Logarithms

iii) Solve Graphically



Exponential and Logarithmic Functions
LESSON THREE- *Logarithmic Functions*
Lesson Notes



$y = \log_b x$

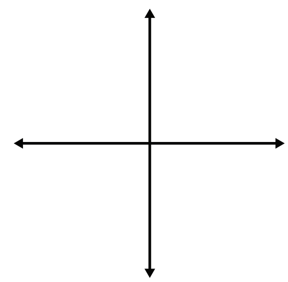
Example 8 Solve each equation by (i) using logarithm laws, and (ii) graphing.

Logarithmic Equations
(solve multiple ways)

a) $\log_3(x+1) = 2$

i) Solve with Logarithm Laws

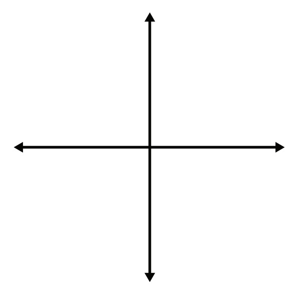
ii) Solve Graphically



b) $\log_5 x^2 + 4\log_5 x = 12$

i) Solve with Logarithm Laws

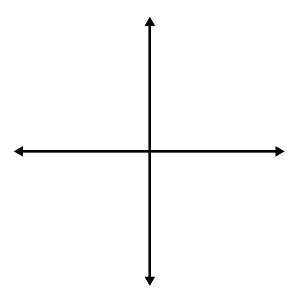
ii) Solve Graphically



c) $\log_2(x-3) + \log_2(x+4) = 3$

i) Solve with Logarithm Laws

ii) Solve Graphically



Example 9 Answer the following questions.

Assorted Mix I

a) The graph of $y = \log_b x$ passes through the point (8, 2). What is the value of b?

b) What are the x- and y-intercepts of $y = \log_2(x + 4)$?

c) What is the equation of the asymptote for $y = \log_3(3x - 8)$?

d) The point (27, 3) lies on the graph of $y = \log_b x$. If the point (4, k) exists on the graph of $y = b^x$, then what is the value of k?

e) What is the domain of $f(x) = \log_x(6 - x)$?

Example 10

Answer the following questions.

Assorted Mix II

a) The graph of $y = \log_3 x$ can be transformed to the graph of $y = \log_3(9x)$ by either a stretch or a translation. What are the two transformation equations?

b) If the point (4, 1) exists on the graph of $y = \log_4 x$, what is the point after the transformation $y = \log_4(2x + 6)$?

c) A vertical translation is applied to the graph of $y = \log_3 x$ so the image has an x-intercept of (9, 0). What is the transformation equation?

d) What is the point of intersection of $f(x) = \log_2 x$ and $g(x) = \log_2(x + 3) - 2$?

e) What is the x-intercept of $y = a\log_b(kx)$?

Example 11

Answer the following questions.

Assorted Mix III

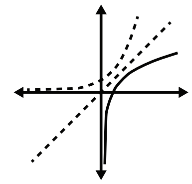
a) What is the equation of the reflection line for the graphs of $f(x) = b^x$ and $g(x) = \left(\frac{1}{b}\right)^x$?

b) If the point (a, 0) exists on the graph of f(x), and the point (0, a) exists on the graph of g(x), what is the transformation equation?

c) What is the inverse of $f(x) = 3^x + 4$?

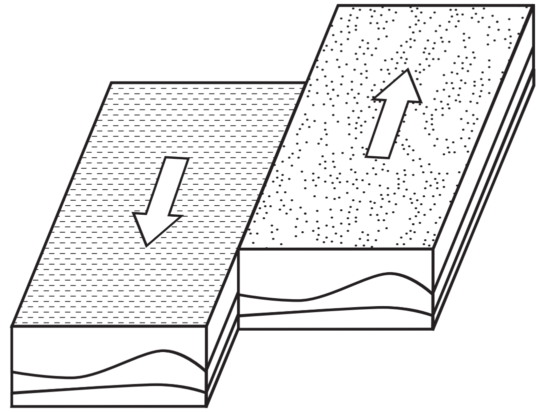
d) If the graph of $f(x) = \log_4 x$ is transformed by the equation $y = f(3x - 12) + 2$, what is the new domain of the graph?

e) The point (k, 3) exists on the inverse of $y = 2^x$. What is the value of k?



$y = \log_b x$

Example 12



The strength of an earthquake is calculated using Richter's formula:

$$\boxed{M = \log \frac{A}{A_0}}$$

where M is the magnitude of the earthquake (unitless), A is the seismograph amplitude of the earthquake being measured (m), and A_0 is the seismograph amplitude of a threshold earthquake (10^{-6} m).

a) An earthquake has a seismograph amplitude of 10^{-2} m.
What is the magnitude of the earthquake?

b) The magnitude of an earthquake is 5.0 on the Richter scale.
What is the seismograph amplitude of this earthquake?

c) Two earthquakes have magnitudes of 4.0 and 5.5.
Calculate the seismograph amplitude ratio for the two earthquakes.

d) The calculation in part (c) required multiple steps because we are comparing each amplitude with A_0, instead of comparing the two amplitudes to each other. It is possible to derive the formula:

$$\boxed{\frac{A_2}{A_1} = 10^{M_2 - M_1}}$$

which compares two amplitudes directly without requiring A_0.
Derive this formula.

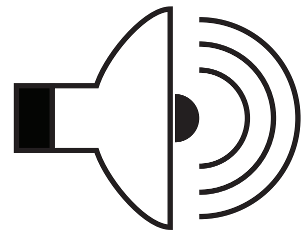
e) What is the ratio of seismograph amplitudes for earthquakes with magnitudes of 5.0 and 6.0?

f) Show that an equivalent form of the equation is: $\boxed{M_2 - M_1 = \log\frac{A_2}{A_1}}$

g) What is the magnitude of an earthquake with triple the seismograph amplitude of a magnitude 5.0 earthquake?

h) What is the magnitude of an earthquake with one-fourth the seismograph amplitude of a magnitude 6.0 earthquake?

Example 13



The loudness of a sound is measured in decibels, and can be calculated using the formula:

$$\boxed{L = 10\log\frac{I}{I_0}}$$

where L is the perceived loudness of the sound (dB), I is the intensity of the sound being measured (W/m^2), and I_0 is the intensity of sound at the threshold of human hearing (10^{-12} W/m^2).

a) The sound intensity of a person speaking in a conversation is 10^{-6} W/m^2. What is the perceived loudness?

b) A rock concert has a loudness of 110 dB. What is the sound intensity?

c) Two sounds have decibel measurements of 85 dB and 105 dB. Calculate the intensity ratio for the two sounds.

d) The calculation in part (c) required multiple steps because we are comparing each sound with I_0, instead of comparing the two sounds to each other. It is possible to derive the formula: $\boxed{\frac{I_2}{I_1} = 10^{\frac{L_2 - L_1}{10}}}$ which compares two sounds directly without requiring I_0. Derive this formula.

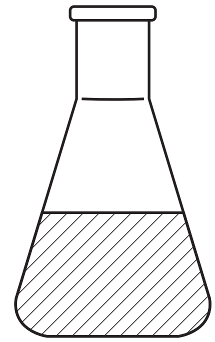
e) How many times more intense is 40 dB than 20 dB?

f) Show that an equivalent form of the equation is: $\boxed{L_2 - L_1 = 10\log\frac{I_2}{I_1}}$

g) What is the loudness of a sound twice as intense as 20 dB?

h) What is the loudness of a sound half as intense as 40 dB?

Example 14



The pH of a solution can be measured with the formula

$$\boxed{pH = -\log\left[H^+\right]}$$

where $[H^+]$ is the concentration of hydrogen ions in the solution (mol/L). Solutions with a pH less than 7 are acidic, and solutions with a pH greater than 7 are basic.

a) What is the pH of a solution with a hydrogen ion concentration of 10^{-4} mol/L? Is this solution acidic or basic?

b) What is the hydrogen ion concentration of a solution with a pH of 11?

c) Two acids have pH values of 3.0 and 6.0. Calculate the hydrogen ion ratio for the two acids.

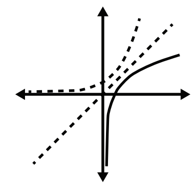
d) The calculation in part (c) required multiple steps. Derive the formulae *(on right)* that can be used to compare the two acids directly.

$$\frac{\left[H^+\right]_2}{\left[H^+\right]_1} = 10^{-(pH_2 - pH_1)}$$ and $$pH_2 - pH_1 = -\log\frac{\left[H^+\right]_2}{\left[H^+\right]_1}$$

e) What is the pH of a solution 1000 times more acidic than a solution with a pH of 5?

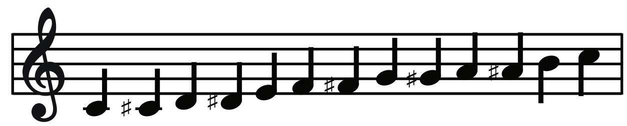
f) What is the pH of a solution with one-tenth the acidity of a solution with a pH of 4?

g) How many times more acidic is a solution with a pH of 2 than a solution with a pH of 4?



$y = \log_b x$

Example 15



In music, a chromatic scale divides an octave into 12 equally-spaced pitches. An octave contains 1200 cents *(a unit of measure for musical intervals)*, and each pitch in the chromatic scale is 100 cents apart. The relationship between cents and note frequency is given by the formula:

$$\boxed{c_2 - c_1 = 1200\left(\log_2 \frac{f_2}{f_1}\right)}$$

a) How many cents are in the interval between A (440 Hz) and B (494 Hz)?

b) There are 100 cents between F# and G. If the frequency of F# is 740 Hz, what is the frequency of G?

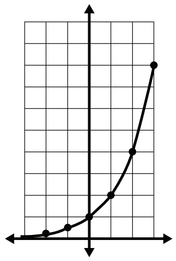
c) How many cents separate two notes, where one note is double the frequency of the other note?

For more practice solving logarithmic equations, return to ***Exponential Functions*** and solve the word problems using logarithms.

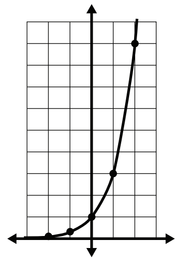
Answer Key

Exponential and Logarithmic Functions Lesson One: Exponential Functions

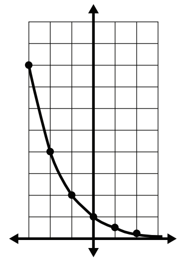
Example 1: **a)**



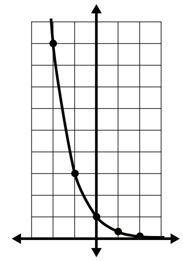
b)



c)



d)

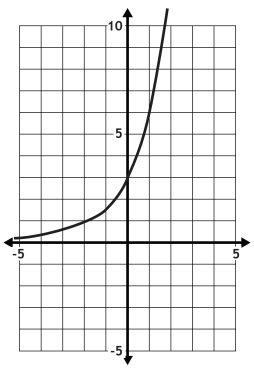


Parts (a-d):
Domain: x ε R or (-∞, ∞)
Range: y > 0 or (0, ∞)
x-intercept: None
y-intercept: (0, 1)
Asymptote: y = 0

An exponential function is defined as $y = b^x$, where b > 0 and b ≠ 1. When b > 1, we get exponential growth. When 0 < b < 1, we get exponential decay. Other b-values, such as -1, 0, and 1, will not form exponential functions.

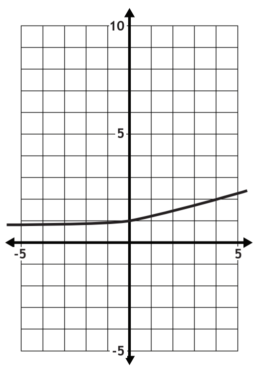
Example 2: **a)** $f(x) = 4^x$; $n = \frac{1}{16}$ **b)** $f(x) = \left(\frac{3}{2}\right)^x$; $n = \frac{8}{27}$ **c)** $f(x) = \left(\frac{1}{5}\right)^x$; $n = \frac{1}{5}$ **d)** $f(x) = \left(\frac{3}{4}\right)^x$; $n = \frac{27}{64}$

Example 3: **a)**



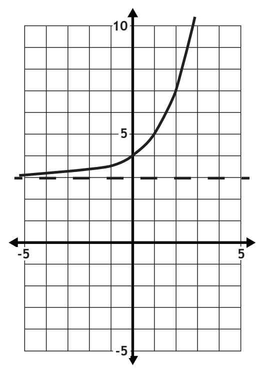
Domain: x ε R or (-∞, ∞)
Range: y > 0 or (0, ∞)
Asymptote: y = 0

b)



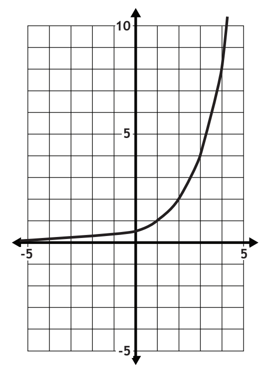
Domain: x ε R or (-∞, ∞)
Range: y > 0 or (0, ∞)
Asymptote: y = 0

c)



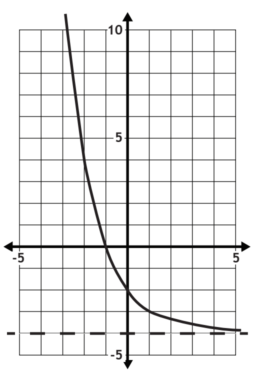
Domain: x ε R or (-∞, ∞)
Range: y > 3 or (3, ∞)
Asymptote: y = 3

d)



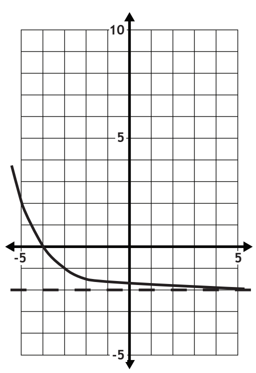
Domain: x ε R or (-∞, ∞)
Range: y > 0 or (0, ∞)
Asymptote: y = 0

Example 4: **a)**



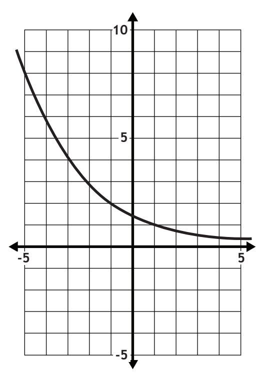
Domain: x ε R or (-∞, ∞)
Range: y > -4 or (-4, ∞)
Asymptote: y = -4

b)



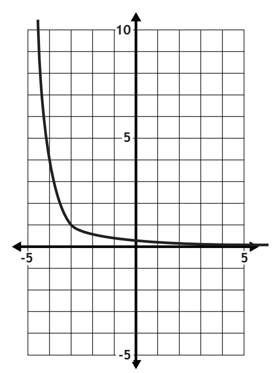
Domain: x ε R or (-∞, ∞)
Range: y > -2 or (-2, ∞)
Asymptote: y = -2

c)



Domain: x ε R or (-∞, ∞)
Range: y > 0 or (0, ∞)
Asymptote: y = 0

d)



Domain: x ε R or (-∞, ∞)
Range: y > 0 or (0, ∞)
Asymptote: y = 0

Example 5: **a)** $f(x) = \left(\frac{2}{3}\right)^x - 3$

$n = \frac{147}{32}$

b) $f(x) = \frac{1}{4}(3)^x + 1$

$n = \frac{973}{972}$

Example 6: **a)** $\left(0, \frac{a}{b^4}\right)$ **b)** $a = \frac{25}{3}$ **c)** $y = \frac{3}{4}\left(\frac{1}{3}\right)^x$ **d)** $y = 2^x - 3$ **e)** V.S. of 9 equals H.T. 2 units left. **f)** See Video

Answer Key

Example 7:

a) $x = 2$
b) $x = 16$
c) $x = \frac{1}{243}$
d) $x = \frac{1}{2}$

Example 8:

a) $x = 4$
b) $x = 5$
c) $x = 5$
d) $x = 6$
e) $x = -2;\ y = \frac{7}{2}$
f) $m = -\frac{11}{6};\ n = -3$

Example 9:

a) $x = -2$
b) $x = 4$
c) $x = 1$
d) $x = \frac{1}{2}$

Example 10:

a) $x = 6$
b) $x = 5$
c) $x = 18$
d) $x = 15$

Example 11:

a) $x = 2$
b) *infinite solutions*
c) $x = 1$
d) $x = 3$

Example 12:

a) $x = \frac{5}{2}$
b) $x = \frac{7}{2}$
c) $x = 3$
d) $x = 1$

Example 13:

a) $x = \frac{1}{2}, 1$
b) $x = -2, 1$
c) $x = 2$
d) $x = 5$

Example 14:

a) $x = 1.77$
b) *no solution*
c) $x = 1.79$
d) $x = 3$

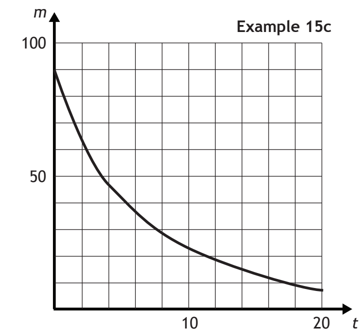
Example 15:

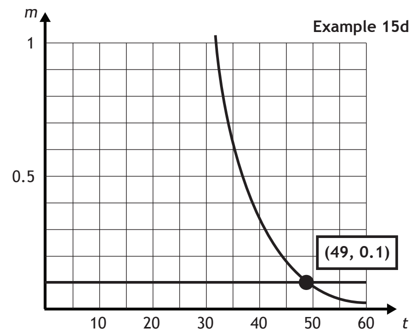
a) $m(t) = 90\left(\frac{1}{2}\right)^{\frac{t}{5}}$

b) 84 g

c) See Graph

d) 49 years





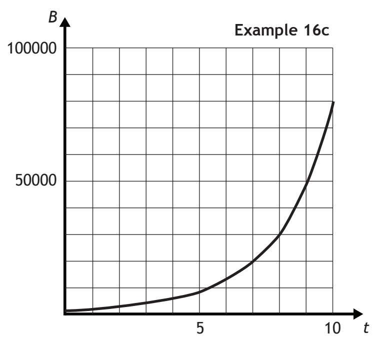
Example 16:

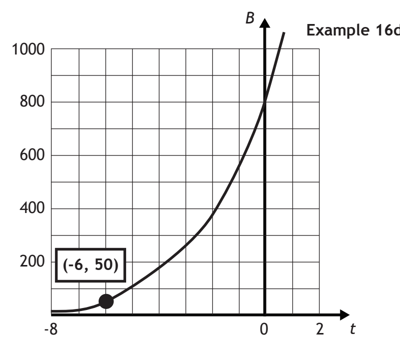
a) $B(t) = 800(2)^{\frac{t}{90}}$

b) 32254 bacteria

c) See Graph

d) 6 hours ago





Watch Out! The graph requires hours on the t-axis, so we can rewrite the exponential function as:

$$B(t) = 800(2)^{\frac{t}{1.5}}$$

Example 17: **a)** $S(t) = 16(1.44)^t$; 69 MHz **b)** $C(t) = 2500(0.70)^t$; $600

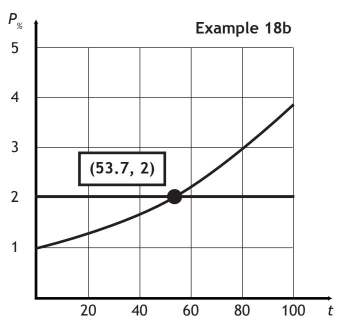
Example 18:

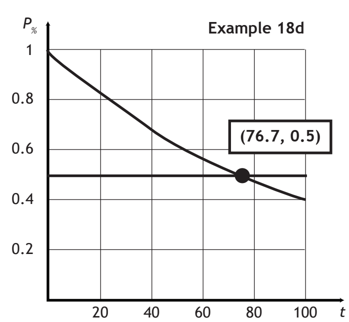
a) 853,370

b) 54 years

c) 21406

d) 77 years





Example 19:

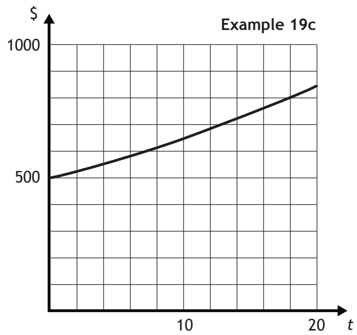
a) $A(t) = 500(1.025)^t$

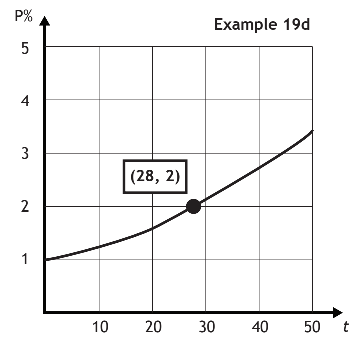
b) $565.70
Interest: $65.70

c) See graph

d) 28 years

e) $566.14; $566.50; $566.57
As the compounding frequency increases, there is less and less of a monetary increase.





Answer Key

Exponential and Logarithmic Functions Lesson Two: Laws of Logarithms

Example 1:

a) The base of the logarithm is ***b***,
a is called the argument of the logarithm,
and ***E*** is the result of the logarithm.

In the exponential form, ***a*** is the result,
b is the base, and ***E*** is the exponent.

b) i. 0; 1; 2; 3 ii. 0; 1; 2; 3

c) i. $\log_4 2$ ii. $\log_9\left(\frac{1}{3}\right)$

Example 2:

a) $\log_9\left(\frac{1}{3}\right)$, $\log_{16}\left(\frac{1}{2}\right)$, $\log_5 1$, $\log 10$, $\log_2 16$

b) $\log_{\frac{1}{3}} 27$, $\log_{\frac{1}{4}} 8$, $\log_{\frac{1}{8}}\left(\frac{1}{2}\right)$, $\log_{\frac{1}{4}}\left(\frac{1}{2}\right)$, $\log_{\frac{1}{8}}\left(\frac{1}{8}\right)$

c) $\log_8 3$, $\log_6 7$, $\log_{\frac{1}{4}}\left(\frac{1}{15}\right)$, $\log_3 25$

Example 3:

a) $y = 2^x$

b) $y = 16$

c) $y = 10^{\frac{x}{a}}$

d) $y = \frac{3^x}{2}$

e) $y = x^{\frac{1}{2}}$

f) $y = 8 + x$

g) $y = (x+1)^2 - 1$

h) $y = 3^{2x-1}$

Example 4:

a) $\log_x y = 2$

b) $\log_x \frac{y}{10} = 4$

c) $\log_{\frac{1}{3}} y = x$

d) $\log_x 3y = \frac{1}{2}$

e) $\log_{\frac{x}{2}} y = \frac{1}{3}$

f) $\log_{x-3} y = 2$

g) $\log_k y = x - 1$

h) $\log a = y - x$

Example 5:

a) 3

b) 3

c) 2

d) 2

e) $\log_{25} 5$

f) $\log_3 \sqrt{3}$

g) $\log_{\frac{1}{3}} \frac{1}{2}$

h) $\log_a b$

Example 6:

a) $\log x + \log y$

b) can't expand

c) $\log 3 + \log(x+1)$

d) $1 + \log x$

e) $\log 12$

f) $\log \frac{1}{2}$

g) $\log x^5$

h) $\log(x^2 - x - 2)$

Example 7:

a) $\log x - \log y$

b) can't expand

c) $\log(x+1) - 2$

d) $\log_3 x - 1 - \log_3(x+1)$

e) $\log 3$

f) $\log \frac{1}{6}$

g) $\log x^3$

h) $\log\left(\frac{2x}{x+3}\right)$

Example 8:

a) $2\log x$

b) can't expand

c) $7\log x$

d) $a\log x + \log x$

e) $\log x^3$

f) $\log(x-1)^2$

g) $\log(8x^6)$

h) $\log x^2$

Example 9:

a) *undefined*

b) *undefined*

c) 0

d) 1

e) x

f) x

g) $2k$

h) $\frac{k}{2}$

Example 10:

a) 160

b) $4a$

c) $2k$

d) $\frac{h}{2}$

e) -7

f) 7

g) $\log(10x)$

h) $\log_2(8x)$

Example 11:

a) $x = \log_3 4$

b) no solution

c) $x = \log_5\left(\frac{7}{2}\right) - 2$

d) $x = \log_{\frac{2}{5}}\left(\frac{1}{3}\right) + 3$

Example 12:

a) $x = \frac{-\log 3}{5\log 6 - 2\log 3}$

b) $x = \frac{-\log 3 - 3\log 2}{\log 2 - 2\log 3}$

c) $x = \frac{\log 4 + \log 3}{2\log 4 - \log 5}$

d) $x = \frac{-\log 2 - 3\log 3}{\log 3 - 3\log 6}$

Example 13:

a) $x = 10$

b) $x = 8$

c) $x = -2$

d) $x = 14$

Example 14:

a) $x = 2$

b) $x = 5$

c) $x = \frac{2}{3}$

d) $x = \pm\sqrt{29}$

Example 15:

a) $x = 6$

b) $x = 5$

c) $x = \frac{1}{10}, 100000$

d) $x = \frac{1}{100}, 100$

Example 16:

a) $\frac{1}{4}$

b) $\log\left(\frac{\sqrt{a}}{b^3c^2}\right)$

c) 16

d) 2

e) $\log_b(2a) = \frac{5}{4}$

f) $\log_5 x$

g) $2x + 24$

h) $\log_3\left(9\sqrt[3]{x}\right)$

Example 17:

a) 12

b) 14

c) 3^{233}

d) 1

e) 100

f) $\log_2\left(a\sqrt{b}\right)$

g) 3

h) 15

Example 18:

a) -3

b) no solution

c) x

d) 1,100

e) a^2

f) $\frac{1}{2}$

g) see video

h) $\log_2(4x+2)$

Example 19:

a) 4

b) $\log(ab)^3$

c) 2

d) $\log(a+1)$

e) -2

f) $x = \pm 1$

g) $\frac{5}{2}$

h) $\log_{16}(4x^3)$

Example 20:

a) 2

b) $\frac{3\log 5 + \log 2}{3\log 2 - 2\log 5}$

c) 199

d) $\log\left(\frac{1}{x}\right)$

e) 9

f) $10^{\frac{8}{5}}$

g) $\log\left(\frac{a^4c}{b^{\frac{1}{2}}}\right)$

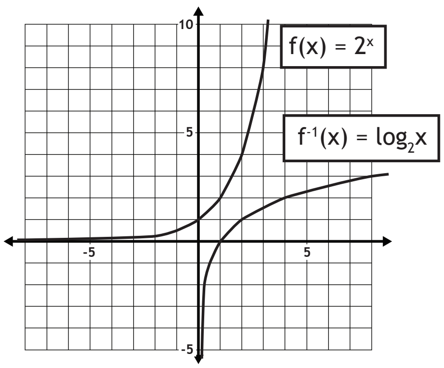
h) 2

Answer Key

Exponential and Logarithmic Functions Lesson Three: Logarithmic Functions

Example 1:

a) See Graph **b)** See Graph **c)** See Video



d)

	$y = 2^x$	$y = \log_2 x$
Domain	x ε R	x > 0
Range	y > 0	y ε R
x-intercept	none	(1, 0)
y-intercept	(0, 1)	none
Asymptote Equation	y = 0	x = 0

e)
i) -1,
ii) 0,
iii) 1,
iv) 2.8

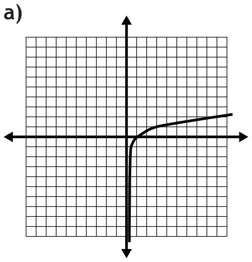
f)
$y = \log_1 x$, $y = \log_0 x$, and $y = \log_{-2} x$ are not functions.
$y = \log_{\frac{1}{10}} x$ is a function.

g) The logarithmic function $y = \log_b x$ is the inverse of the exponential function $y = b^x$. It is defined for all real numbers such that b>0 and x>0.

h) Graph $\log_2 x$ using logx/log2

Example 2:

a)

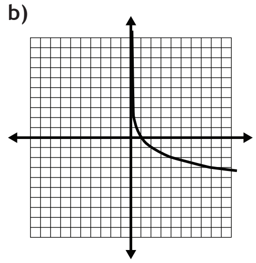


D: x > 0
or (0, ∞)

R: y ε R
or (-∞, ∞)

A: x = 0

b)

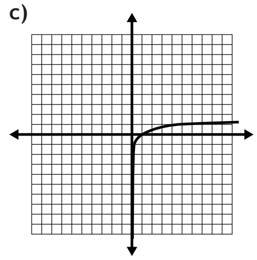


D: x > 0
or (0, ∞)

R: y ε R
or (-∞, ∞)

A: x = 0

c)

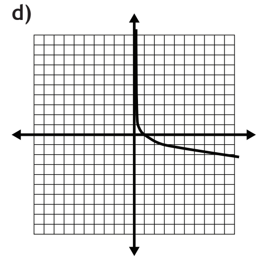


D: x > 0
or (0, ∞)

R: y ε R
or (-∞, ∞)

A: x = 0

d)



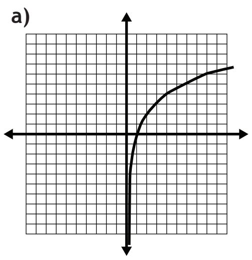
D: x > 0
or (0, ∞)

R: y ε R
or (-∞, ∞)

A: x = 0

Example 3:

a)

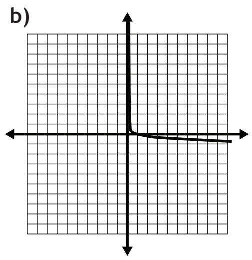


D: x > 0
or (0, ∞)

R: y ε R
or (-∞, ∞)

A: x = 0

b)

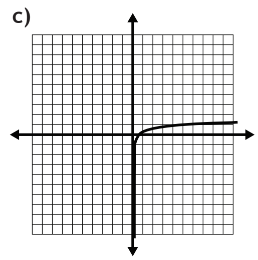


D: x > 0
or (0, ∞)

R: y ε R
or (-∞, ∞)

A: x = 0

c)

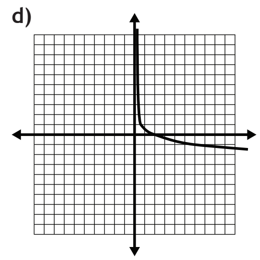


D: x > 0
or (0, ∞)

R: y ε R
or (-∞, ∞)

A: x = 0

d)



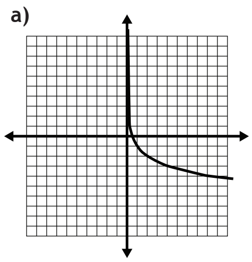
D: x > 0
or (0, ∞)

R: y ε R
or (-∞, ∞)

A: x = 0

Example 4:

a)

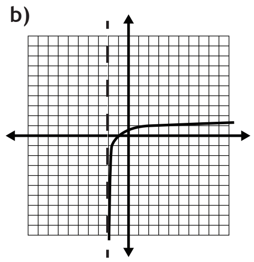


D: x > 0
or (0, ∞)

R: y ε R
or (-∞, ∞)

A: x = 0

b)

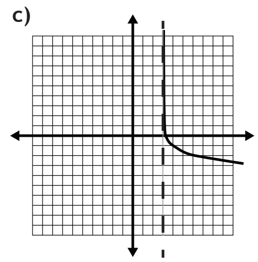


D: x > -2
or (-2, ∞)

R: y ε R
or (-∞, ∞)

A: x = -2

c)

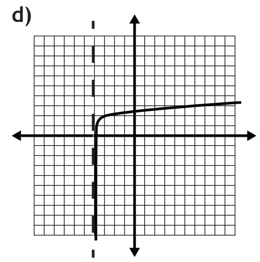


D: x > 3
or (3, ∞)

R: y ε R
or (-∞, ∞)

A: x = 3

d)



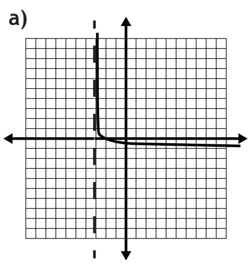
D: x > -4
or (-4, ∞)

R: y ε R
or (-∞, ∞)

A: x = -4

Example 5:

a)

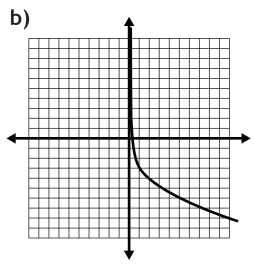


D: x > -3
or (-3, ∞)

R: y ε R
or (-∞, ∞)

A: x = -3

b)

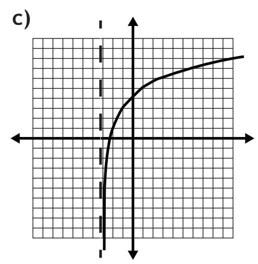


D: x > 0
or (0, ∞)

R: y ε R
or (-∞, ∞)

A: x = 0

c)

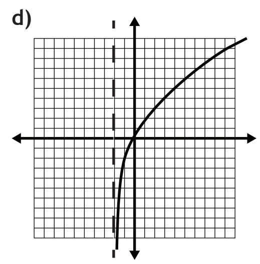


D: x > -3
or (-3, ∞)

R: y ε R
or (-∞, ∞)

A: x = -3

d)



D: x > -2
or (-2, ∞)

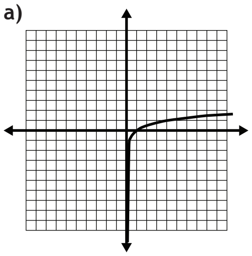
R: y ε R
or (-∞, ∞)

A: x = -2

Answer Key

Example 6:

a)

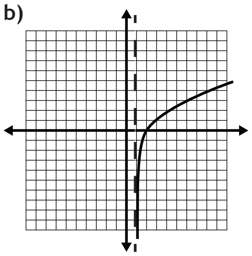


D: x > 0
or (0, ∞)

R: y ε R
or (-∞, ∞)

A: x = 0

b)

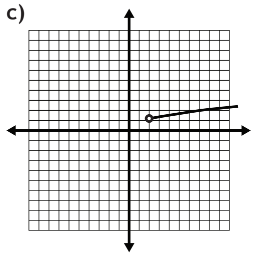


D: x > 1
or (1, ∞)

R: y ε R
or (-∞, ∞)

A: x = 1

c)

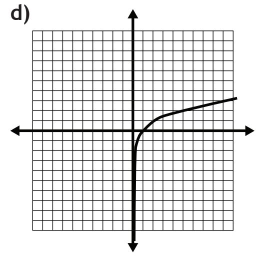


D: x > 2
or (2, ∞)

R: y > $\log_3 4$
or ($\log_3 4$, ∞)

A: none

d)



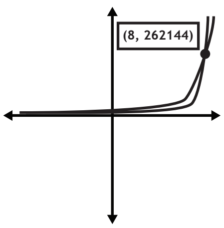
D: x > 0
or (0, ∞)

R: y ε R
or (-∞, ∞)

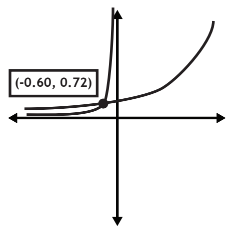
A: x = 0

Example 7:

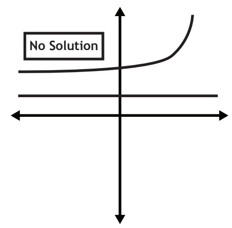
a) x = 8



b) $x = \frac{-2\log 5}{4\log 5 - \log 3}$

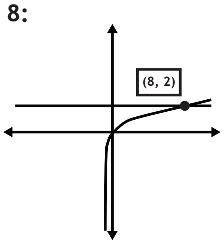


c) No Solution

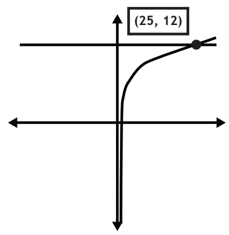


Example 8:

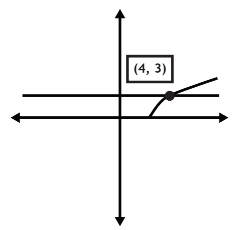
a) x = 8



b) x = 25



c) x = 4



Example 9:

a) $b = 2\sqrt{2}$

b) (-3, 0) and (0, 2)

c) $x = \frac{8}{3}$

d) $k = 81$

e) $0 < x < 6,\ x \neq 1$

Example 10:

a) $y = f(9x)$; $y = f(x) + 2$

b) $(-1, 1)$

c) $y = f(x) - 2$

d) $(1, 0)$

e) $x = \frac{1}{k}$

Example 11:

a) $x = 0$ *(y-axis)*

b) $g(x) = f^{-1}(x)$

c) $f^{-1}(x) = \log_3(x - 4)$

d) $x > 4$

e) $k = 8$

Example 12:

a) 4

b) 0.1 m

c) 31.6 times stronger

d) See Video

e) 10 times stronger

f) See Video

g) 5.5

h) 5.4

Example 13:

a) 60 dB

b) 0.1 W/m^2

c) 100 times more intense

d) See Video

e) 100 times more intense

f) See Video

g) 23 dB

h) 37 dB

Example 14:

a) pH = 4

b) 10^{-11} mol/L

c) 1000 times stronger

d) See Video

e) pH = 2

f) pH = 5

g) 100 times more acidic

Example 15:

a) 200 cents

b) 784 Hz

c) 1200 cents separate
the two notes