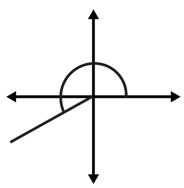
Mathematics 30-1

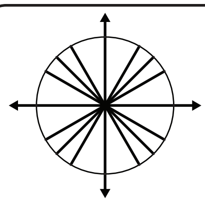
Student Workbook

Unit 4



$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$

Lesson 1: Degrees and Radians
Approximate Completion Time: 4 Days

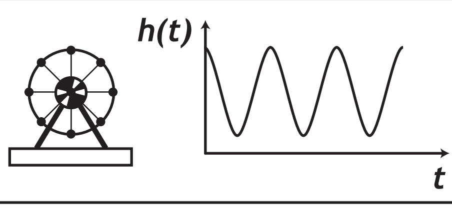


$(\cos\theta, \sin\theta)$

Lesson 2: The Unit Circle
Approximate Completion Time: 4 Days

$y = a\sin b(\theta - c) + d$

Lesson 3: Trigonometric Functions I
Approximate Completion Time: 4 Days



Lesson 4: Trigonometric Functions II
Approximate Completion Time: 4 Days

UNIT FOUR
Trigonometry I

Mathematics 30-1

Student Workbook

Unit
4

Complete this workbook by watching the videos on **www.math30.ca**.
Work neatly and use proper mathematical form in your notes.

UNIT FOUR

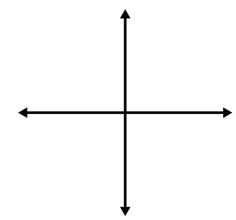
Trigonometry I

$$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$$

Trigonometry
LESSON ONE - *Degrees and Radians*
Lesson Notes

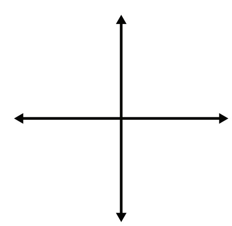
Example 1 Define each term or phrase and draw a sample angle. Angle Definitions

a) angle in standard position:

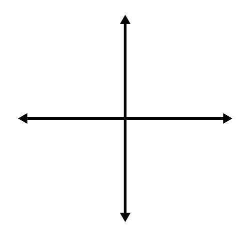


Draw a standard position angle, θ.

b) positive and negative angles:

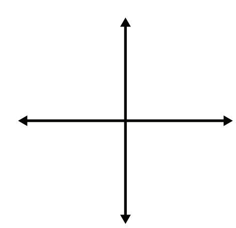


Draw θ = 120°



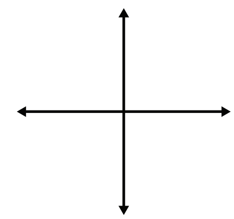
Draw θ = -120°

c) reference angle:



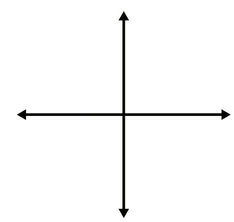
Find the reference angle of θ = 150°.

d) co-terminal angles:



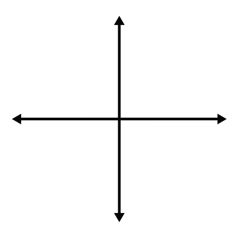
Draw the first positive co-terminal angle of 60°.

e) principal angle:

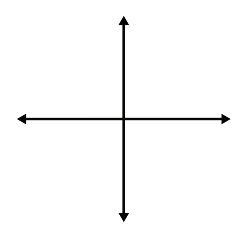


Find the principal angle of θ = 420°.

f) general form of co-terminal angles:



Find the first four ***positive*** *co-terminal angles of θ = 45°.*



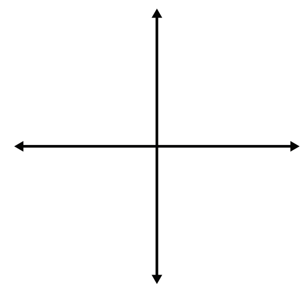
Find the first four ***negative*** *co-terminal angles of θ = 45°.*

Example 2 Three Angle Types:
***Degrees*, *Radians*, *and Revolutions*.**

Angle Types and Conversion Multipliers

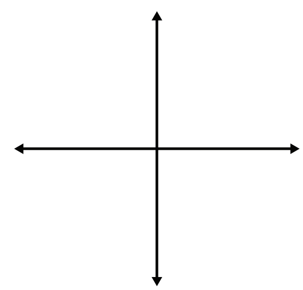
a) Define degrees, radians, and revolutions.

i) Degrees:



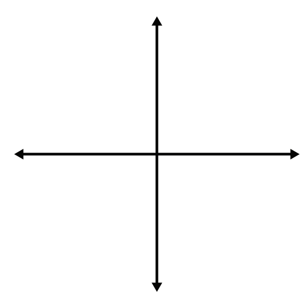
Draw θ = 1°

ii) Radians:



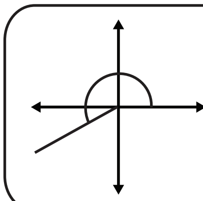
Draw θ = 1 rad

iii) Revolutions:



Draw θ = 1 rev

Trigonometry
LESSON ONE - *Degrees and Radians*
Lesson Notes



$$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$$

b) Use conversion multipliers to answer the questions and fill in the reference chart.
Round all decimals to the nearest hundredth.

i) $23^\circ \times \boxed{\quad\quad} =$ ________ rad

ii) $23^\circ \times \boxed{\quad\quad} =$ ________ rev

iii) $2.6 \times \boxed{\quad\quad} =$ ________ °

iv) $2.6 \times \boxed{\quad\quad} =$ ________ rev

v) $0.75 \text{ rev} \times \boxed{\quad\quad} =$ ________ °

vi) $0.75 \text{ rev} \times \boxed{\quad\quad} =$ ________ rad

Conversion Multiplier Reference Chart

	degree	radian	revolution
degree			
radian			
revolution			

c) Contrast the decimal approximation of a radian with the exact value of a radian.

i) Decimal Approximation: $45^\circ \times \boxed{\quad\quad} =$ ________ rad

ii) Exact Value: $45^\circ \times \boxed{\quad\quad} =$ ________ rad

Example 3

Convert each angle to the requested form.
Round all decimals to the nearest hundredth.

Angle Conversion Practice

a) convert 175° to an approximate radian decimal.

b) convert 210° to an exact-value radian.

c) convert 120° to an exact-value revolution.

d) convert 2.5 to degrees.

e) convert $\frac{3\pi}{2}$ to degrees.

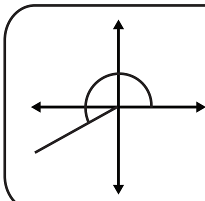
f) write $\frac{3\pi}{2}$ as an approximate radian decimal.

g) convert $\frac{\pi}{2}$ to an exact-value revolution.

h) convert 0.5 rev to degrees.

i) convert 3 rev to radians.

Trigonometry
LESSON ONE - *Degrees and Radians*
Lesson Notes



$$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$$

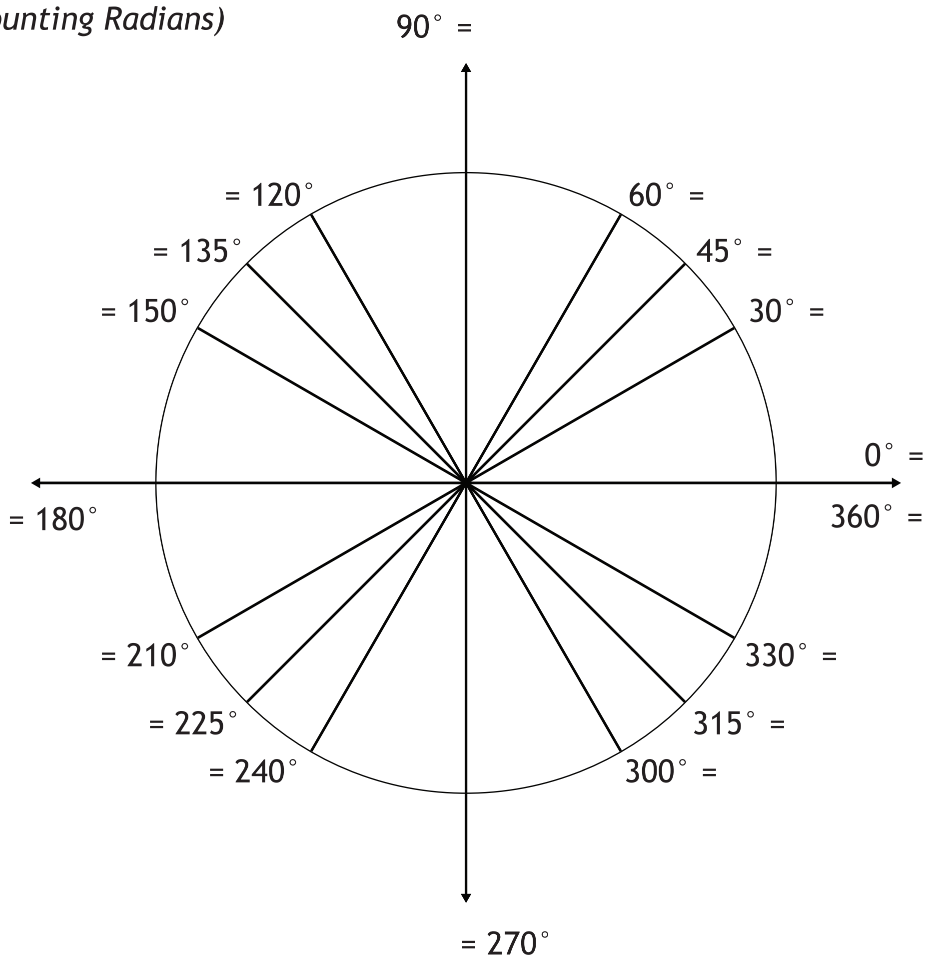
Example 4

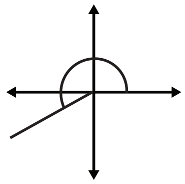
The diagram shows commonly used degrees. Find exact-value radians that correspond to each degree. When complete, memorize the diagram.

Commonly Used Degrees and Radians

a) ***Method One:*** Find all exact-value radians using a conversion multiplier.

b) ***Method Two:*** Use a shortcut. *(Counting Radians)*





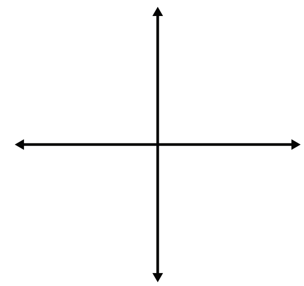
$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$

Trigonometry
LESSON ONE - *Degrees and Radians*
Lesson Notes

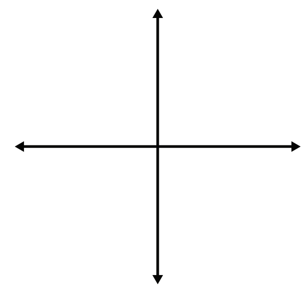
Example 5 Draw each of the following angles in standard position. State the reference angle.

Reference Angles

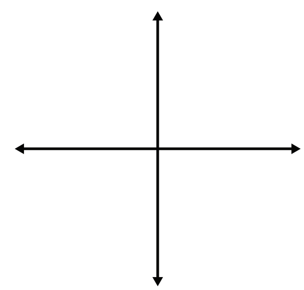
a) 210°



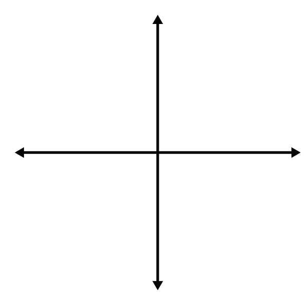
b) -260°



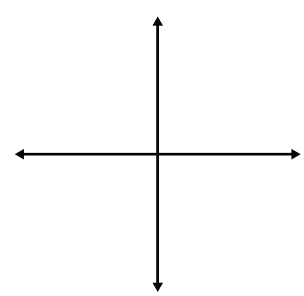
c) 5.3

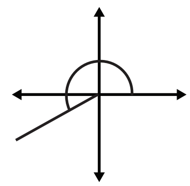


d) $-\frac{5\pi}{4}$



e) $\frac{12\pi}{7}$



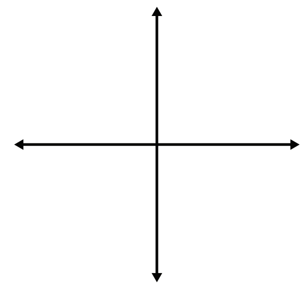


$$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$$

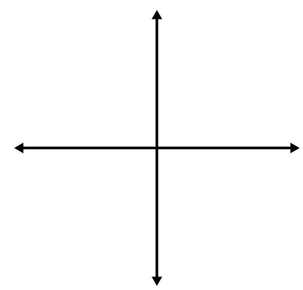
Example 6 Draw each of the following angles in standard position. State the principal and reference angles.

Principal and Reference Angles

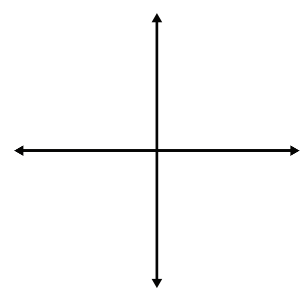
a) 930°



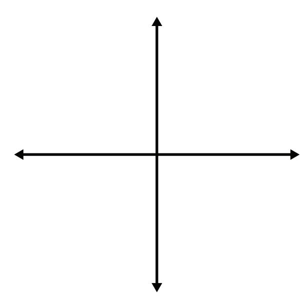
b) -855°



c) 9



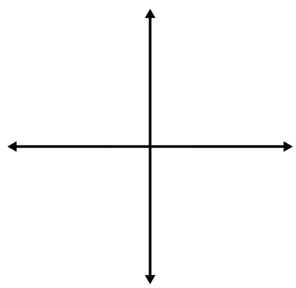
d) $-\frac{10\pi}{3}$



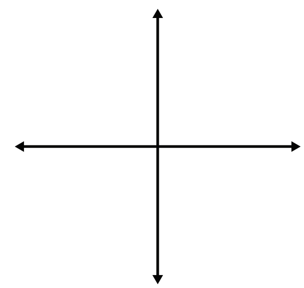
Example 7 For each angle, find all co-terminal angles within the stated domain.

Co-terminal Angles

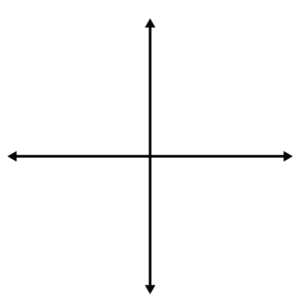
a) 60°, Domain: $-360^\circ \le \theta < 1080^\circ$



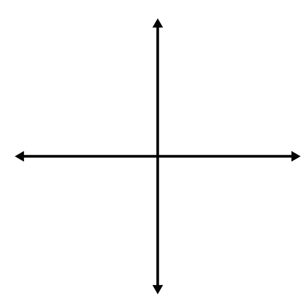
b) -495°, Domain: $-1080^\circ \le \theta < 720^\circ$



c) 11.78, Domain: $-2\pi \le \theta < 4\pi$



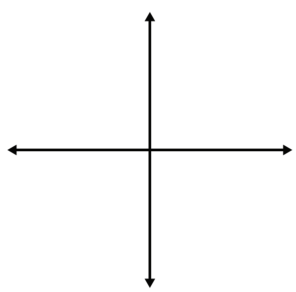
d) $\frac{8\pi}{3}$, Domain: $-\frac{13\pi}{2} \le \theta < \frac{37\pi}{5}$



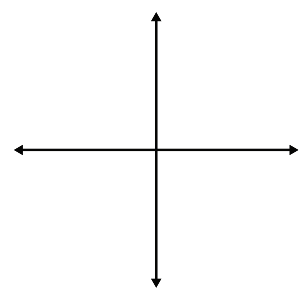
Example 8 For each angle, use estimation to find the principal angle.

Principal Angle of a Large Angle

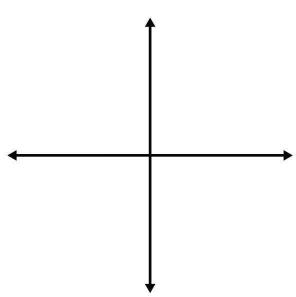
a) 1893°



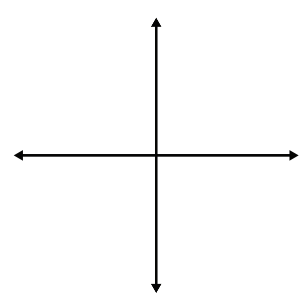
b) -437.24



c) $\frac{912\pi}{15}$



d) $\frac{95\pi}{6}$



Example 9

Use the general form of co-terminal angles to find the specified angle.

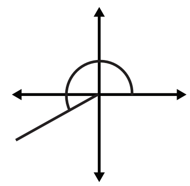
General Form of Co-terminal Angles

a) principal angle = 300°
(find co-terminal angle 3 rotations counter-clockwise)

b) principal angle = $\frac{2\pi}{5}$
(find co-terminal angle 14 rotations clockwise)

c) How many rotations are required to find the principal angle of -4300°? State the principal angle.

d) How many rotations are required to find the principal angle of $\frac{32\pi}{3}$? State the principal angle.

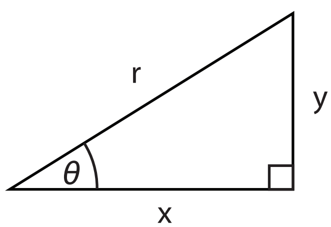


$$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$$

Example 10

Six Trigonometric Ratios

In addition to the three primary trigonometric ratios (sinθ, cosθ, and tanθ), there are three reciprocal ratios (cscθ, secθ, and cotθ). Given a triangle with side lengths of x and y, and a hypotenuse of length r, the six trigonometric ratios are as follows:

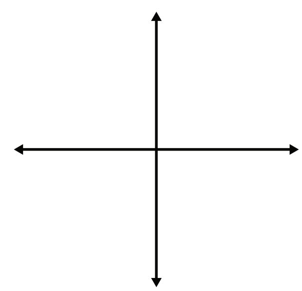


$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{r}{y}$$

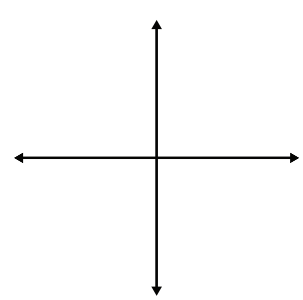
$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$$

a) If the point P(-5, 12) exists on the terminal arm of an angle θ in standard position, determine the exact values of all six trigonometric ratios. State the reference angle and the standard position angle.



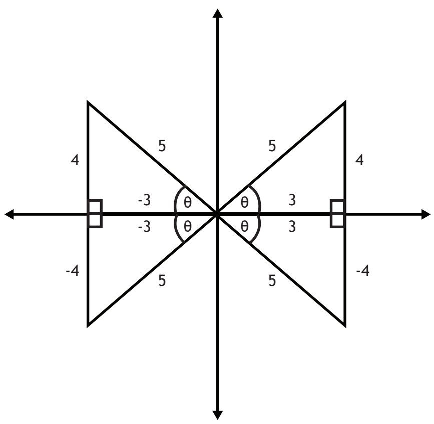
b) If the point P(2, -3) exists on the terminal arm of an angle θ in standard position, determine the exact values of all six trigonometric ratios. State the reference angle and the standard position angle.



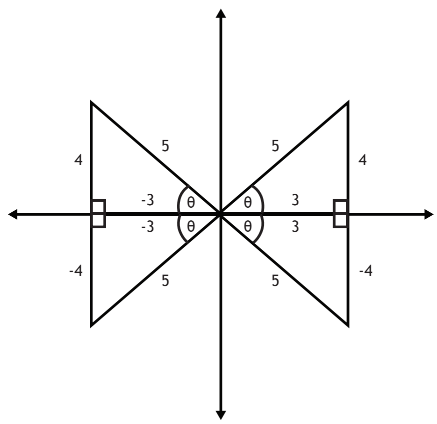
Example 11 Determine the sign of each trigonometric ratio in each quadrant.

Signs of Trigonometric Ratios

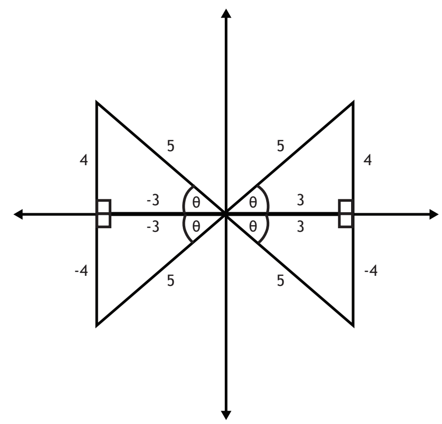
a) sinθ



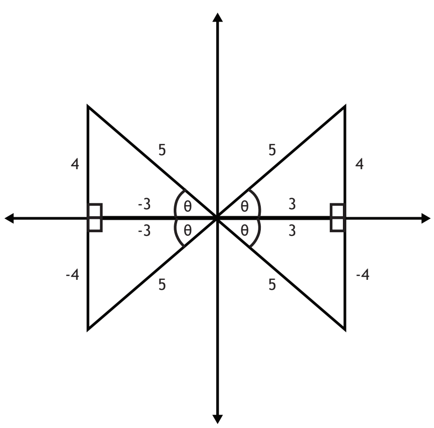
b) cosθ



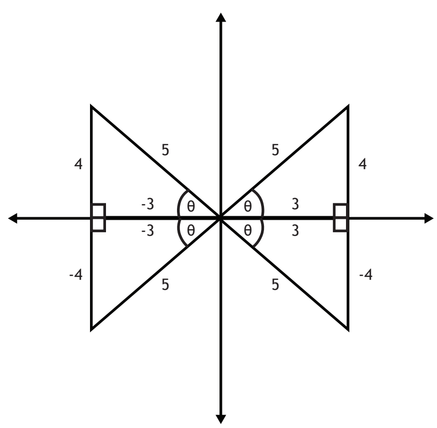
c) tanθ



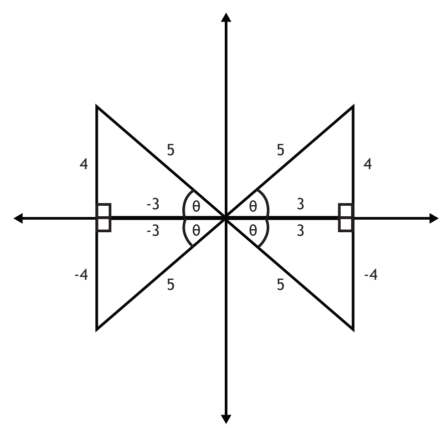
d) cscθ



e) secθ



f) cotθ



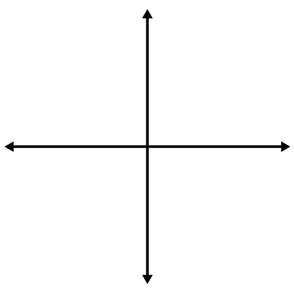
g) How do the quadrant signs of the reciprocal trigonometric ratios (cscθ, secθ, and cotθ) compare to the quadrant signs of the primary trigonometric ratios (sinθ, cosθ, and tanθ)?

Example 12 Given the following conditions, find the quadrant(s) where the angle θ could potentially exist.

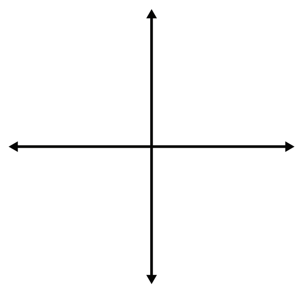
What Quadrant(s) is the Angle in?

a)

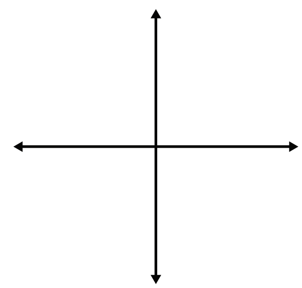
i) $\sin\theta < 0$



ii) $\cos\theta > 0$

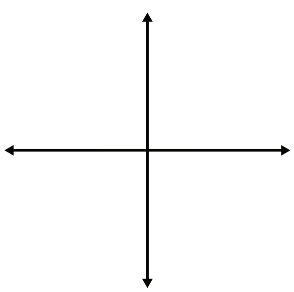


iii) $\tan\theta > 0$

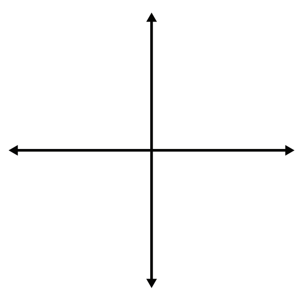


b)

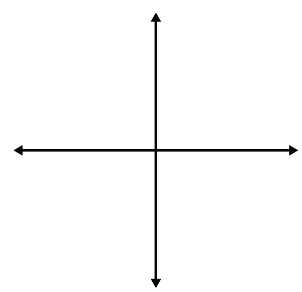
i) $\sin\theta > 0$ and $\cos\theta > 0$



ii) $\sec\theta > 0$ and $\tan\theta < 0$

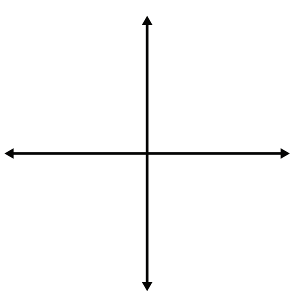


iii) $\csc\theta < 0$ and $\cot\theta > 0$

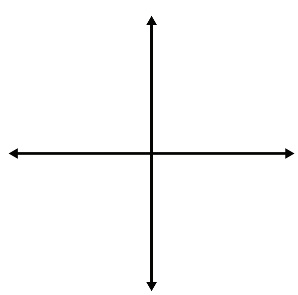


c)

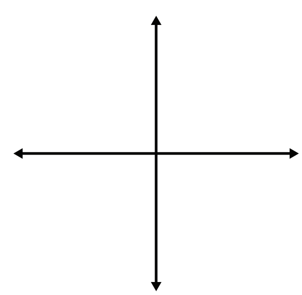
i) $\sin\theta < 0$ and $\csc\theta = \frac{1}{2}$



ii) $\cos\theta = -\frac{\sqrt{3}}{2}$ and $\csc\theta < 0$



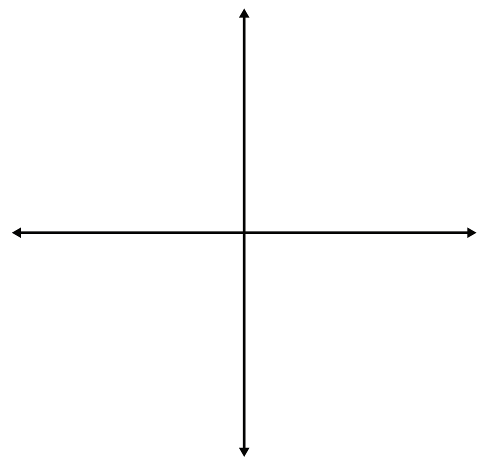
iii) $\sec\theta > 0$ and $\tan\theta = 1$



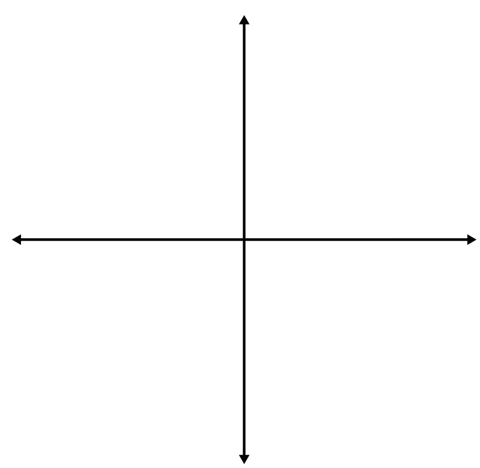
Example 13 Given one trigonometric ratio, find the exact values of the other five trigonometric ratios. State the reference angle and the standard position angle, to the nearest hundredth of a radian.

Exact Values of Trigonometric Ratios

a) $\cos\theta = -\frac{12}{13}, \quad \pi \le \theta < \frac{3\pi}{2}$



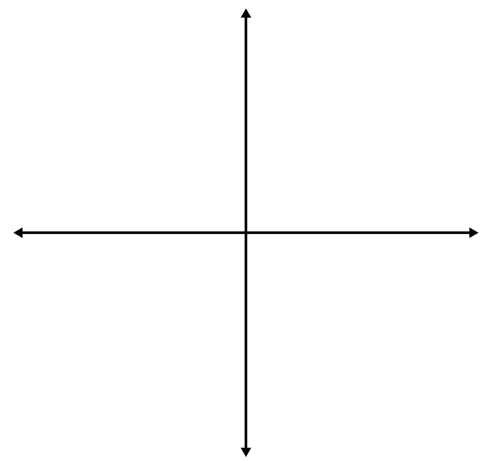
b) $\csc\theta = \frac{7}{3}, \quad \frac{\pi}{2} \le \theta < \pi$



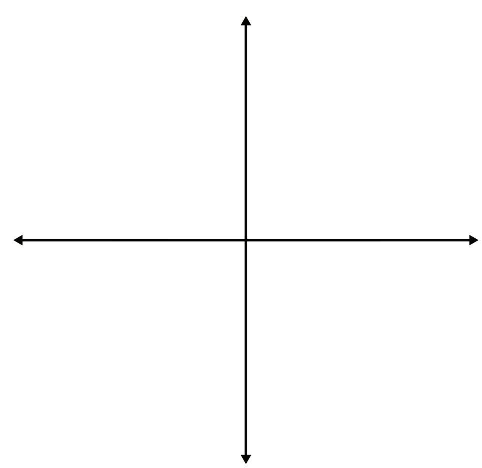
Example 14 Given one trigonometric ratio, find the exact values of the other five trigonometric ratios. State the reference angle and the standard position angle, to the nearest hundredth of a degree.

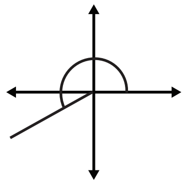
Exact Values of Trigonometric Ratios

a) $\sec\theta = \frac{5}{4}$, $\sin\theta < 0$



b) $\tan\theta = -\frac{2}{3}$, $\sec\theta > 0$





$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$

Example 15

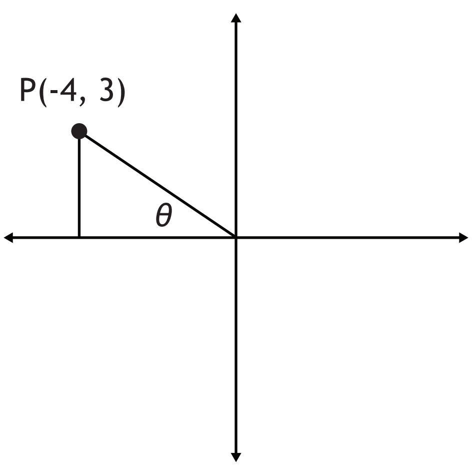
Calculating θ with a calculator.

Calculator Concerns

a) When you solve a trigonometric equation in your calculator, the answer you get for θ can seem unexpected. Complete the following chart to learn how the calculator processes your attempt to solve for θ.

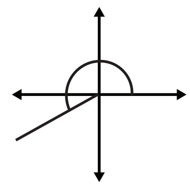
If the angle θ could exist in either quadrant ___ or ___ ...	The calculator always picks quadrant
I or II	
I or III	
I or IV	
II or III	
II or IV	
III or IV	

b) Given the point P(-4, 3), Mark tries to find the reference angle using a sine ratio, Jordan tries to find it using a cosine ratio, and Dylan tries to find it using a tangent ratio. Why does each person get a different result from their calculator?



Mark's Calculation of θ (using sine)	Jordan's Calculation of θ (using cosine)	Dylan's Calculation of θ (using tan)
$\sin\theta = \frac{3}{5}$ $\theta = 36.87^\circ$	$\cos\theta = \frac{-4}{5}$ $\theta = 143.13^\circ$	$\tan\theta = \frac{3}{-4}$ $\theta = -36.87^\circ$

Trigonometry
LESSON ONE - *Degrees and Radians*
Lesson Notes

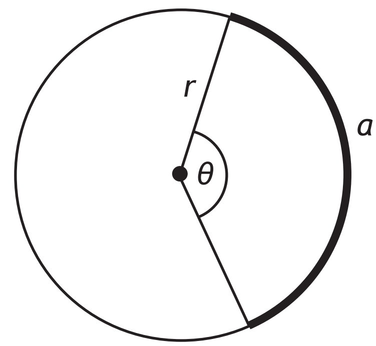


$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$

Example 16

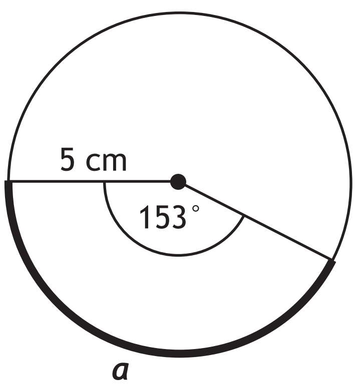
Arc Length

The formula for arc length is $a = r\theta$, where a is the arc length, θ is the central angle in radians, and r is the radius of the circle. The radius and arc length must have the same units.

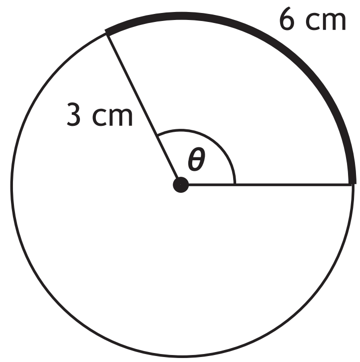


a) Derive the formula for arc length, a = rθ.

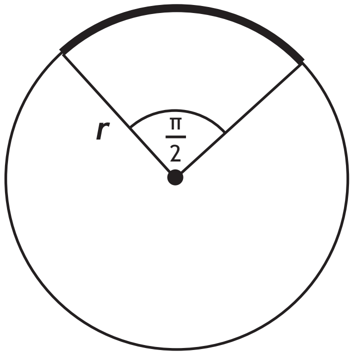
b) Solve for a, to the nearest hundredth.



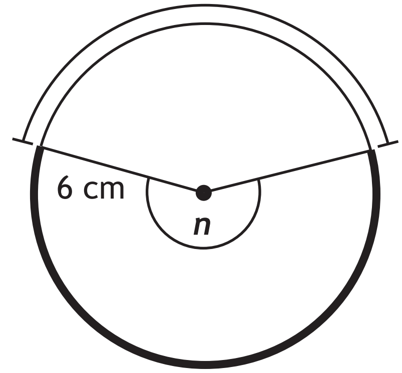
c) Solve for θ. *(express your answer as a degree, to the nearest hundredth.)*



d) Solve for r, to the nearest hundredth.



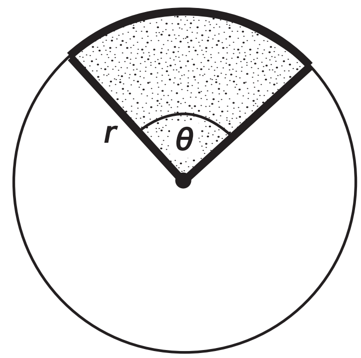
e) Solve for n. *(express your answer as an exact-value radian.)*



Example 17 Area of a circle sector.

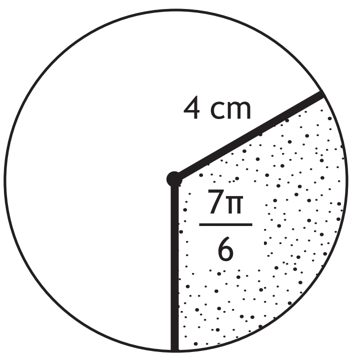
Sector Area

a) Derive the formula for the area of a circle sector, $A = \frac{r^2\theta}{2}$.

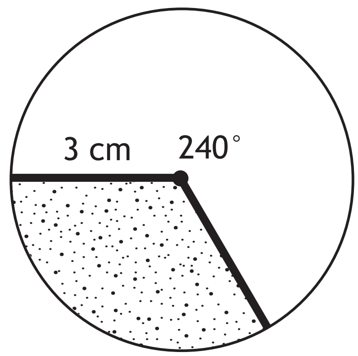


In parts (b - e), find the area of each shaded region.

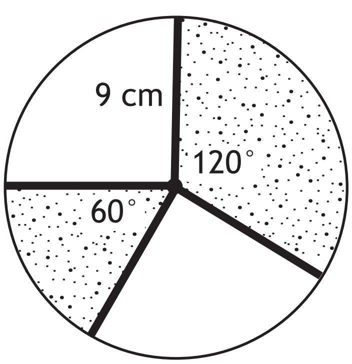
b)



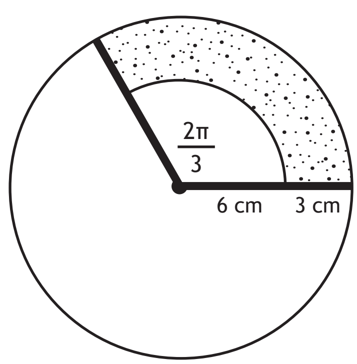
c)



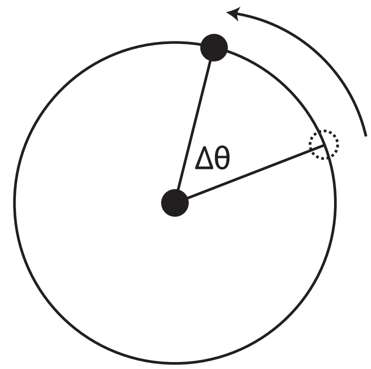
d)



e)



Example 18



The formula for angular speed is $\omega = \frac{\Delta\theta}{\Delta T}$, where ω (Greek: Omega) is the angular speed, $\Delta\theta$ is the change in angle, and ΔT is the change in time. Calculate the requested quantity in each scenario. Round all decimals to the nearest hundredth.

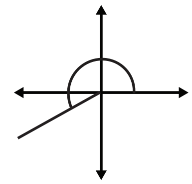
a) A bicycle wheel makes 100 complete revolutions in 1 minute. Calculate the angular speed in degrees per second.

b) A Ferris wheel rotates 1020° in 4.5 minutes. Calculate the angular speed in radians per second.

c) The moon orbits Earth once every 27 days. Calculate the angular speed in revolutions per second. If the average distance from the Earth to the moon is 384 400 km, how far does the moon travel in one second?

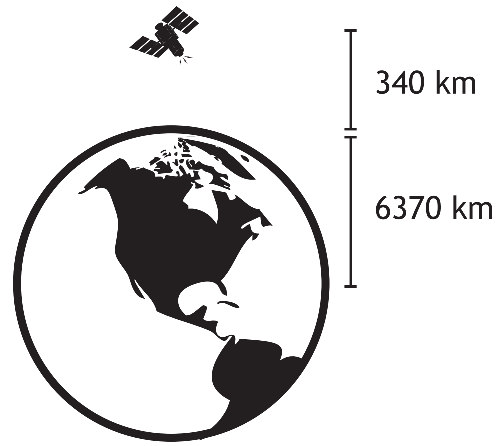
d) A cooling fan rotates with an angular speed of 4200 rpm. What is the speed in rps?

e) A bike is ridden at a speed of 20 km/h, and each wheel has a diameter of 68 cm. Calculate the angular speed of one of the bicycle wheels and express the answer using revolutions per second.



$$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$$

Example 19



A satellite orbiting Earth 340 km above the surface makes one complete revolution every 90 minutes. The radius of Earth is approximately 6370 km.

a) Calculate the angular speed of the satellite. Express your answer as an exact value, in radians/second.

b) How many kilometres does the satellite travel in one minute? Round your answer to the nearest hundredth of a kilometre.

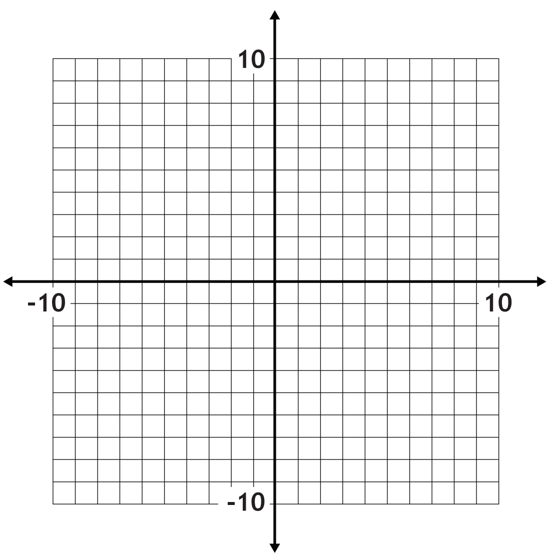
Trigonometry
LESSON TWO - *The Unit Circle*
Lesson Notes

Example 1 Introduction to Circle Equations.

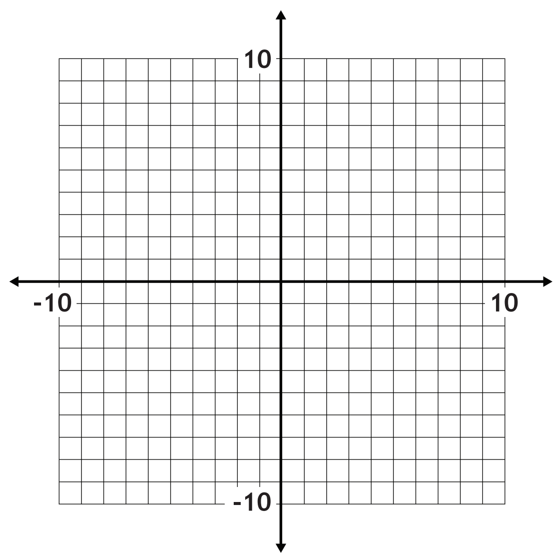
Equation of a Circle

a) A circle centered at the origin can be represented by the relation $x^2 + y^2 = r^2$, where r is the radius of the circle. Draw each circle:

i) $x^2 + y^2 = 4$



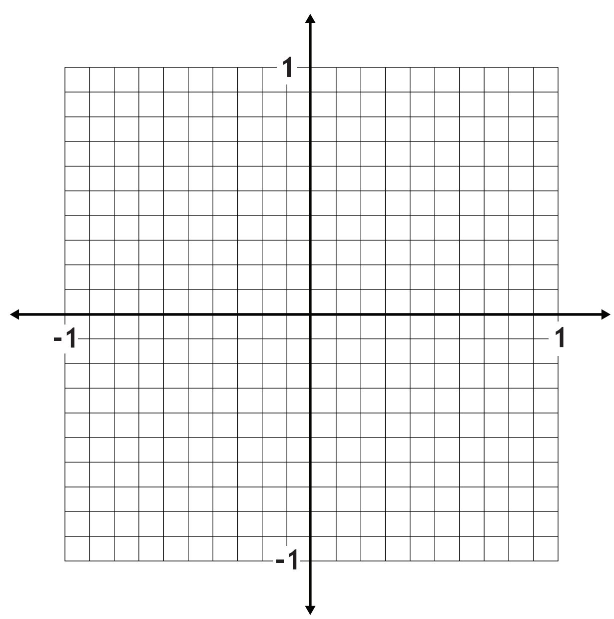
ii) $x^2 + y^2 = 49$



b) A circle centered at the origin with a radius of 1 has the equation $x^2 + y^2 = 1$. This special circle is called the *unit circle*. Draw the unit circle and determine if each point exists on the circumference of the unit circle.

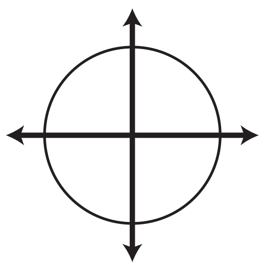
i) (0.6, 0.8)

ii) (0.5, 0.5)

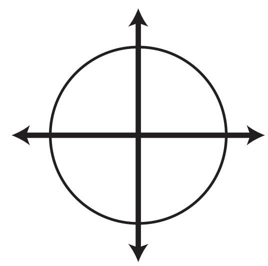


c) Using the equation of the unit circle, $x^2 + y^2 = 1$, find the unknown coordinate of each point. Is there more than one unique answer?

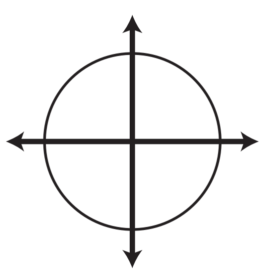
i) $\left(\frac{1}{2}, y\right)$



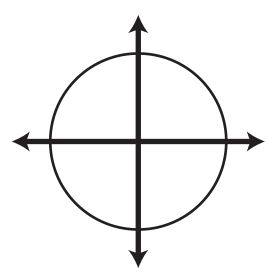
ii) $\left(x, \frac{\sqrt{3}}{2}\right)$, quadrant II.



iii) $(-1, y)$



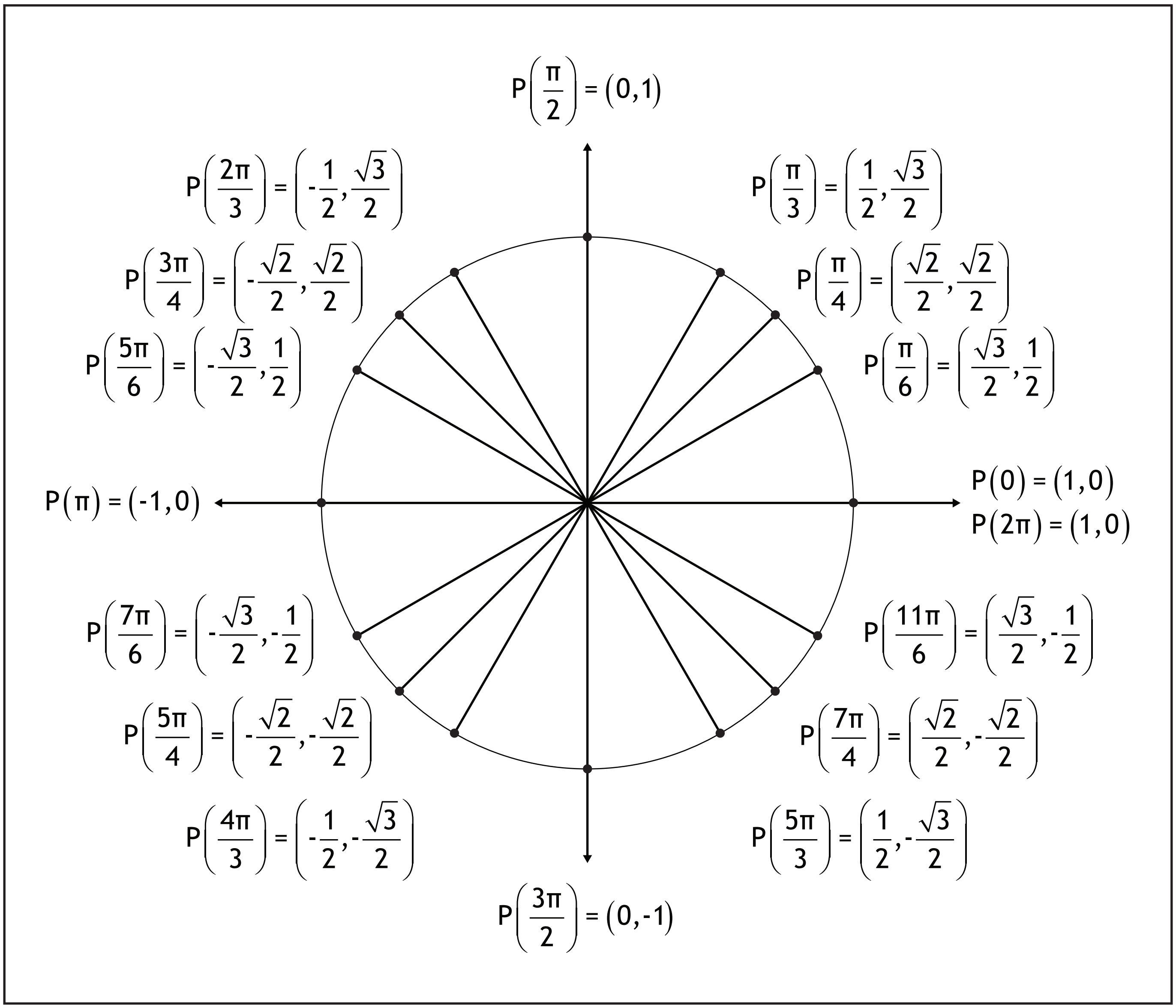
iv) $\left(x, -\frac{\sqrt{2}}{2}\right)$, $\cos\theta > 0$.



Example 2 The Unit Circle.

The Unit Circle

The following diagram is called *the unit circle*. Commonly used angles are shown as radians, and their exact-value coordinates are in brackets. Take a few moments to memorize this diagram. When you are done, use the blank unit circle on the next page to practice drawing the unit circle from memory.



questions on next page.

Trigonometry

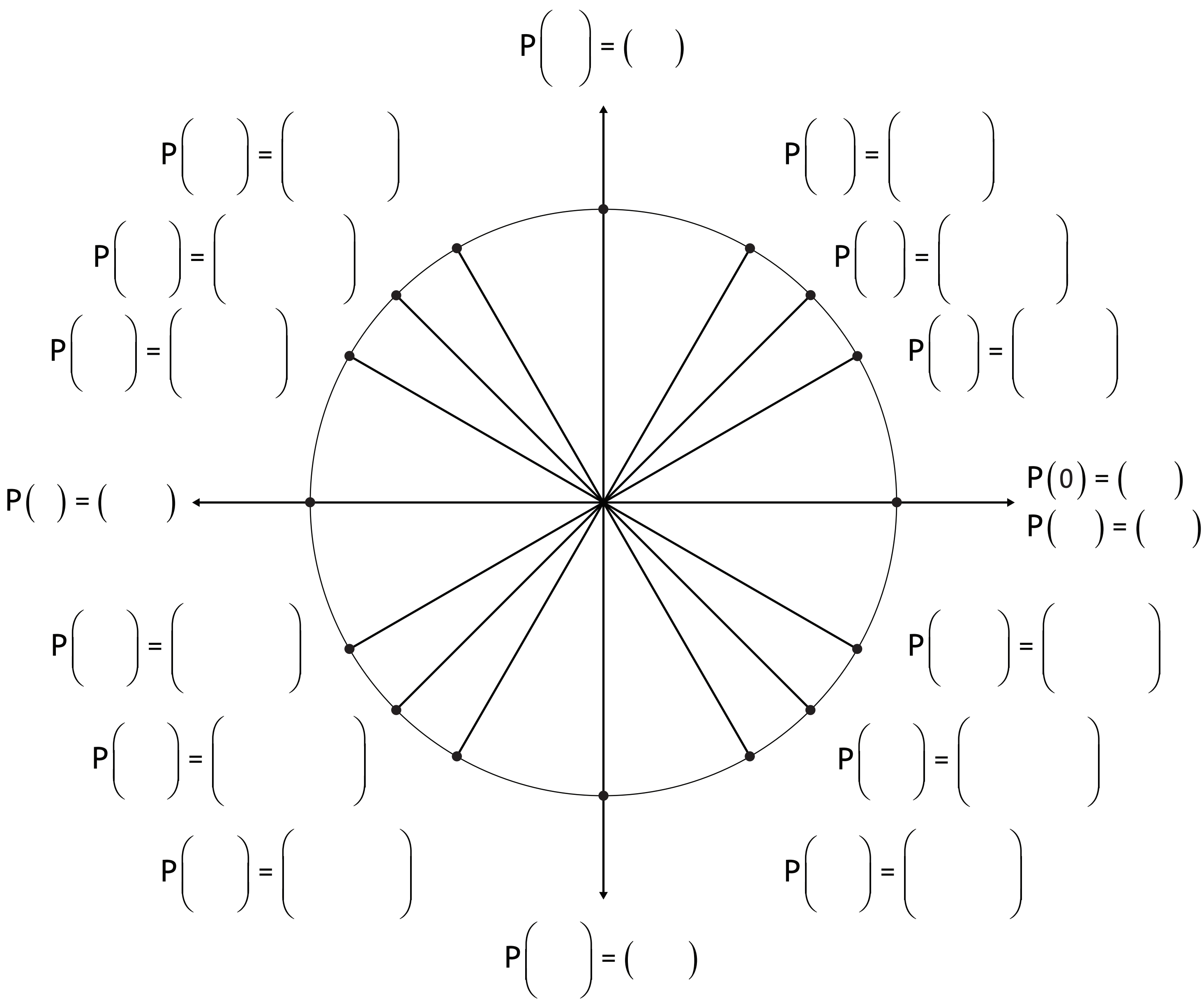
LESSON TWO - *The Unit Circle*

Lesson Notes

$(\cos\theta, \sin\theta)$

a) What are some useful tips to memorize the unit circle?

b) Draw the unit circle from memory using a partially completed template.



Example 3

Use the unit circle to find the exact value of each trigonometric ratio.

Finding Primary Trigonometric Ratios with the Unit Circle

a) $\sin \frac{\pi}{6}$

b) $\cos 180^\circ$

c) $\cos \frac{3\pi}{4}$

d) $\sin \frac{11\pi}{6}$

e) $\sin 0$

f) $\cos -\frac{\pi}{2}$

g) $\sin \frac{4\pi}{3}$

h) $\cos -120^\circ$

Example 4

Use the unit circle to find the exact value of each trigonometric ratio.

a) $\cos 420^\circ$

b) $-\cos 3\pi$

c) $\sin \frac{13\pi}{6}$

d) $\cos -\frac{2\pi}{3}$

e) $\sin -\frac{5\pi}{2}$

f) $-\sin \frac{9\pi}{4}$

g) $\cos^2 (-840^\circ)$

h) $\cos -\frac{7\pi}{3}$

Trigonometry
LESSON TWO - *The Unit Circle*
Lesson Notes

$(\cos\theta, \sin\theta)$

Example 5 Other Trigonometric Ratios.

Other Trigonometric Ratios

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

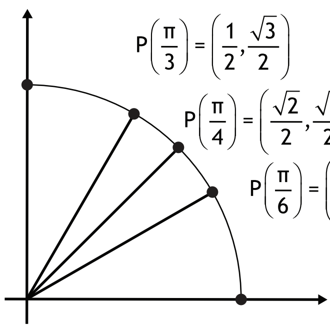
$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

The unit circle contains values for $\cos\theta$ and $\sin\theta$ only. The other four trigonometric ratios can be obtained using the identities on the right.

Given angles from the first quadrant of the unit circle, find the exact values of $\sec\theta$ and $\csc\theta$.

a) $\sec\theta$

$P\left(\frac{\pi}{2}\right) = (0,1)$ **sec** $\frac{\pi}{2}$ =



$P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **sec** $\frac{\pi}{3}$ =

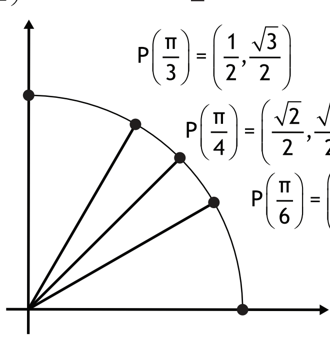
$P\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ **sec** $\frac{\pi}{4}$ =

$P\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ **sec** $\frac{\pi}{6}$ =

$P(0) = (1,0)$ **sec 0** =

b) $\csc\theta$

$P\left(\frac{\pi}{2}\right) = (0,1)$ **csc** $\frac{\pi}{2}$ =



$P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **csc** $\frac{\pi}{3}$ =

$P\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ **csc** $\frac{\pi}{4}$ =

$P\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ **csc** $\frac{\pi}{6}$ =

$P(0) = (1,0)$ **csc 0** =

Example 6 Other Trigonometric Ratios.

The unit circle contains values for $\cos\theta$ and $\sin\theta$ only. The other four trigonometric ratios can be obtained using the identities on the right.

Given angles from the first quadrant of the unit circle, find the exact values of $\tan\theta$ and $\cot\theta$.

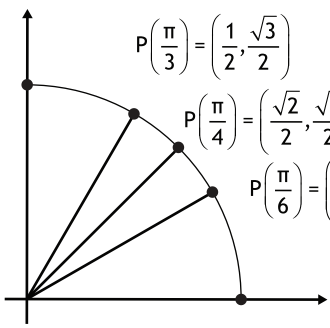
Other Trigonometric Ratios

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

a) $\tan\theta$

$P\left(\frac{\pi}{2}\right) = (0,1)$ **$\tan\frac{\pi}{2}$ =**



$P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **$\tan\frac{\pi}{3}$ =**

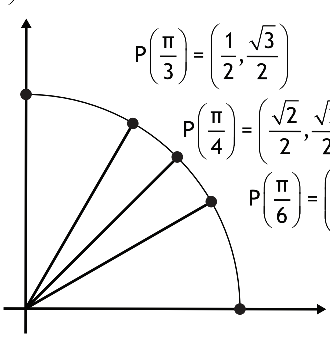
$P\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ **$\tan\frac{\pi}{4}$ =**

$P\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ **$\tan\frac{\pi}{6}$ =**

$P(0) = (1,0)$ **$\tan 0$ =**

b) $\cot\theta$

$P\left(\frac{\pi}{2}\right) = (0,1)$ **$\cot\frac{\pi}{2}$ =**



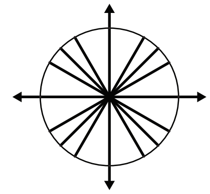
$P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **$\cot\frac{\pi}{3}$ =**

$P\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ **$\cot\frac{\pi}{4}$ =**

$P\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ **$\cot\frac{\pi}{6}$ =**

$P(0) = (1,0)$ **$\cot 0$ =**

Trigonometry
LESSON TWO - *The Unit Circle*
Lesson Notes



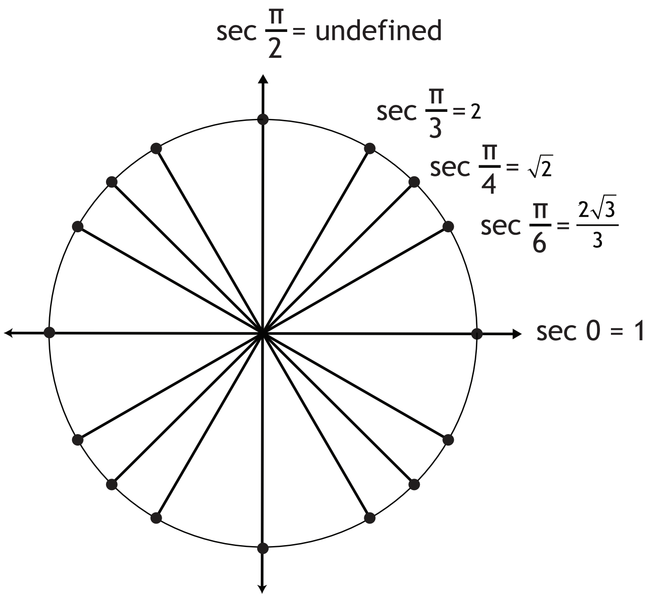
$(\cos\theta, \sin\theta)$

Example 7

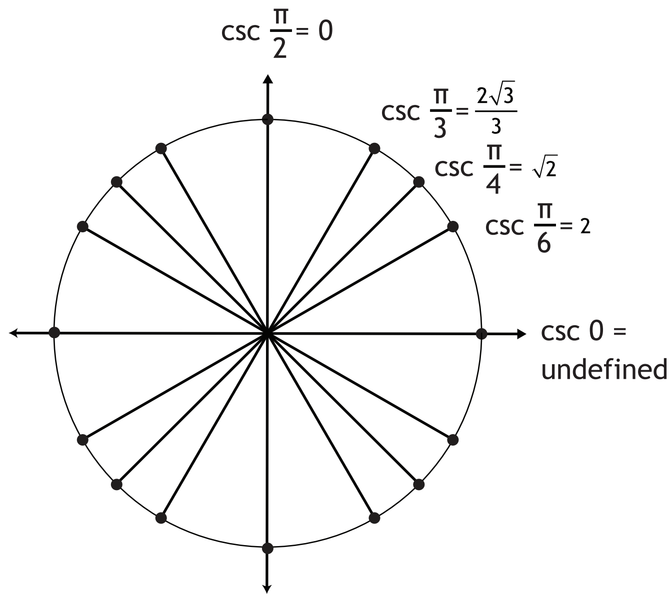
Use symmetry to fill in quadrants II, III, and IV for each unit circle.

Symmetry of the Unit Circle

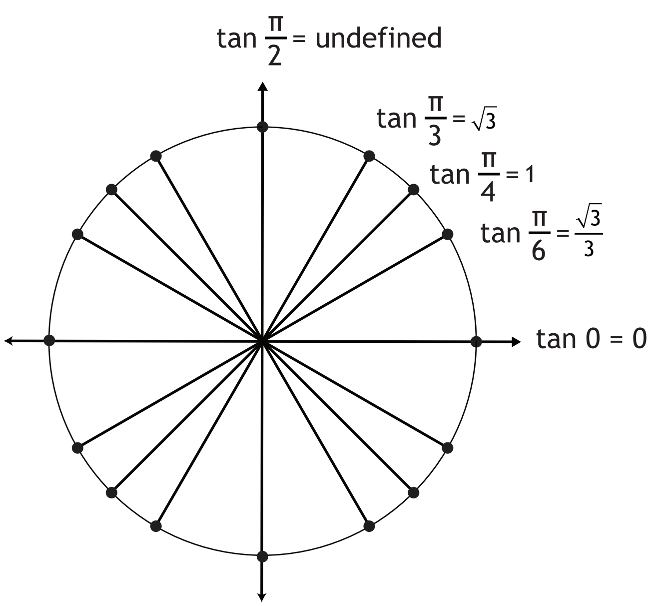
a) $\sec\theta$



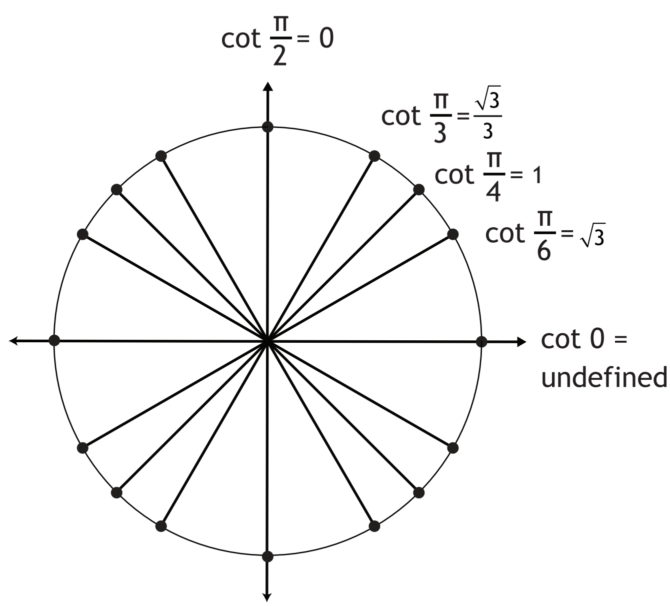
b) $\csc\theta$



c) $\tan\theta$



d) $\cot\theta$



Example 8 Find the exact value of each trigonometric ratio.

Finding Reciprocal Trigonometric Ratios with the Unit Circle

a) $\sec 120^\circ$

b) $\sec \frac{3\pi}{2}$

c) $\csc \frac{\pi}{3}$

d) $\csc -\frac{3\pi}{4}$

e) $\tan \frac{\pi}{6}$

f) $-\tan \frac{5\pi}{4}$

g) $\cot^2(270^\circ)$

h) $\cot -\frac{5\pi}{6}$

Trigonometry
LESSON TWO - *The Unit Circle*
Lesson Notes

$(\cos\theta, \sin\theta)$

Example 9 Find the exact value of each trigonometric expression.

Evaluating Complex Expressions with the Unit Circle

a) $\sin\left(-\frac{\pi}{3}\right) + \cos\left(\frac{5\pi}{4}\right)$

b) $\cos^2\frac{\pi}{4} + \sin^2\frac{\pi}{4}$

c) $\cot^2\frac{\pi}{3} + 1$

d) $\dfrac{\sec\frac{\pi}{6}}{\tan\frac{\pi}{6} + \cot\frac{\pi}{6}}$

Example 10 Find the exact value of each trigonometric expression.

Evaluating Complex Expressions with the Unit Circle

a) $\sin\frac{\pi}{3}\cos\frac{\pi}{6}+\cos\frac{\pi}{3}\sin\frac{\pi}{6}$

b) $\cos\frac{\pi}{4}\cos\frac{\pi}{6}-\sin\frac{\pi}{4}\sin\frac{\pi}{6}$

c) $\dfrac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}}$

d) $\dfrac{\tan\frac{3\pi}{4}-\tan\frac{\pi}{6}}{1+\tan\frac{3\pi}{4}\tan\frac{\pi}{6}}$

Example 11

Find the exact value of each trigonometric ratio.

Finding the Trigonometric Ratios of Large Angles with the Unit Circle

a) $\csc\left(-\dfrac{9\pi}{2}\right)$

b) $-\tan^2\left(\dfrac{617\pi}{6}\right)$

c) $\sec\left(\dfrac{61\pi}{2}\right)$

d) $\cot\left(-1980^\circ\right)$

Example 12

Verify each trigonometric statement with a calculator. *Note: Every question in this example has already been seen earlier in the lesson.*

Evaluating Trigonometric Ratios with a Calculator

a) $\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

b) $\cos^2\left(-840^\circ\right) = \frac{1}{4}$

c) $\sec\frac{3\pi}{2} = \textit{undefined}$

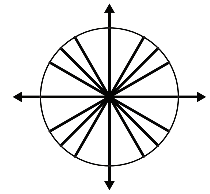
d) $\csc\left(-\frac{3\pi}{4}\right) = -\sqrt{2}$

e) $-\tan^2\left(\frac{617\pi}{6}\right) = -\frac{1}{3}$

f) $\cot^2\frac{\pi}{3} + 1 = \frac{4}{3}$

g) $\dfrac{\sec\frac{\pi}{6}}{\tan\frac{\pi}{6} + \cot\frac{\pi}{6}} = \frac{1}{2}$

h) $\dfrac{\tan\frac{3\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{3\pi}{4}\tan\frac{\pi}{6}} = -2 - \sqrt{3}$



$(\cos\theta, \sin\theta)$

Example 13 Answer each of the following questions related to the unit circle.

Coordinate Relationships on the Unit Circle

a) What is meant when you are asked to find $P\left(\frac{\pi}{3}\right)$ on the unit circle?

b) Find one positive and one negative angle such that $P(\theta) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

c) How does a half-rotation around the unit circle change the coordinates?
If $\theta = \frac{\pi}{6}$, find the coordinates of the point halfway around the unit circle.

d) How does a quarter-rotation around the unit circle change the coordinates?
If $\theta = \frac{2\pi}{3}$, find the coordinates of the point a quarter-revolution (clockwise) around the unit circle.

e) What are the coordinates of P(3)? Express coordinates to four decimal places.

Example 14

Answer each of the following questions related to the unit circle.

Circumference and Arc Length of the Unit Circle

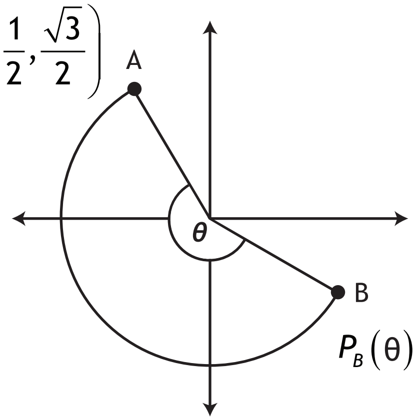
a) What is the circumference of the unit circle?

b) How is the central angle of the unit circle related to its corresponding arc length?

c) If a point on the terminal arm rotates from P(θ) = (1, 0) to P(θ) = $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, what is the arc length?

d) What is the arc length from point A to point B on the unit circle?

$P_A(\theta) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

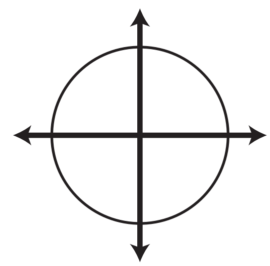


$P_B(\theta) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Example 15 Answer each of the following questions related to the unit circle.

Domain and Range of the Unit Circle

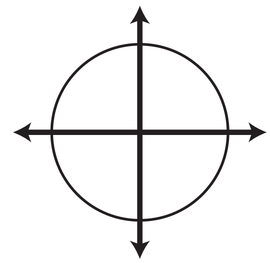
a) Is sinθ = 2 possible? Explain, using the unit circle as a reference.



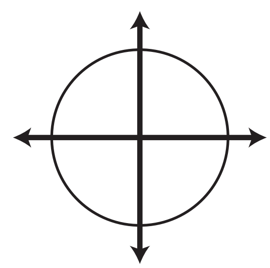
b) Which trigonometric ratios are restricted to a range of $-1 \le y \le 1$? Which trigonometric ratios exist outside that range?

	Range	Number Line
cosθ & sinθ		
cscθ & secθ		
tanθ & cotθ		

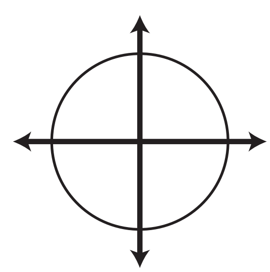
c) If $\sin\theta = -\frac{\sqrt{2}}{2}$ exists on the unit circle, how can the unit circle be used to find $\cos\theta$?
How many values for $\cos\theta$ are possible?

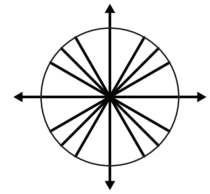


d) If $\cos\theta = \frac{3}{5}$ exists on the unit circle, how can the equation of the unit circle be used to find $\sin\theta$?
How many values for $\sin\theta$ are possible?



e) If $\cos\theta = 0$, and $0 \le \theta < \pi$, how many values for $\sin\theta$ are possible?

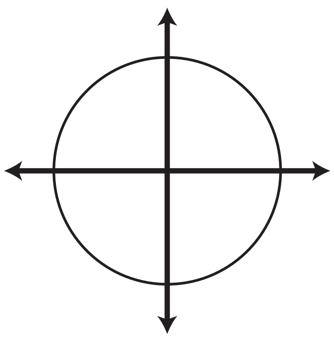




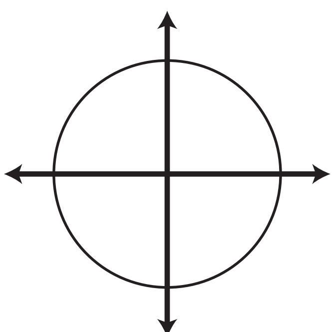
Example 16 Complete the following questions related to the unit circle.

Unit Circle Proofs

a) Use the Pythagorean Theorem to prove that the equation of the unit circle is $x^2 + y^2 = 1$.

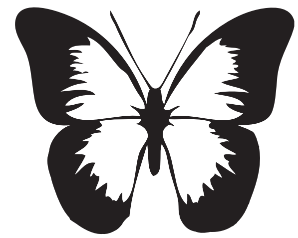


b) Prove that the point where the terminal arm intersects the unit circle, $P(\theta)$, has coordinates of ($\cos\theta$, $\sin\theta$).



c) If the point $P(\theta) = \left(-\frac{40}{41}, \frac{9}{41}\right)$ exists on the terminal arm of a unit circle, find the exact values of the six trigonometric ratios. State the reference angle and standard position angle to the nearest hundredth of a degree.

Example 17



In a video game, the graphic of a butterfly needs to be rotated. To make the butterfly graphic rotate, the programmer uses the equations:

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

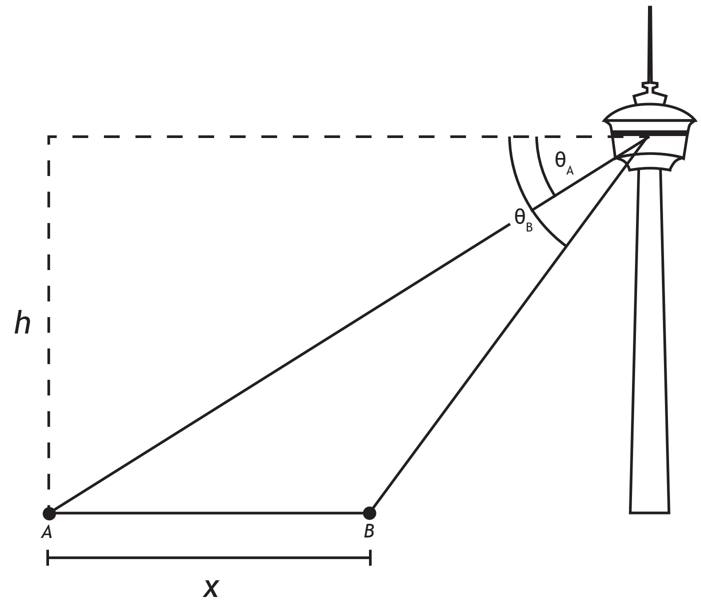
to transform each pixel of the graphic from its original coordinates, (x, y), to its new coordinates, (x', y'). Pixels may have positive or negative coordinates.

a) If a particular pixel with coordinates of (250, 100) is rotated by $\frac{\pi}{6}$, what are the new coordinates? Round coordinates to the nearest whole pixel.

b) If a particular pixel has the coordinates (640, 480) after a rotation of $\frac{5\pi}{4}$, what were the original coordinates? Round coordinates to the nearest whole pixel.

Example 18

From the observation deck of the Calgary Tower, an observer has to tilt their head θ_A down to see point A, and θ_B down to see point B.



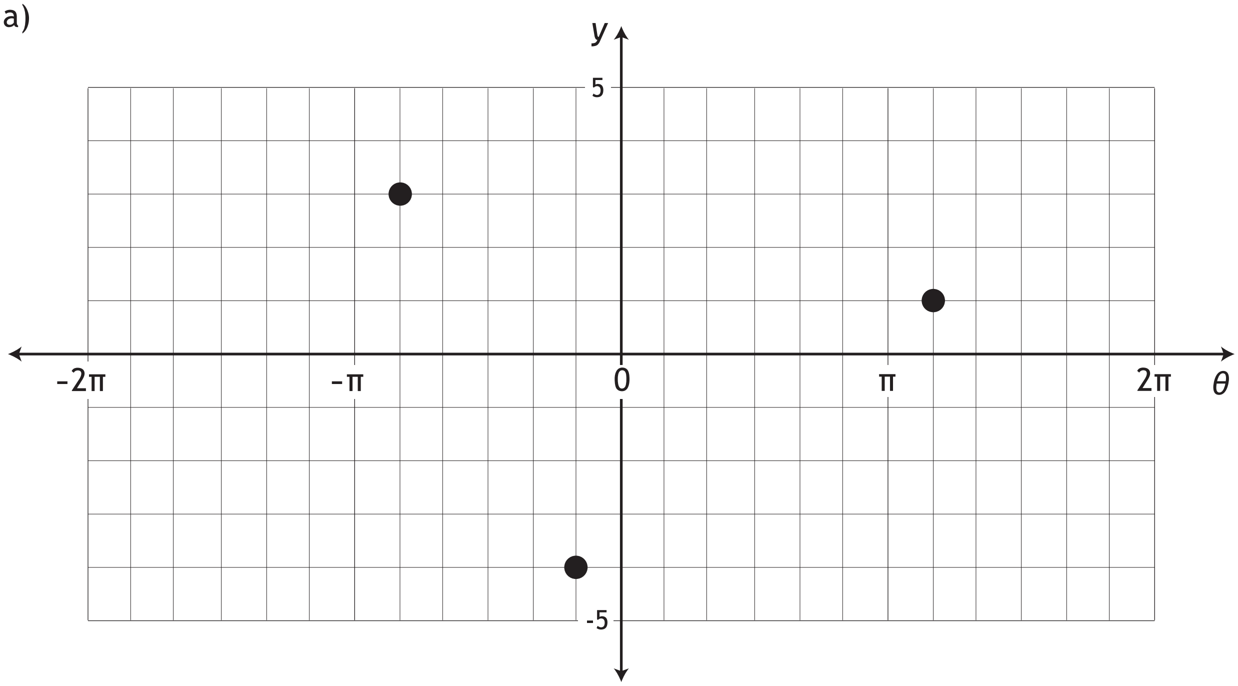
a) Show that the height of the observation deck is $h = \dfrac{x}{\cot\theta_A - \cot\theta_B}$.

b) If $\theta_A = \dfrac{131}{900}\pi$, $\theta_B = \dfrac{61}{200}\pi$, and x = 212.92 m, how high is the observation deck above the ground, to the nearest metre?

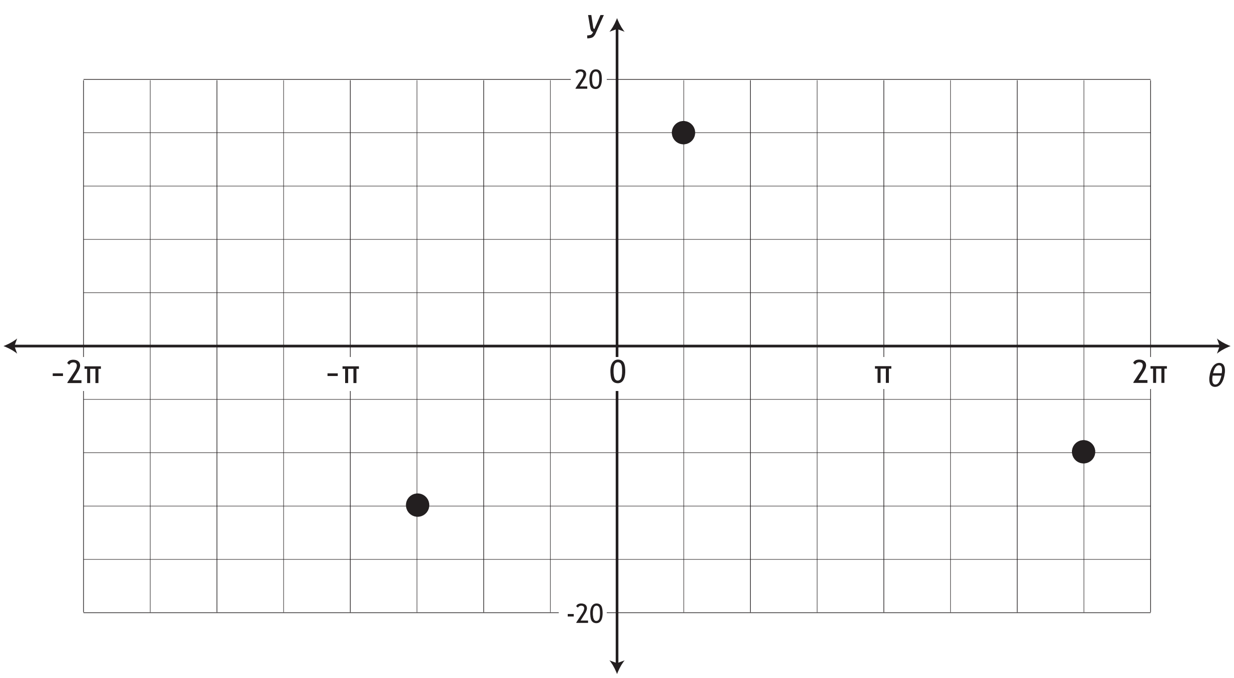
Example 1 Label all tick marks in the following grids and state the coordinates of each point.

Trigonometric Coordinate Grids

a)

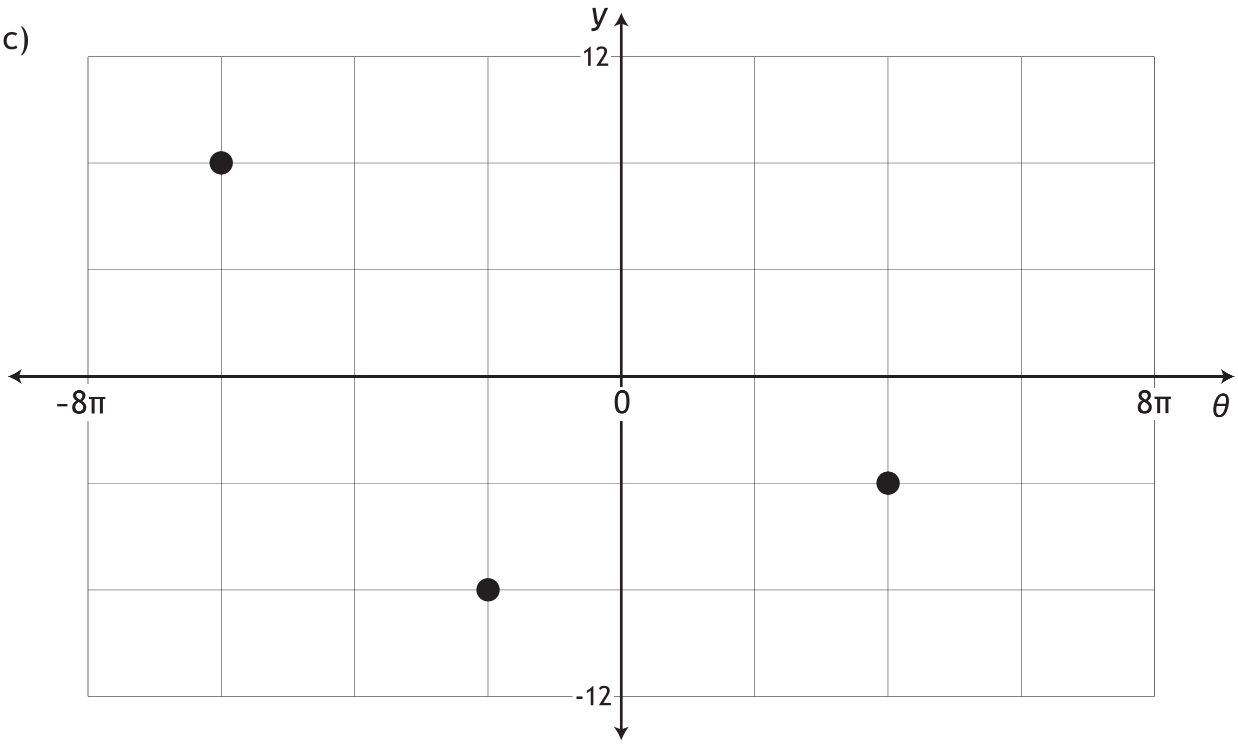


b)

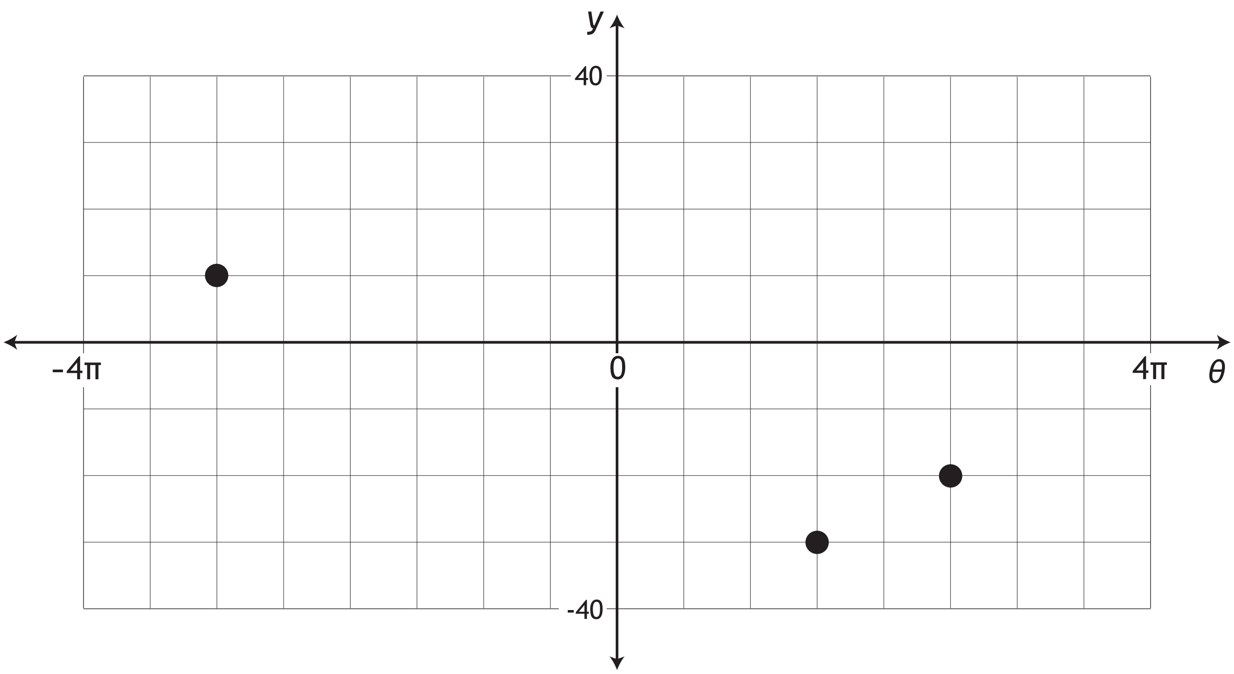


$y = a\sin b(\theta - c) + d$

c)



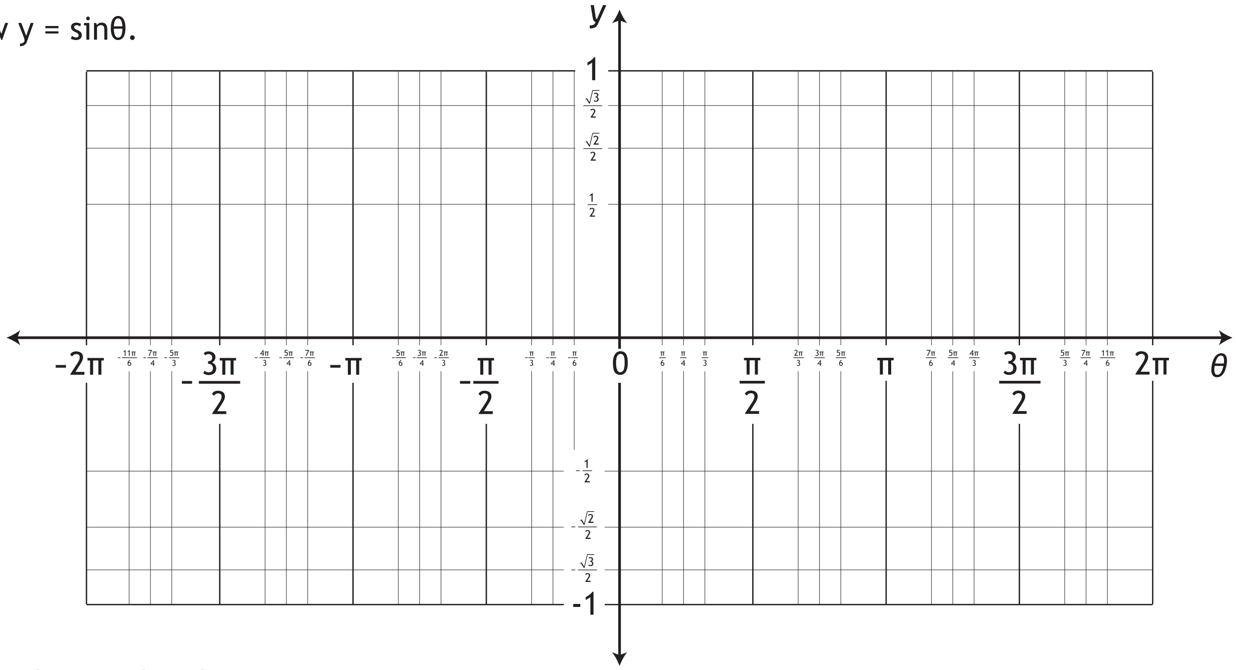
d)



Example 2 Exploring the graph of y = sinθ.

y = sinθ

a) Draw y = sinθ.



b) State the amplitude.

c) State the period.

d) State the horizontal displacement (phase shift).

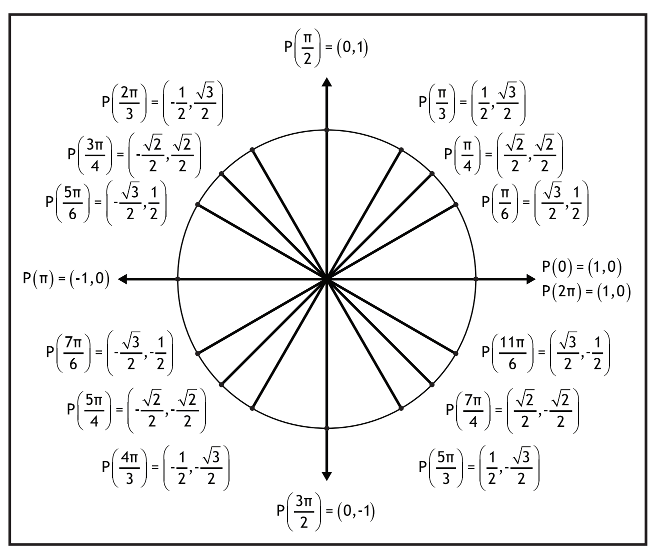
e) State the vertical displacement.

f) State the θ-intercepts. Write your answer using a general form expression.

g) State the y-intercept.

h) State the domain and range.

Unit Circle Reference



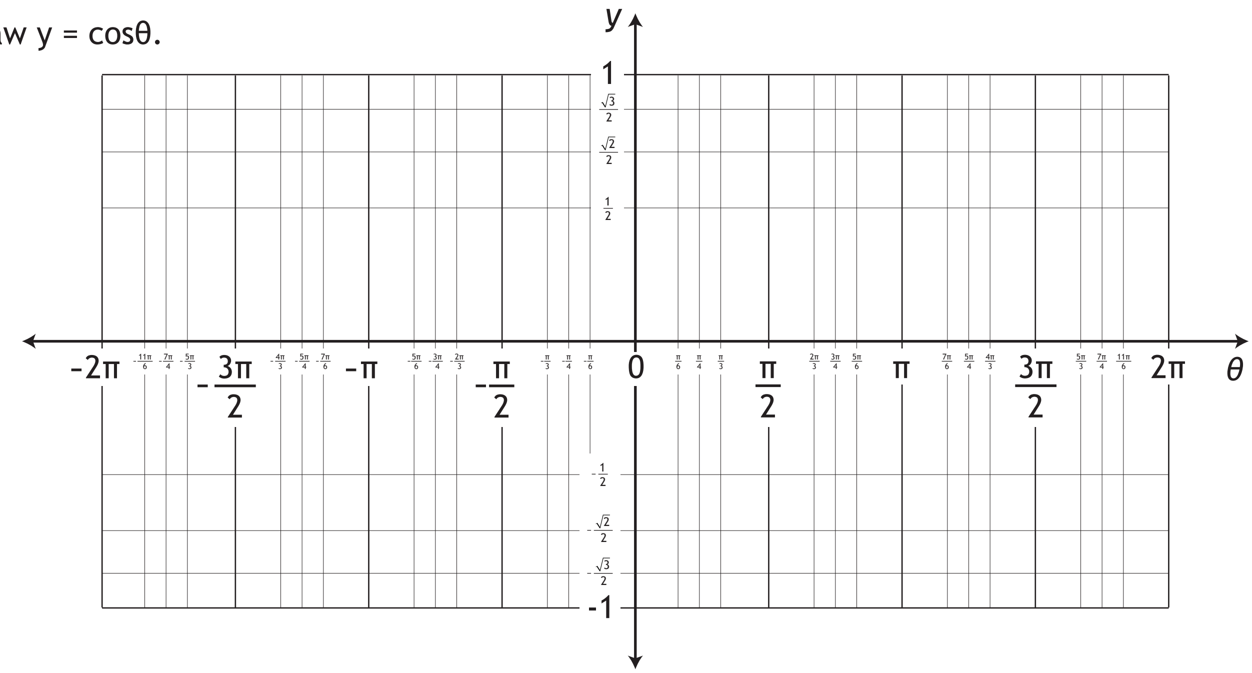
Trigonometry
LESSON THREE - *Trigonometric Functions I*
Lesson Notes

$y = a\sin b(\theta - c) + d$

Example 3 Exploring the graph of y = cosθ.

y = cosθ

a) Draw y = cosθ.



b) State the amplitude.

c) State the period.

d) State the horizontal displacement (phase shift).

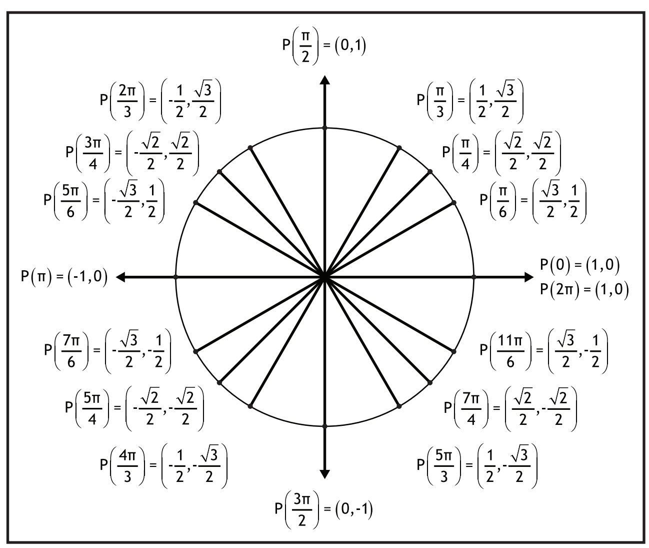
e) State the vertical displacement.

f) State the θ-intercepts. Write your answer using a general form expression.

g) State the y-intercept.

h) State the domain and range.

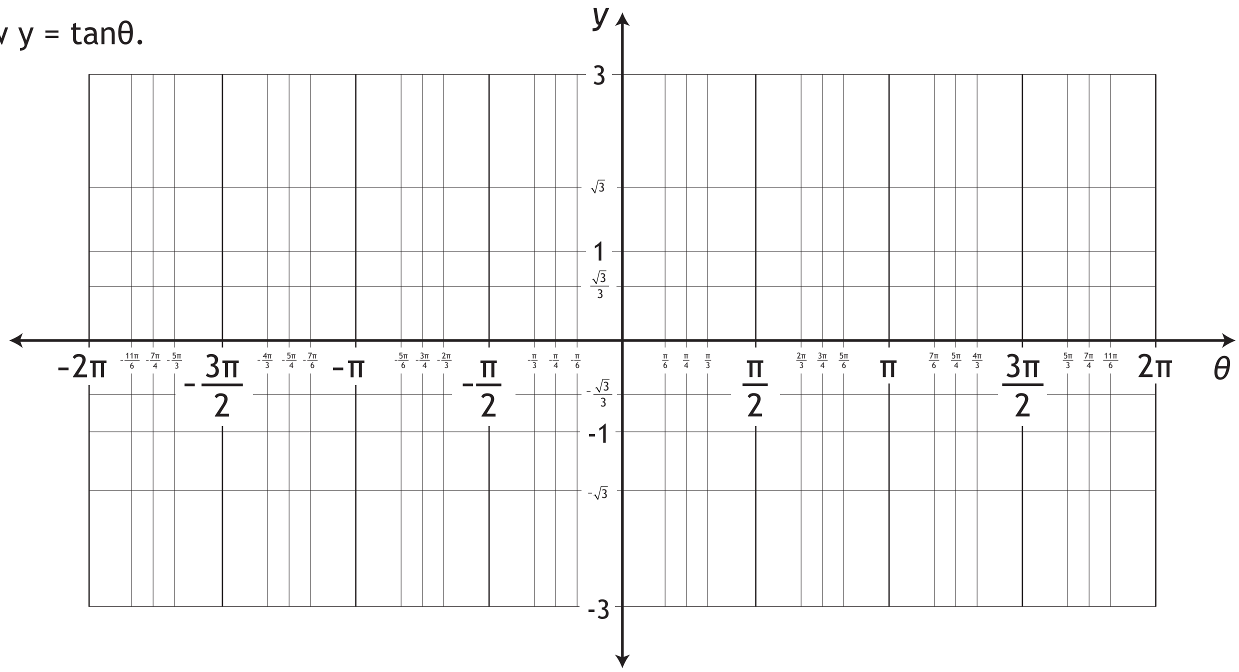
Unit Circle Reference



Example 4 Exploring the graph of y = tanθ.

y = tanθ

a) Draw y = tanθ.



b) Is it correct to say a tangent graph has an amplitude?

c) State the period.

d) State the horizontal displacement (phase shift).

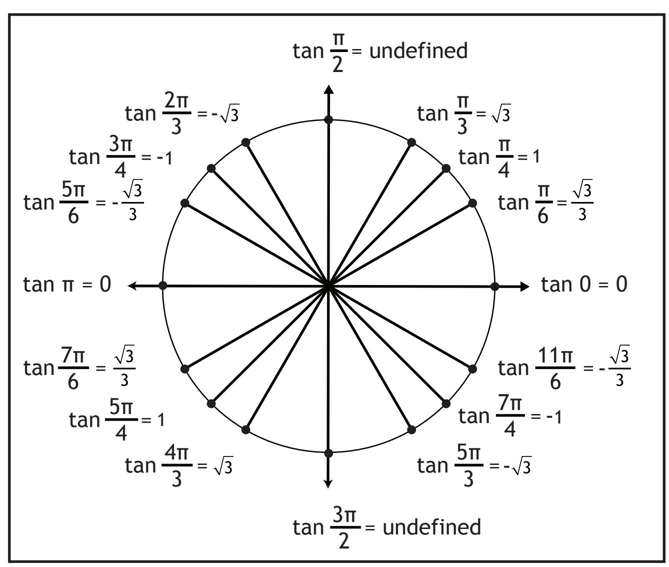
e) State the vertical displacement.

f) State the θ-intercepts. Write your answer using a general form expression.

g) State the y-intercept.

h) State the domain and range.

Unit Circle Reference



Trigonometry

LESSON THREE - *Trigonometric Functions I*

Lesson Notes

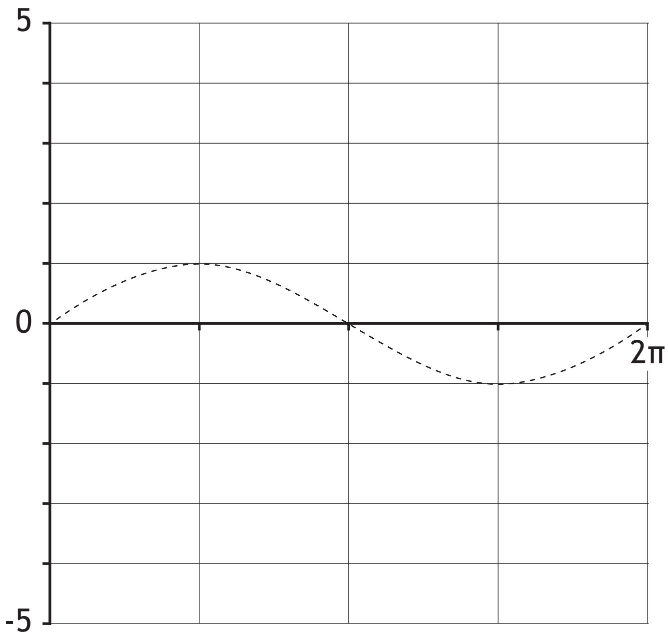
$$y = a\sin b(\theta - c) + d$$

Example 5

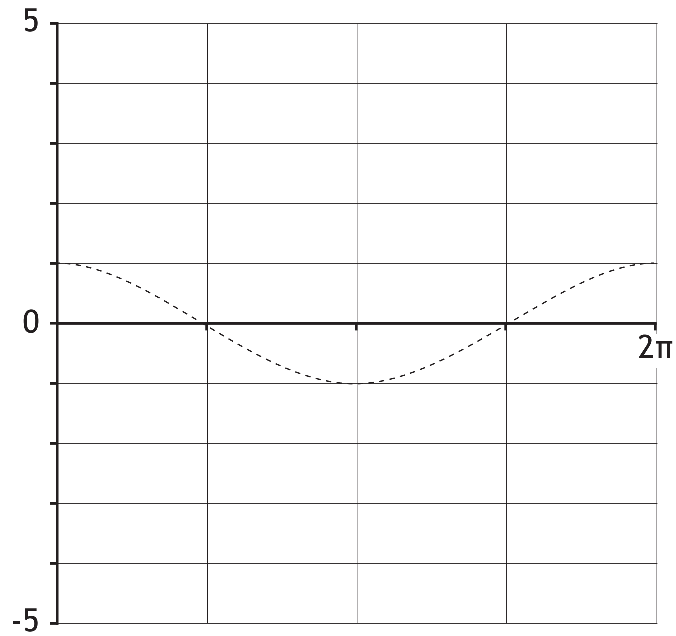
Graph each function over the domain $0 \le \theta \le 2\pi$.
The base graph is provided as a convenience.

The ***a*** Parameter

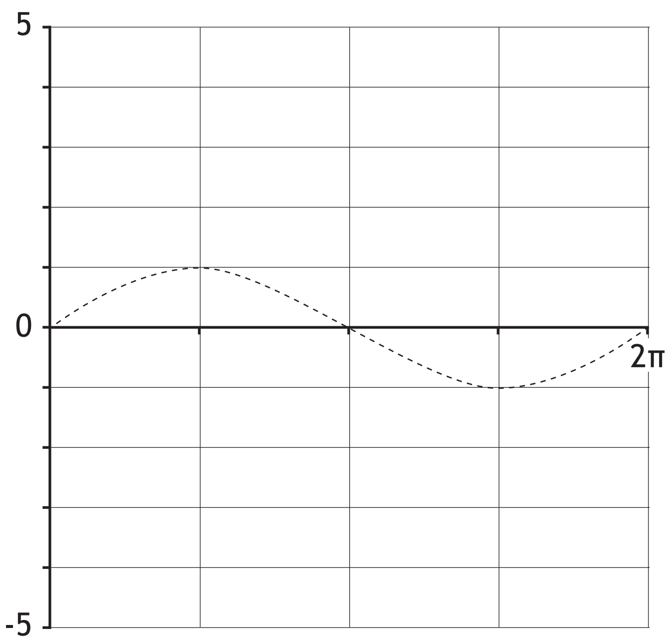
a) $y = 3\sin\theta$



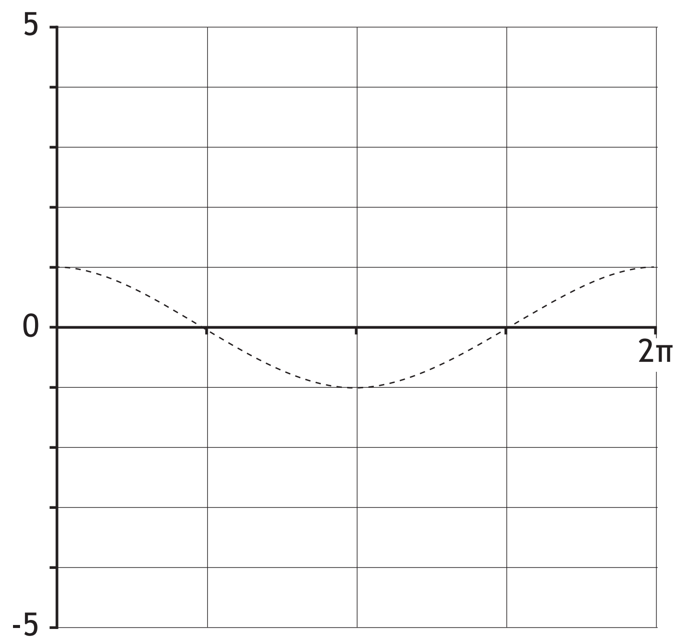
b) $y = -2\cos\theta$



c) $y = -\frac{1}{2}\sin\theta$



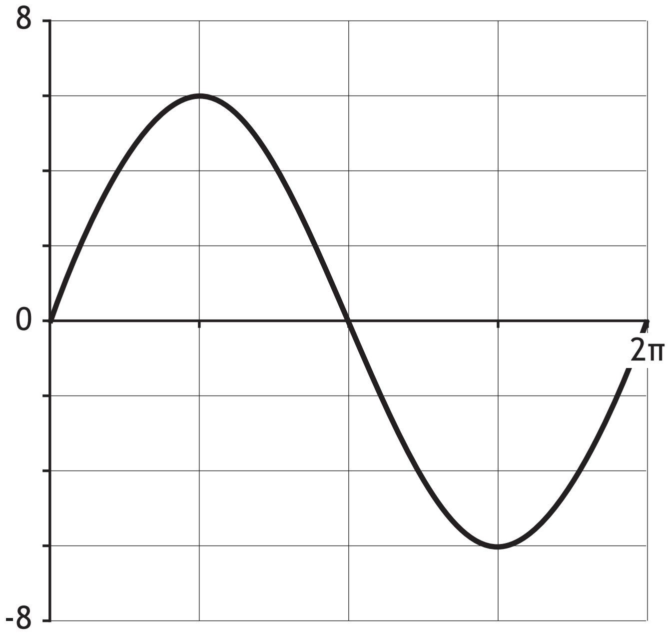
d) $y = \frac{5}{2}\cos\theta$



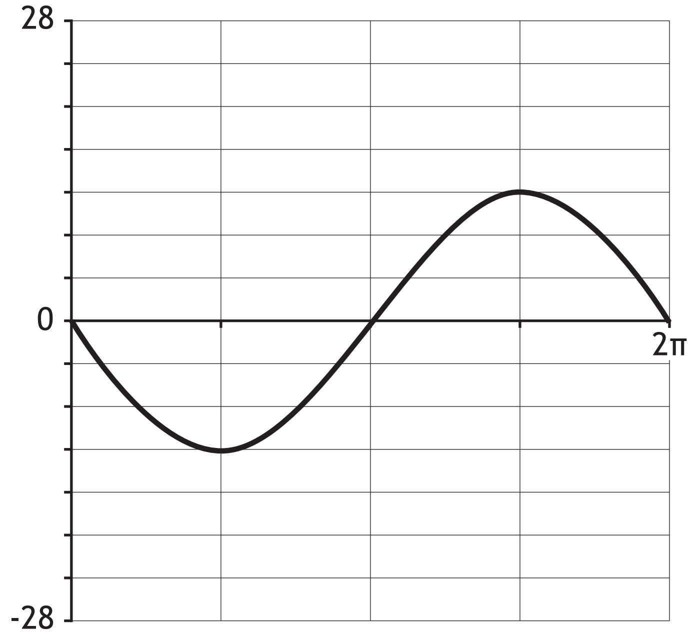
Example 6 Determine the trigonometric function corresponding to each graph.

The ***a*** Parameter

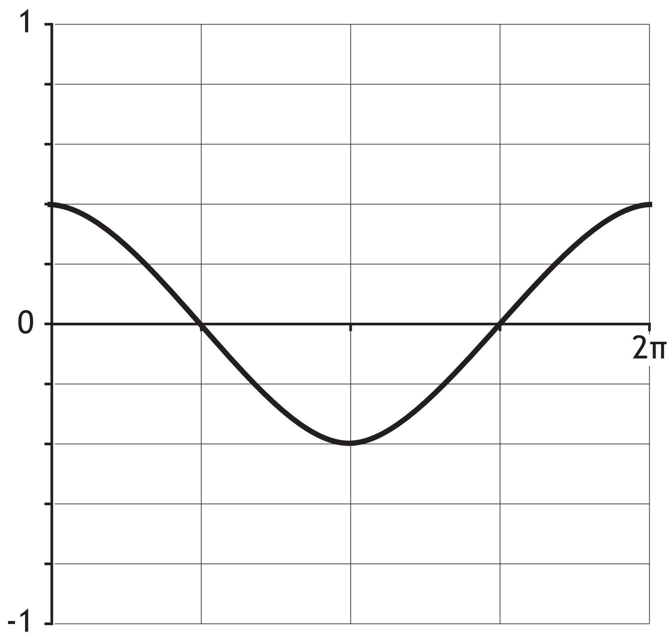
a) write a **sine** function.



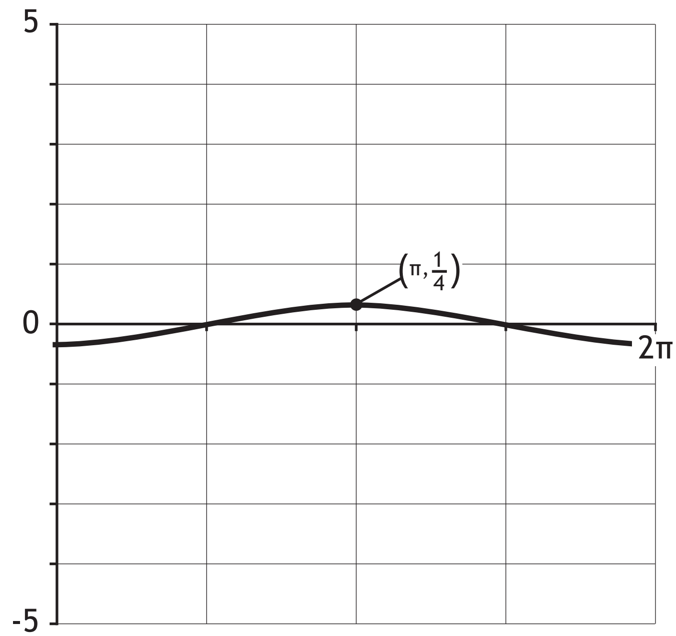
b) write a **sine** function.



c) write a **cosine** function.



d) write a **cosine** function.



Trigonometry

LESSON THREE - *Trigonometric Functions I*

Lesson Notes

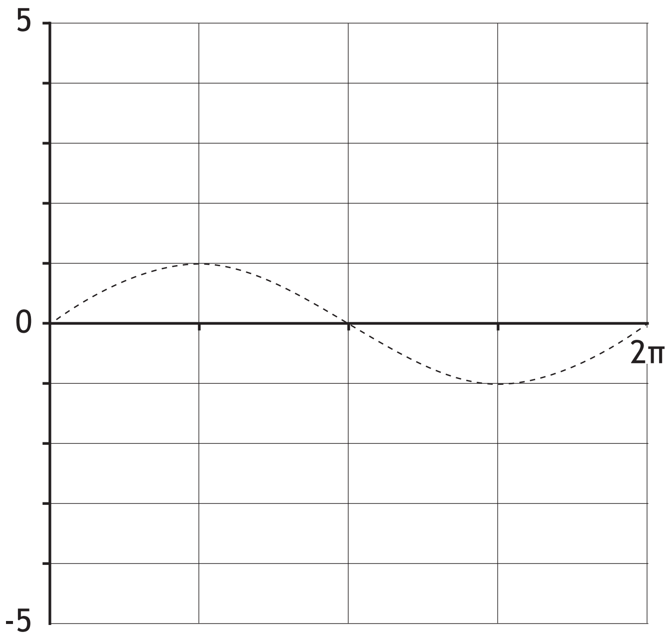
$$y = a\sin b(\theta - c) + d$$

Example 7

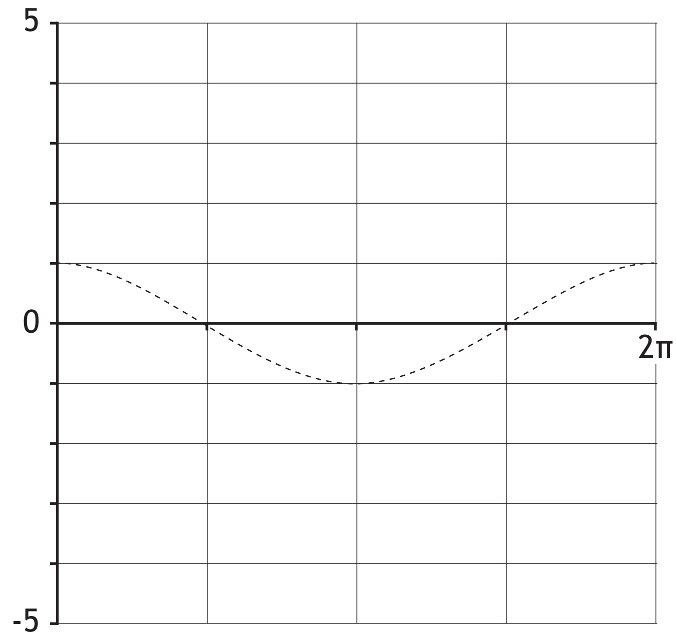
Graph each function over the domain $0 \le \theta \le 2\pi$.
The base graph is provided as a convenience.

The ***d*** Parameter

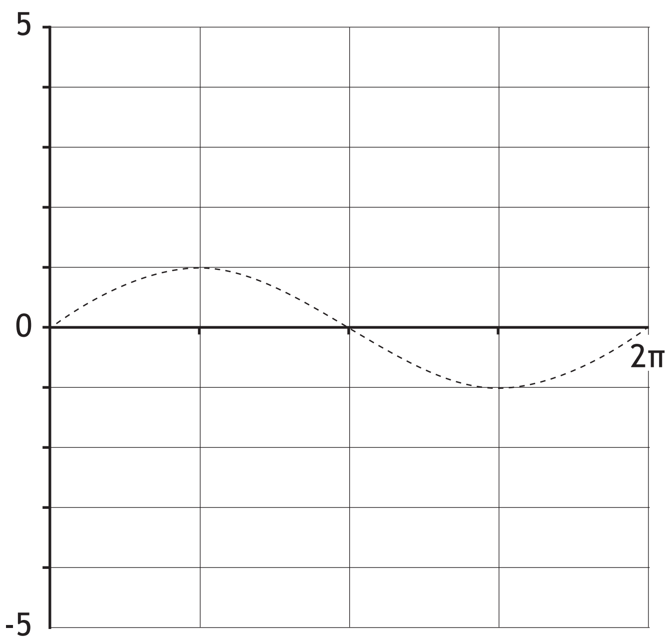
a) $y = \sin\theta - 2$



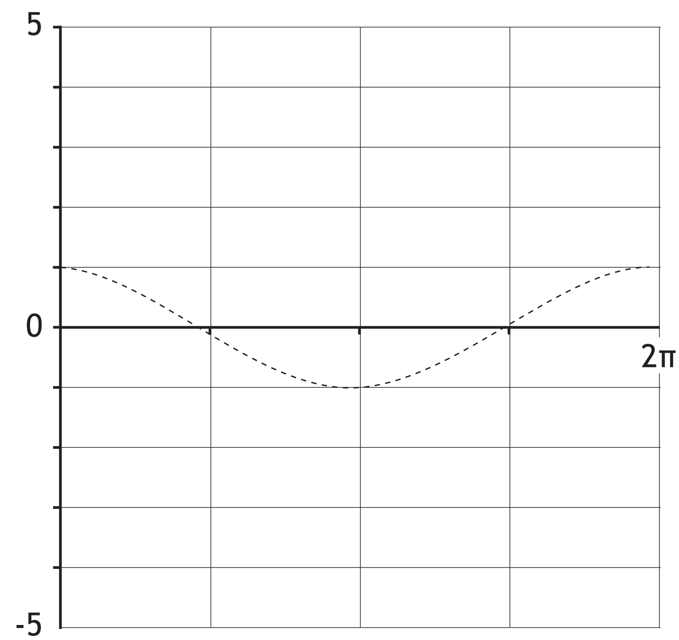
b) $y = \cos\theta + 4$



c) $y = -\frac{1}{2}\sin\theta + 2$



d) $y = \frac{1}{2}\cos\theta - \frac{1}{2}$

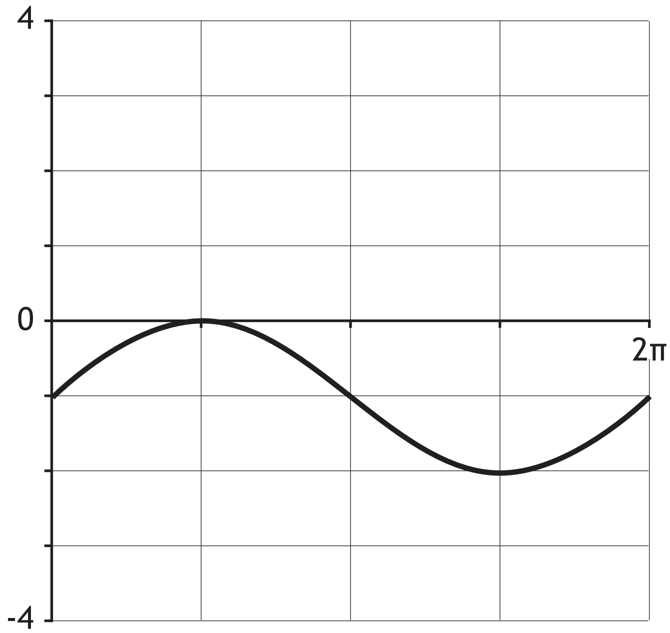


Example 8

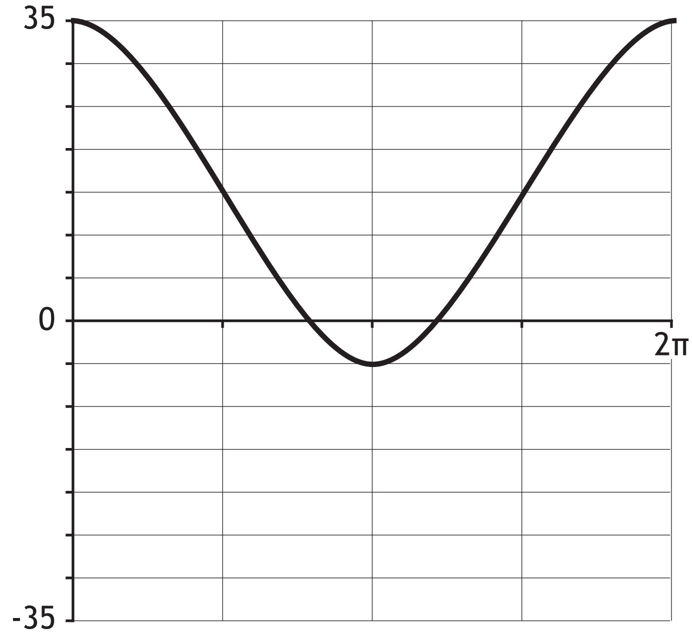
Determine the trigonometric function corresponding to each graph.

The ***d*** Parameter

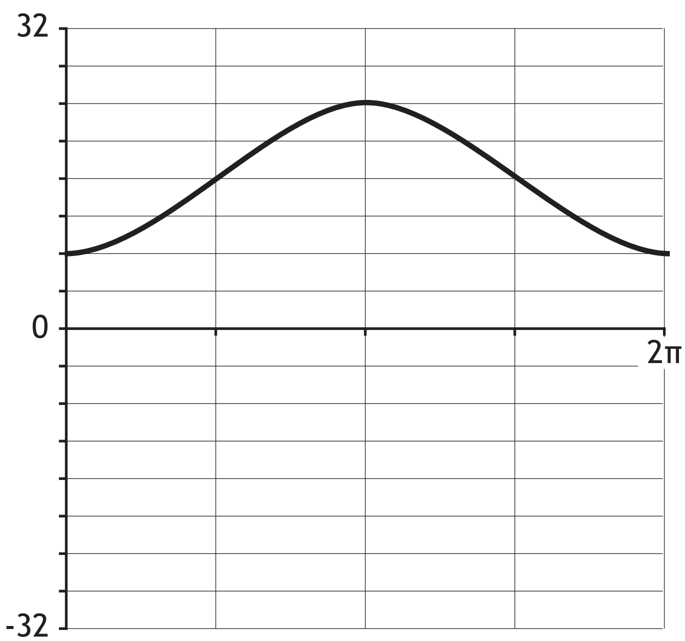
a) write a **sine** function.



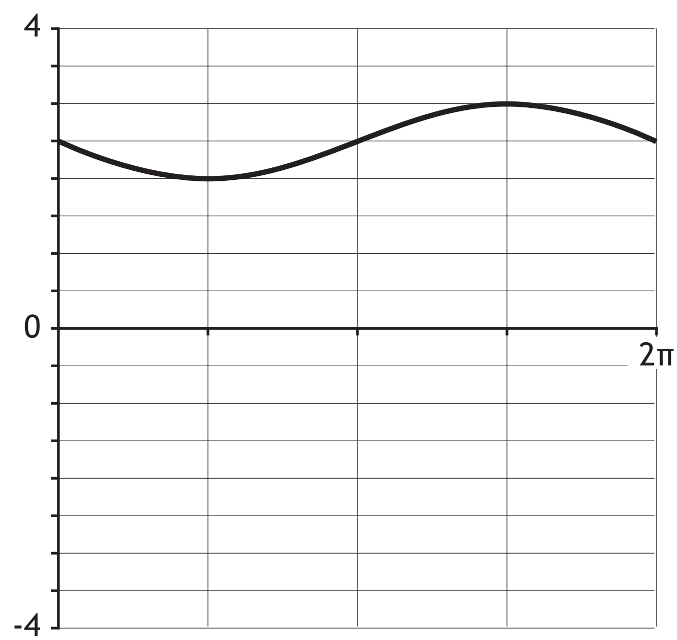
b) write a **cosine** function.



c) write a **cosine** function.



d) write a **sine** function.



Trigonometry

LESSON THREE - *Trigonometric Functions I*

Lesson Notes

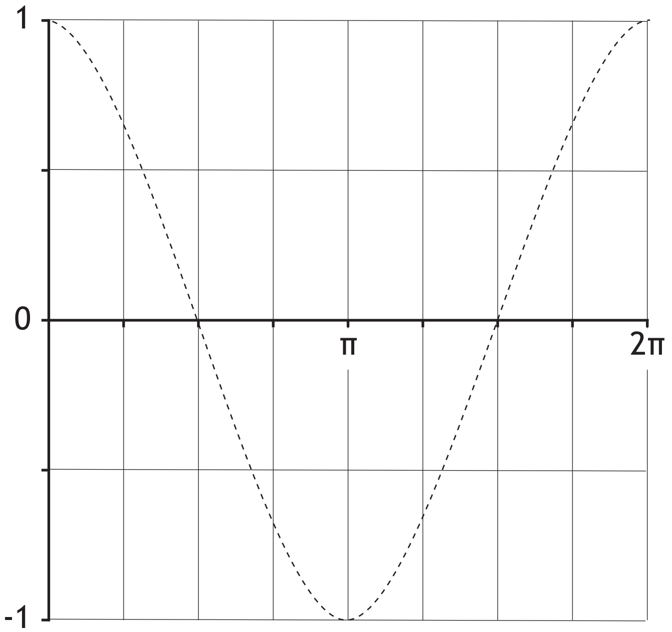
$$y = a\sin b(\theta - c) + d$$

Example 9

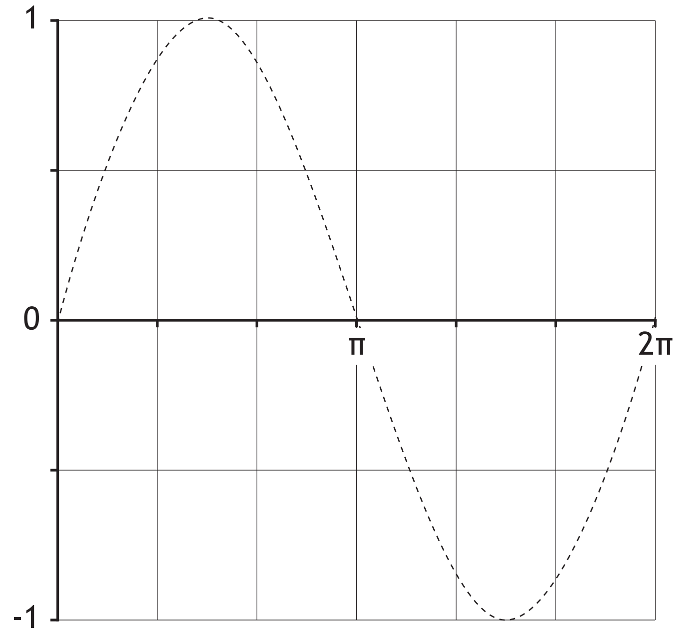
Graph each function over the stated domain.
The base graph is provided as a convenience.

The ***b*** Parameter

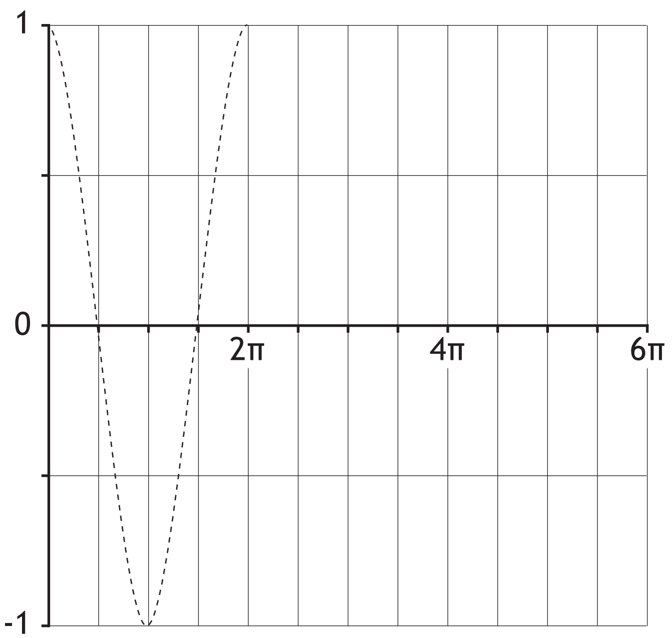
a) $y = \cos 2\theta$ *($0 \le \theta \le 2\pi$)*



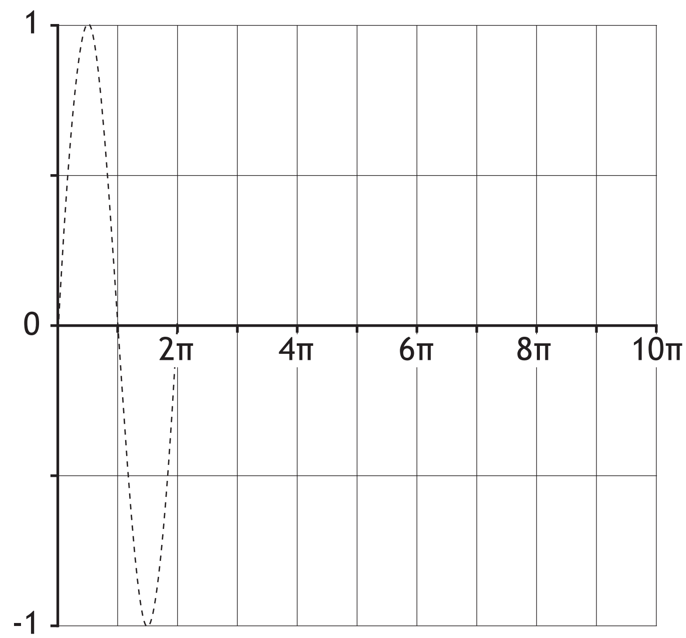
b) $y = \sin 3\theta$ *($0 \le \theta \le 2\pi$)*



c) $y = \cos \frac{1}{3}\theta$ *($0 \le \theta \le 6\pi$)*



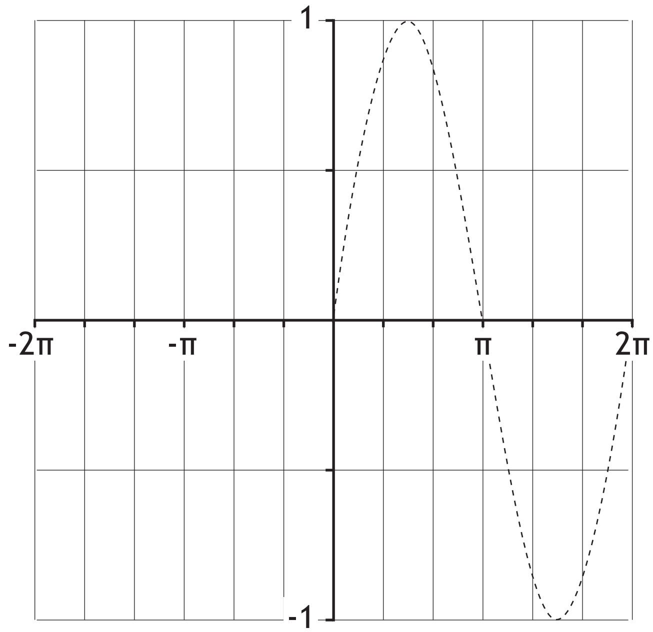
d) $y = \sin \frac{1}{5}\theta$ *($0 \le \theta \le 10\pi$)*



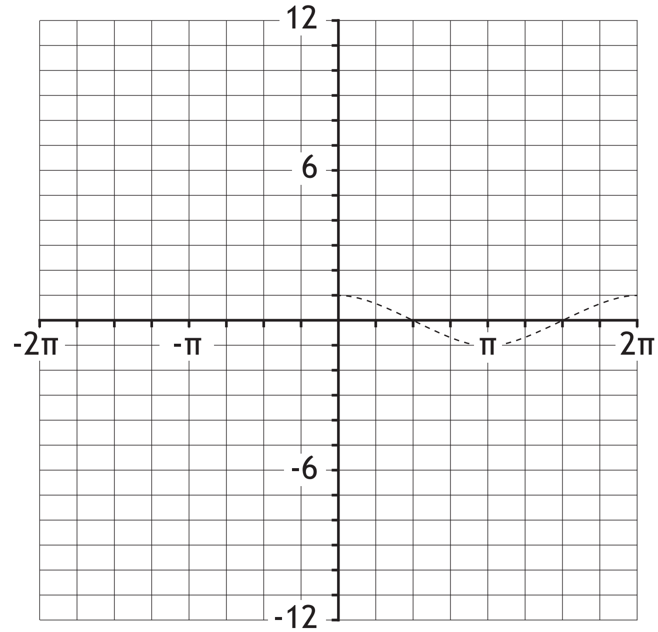
Example 10 Graph each function over the stated domain. The base graph is provided as a convenience.

The ***b*** Parameter

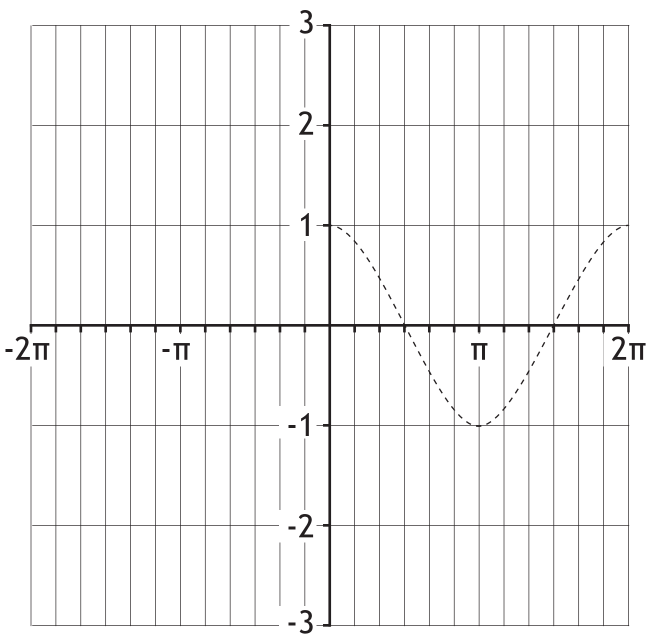
a) $y = -\sin(3\theta)$ $(-2\pi \le \theta \le 2\pi)$



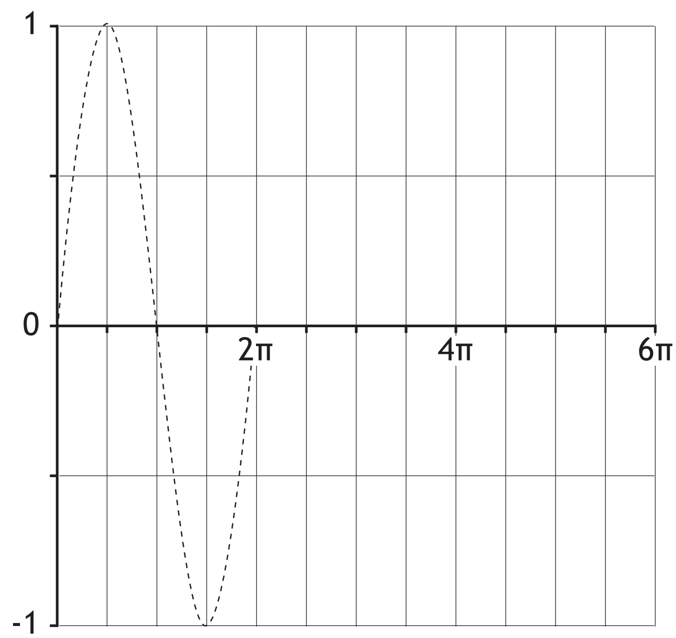
b) $y = 4\cos 2\theta + 6$ $(-2\pi \le \theta \le 2\pi)$



c) $y = 2\cos \frac{1}{2}\theta - 1$ $(-2\pi \le \theta \le 2\pi)$



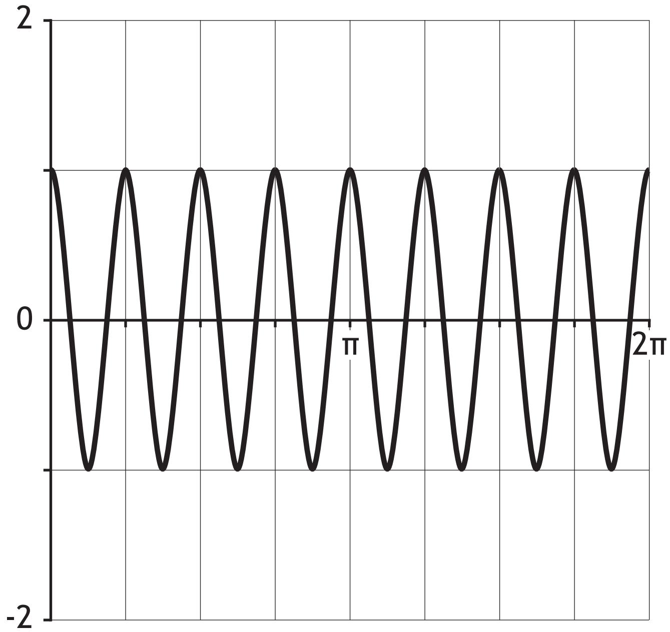
d) $y = \sin \frac{4}{3}\theta$ $(0 \le \theta \le 6\pi)$



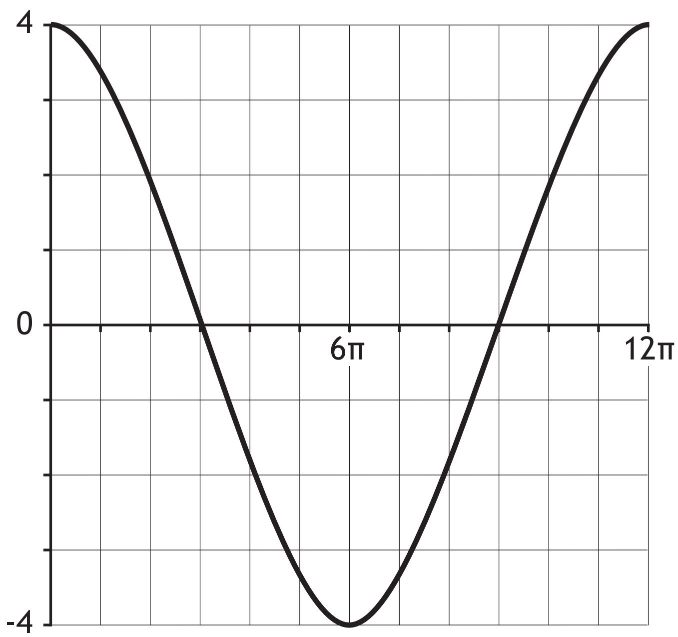
Example 11 Determine the trigonometric function corresponding to each graph.

The *b* Parameter

a) write a **cosine** function.

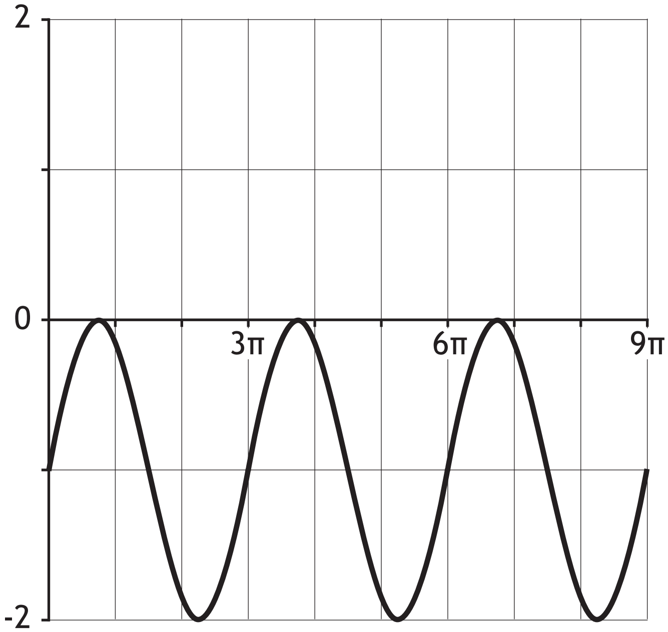


b) write a **cosine** function.

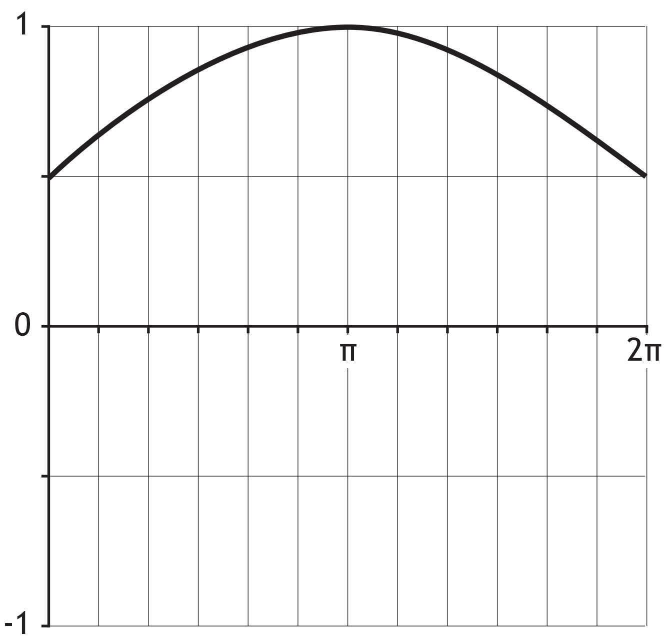


The ***b*** Parameter

c) write a **sine** function.



d) write a **sine** function.



Trigonometry

LESSON THREE - *Trigonometric Functions I*

Lesson Notes

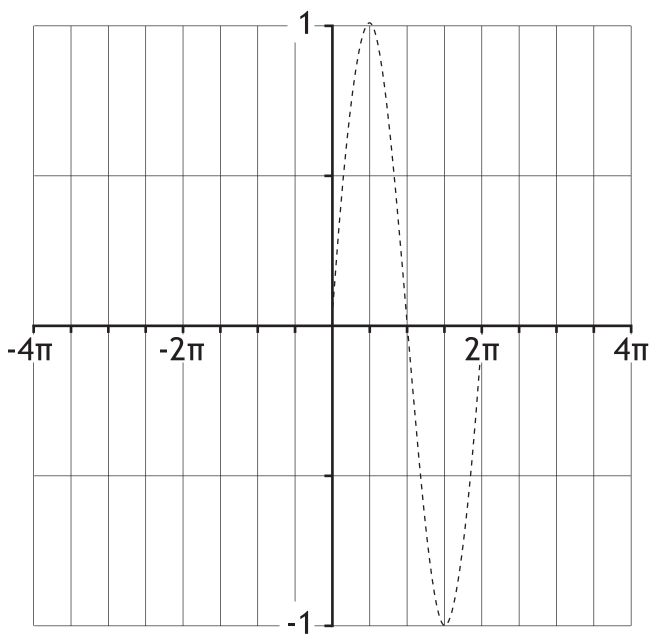
$$y = a\sin b(\theta - c) + d$$

Example 12

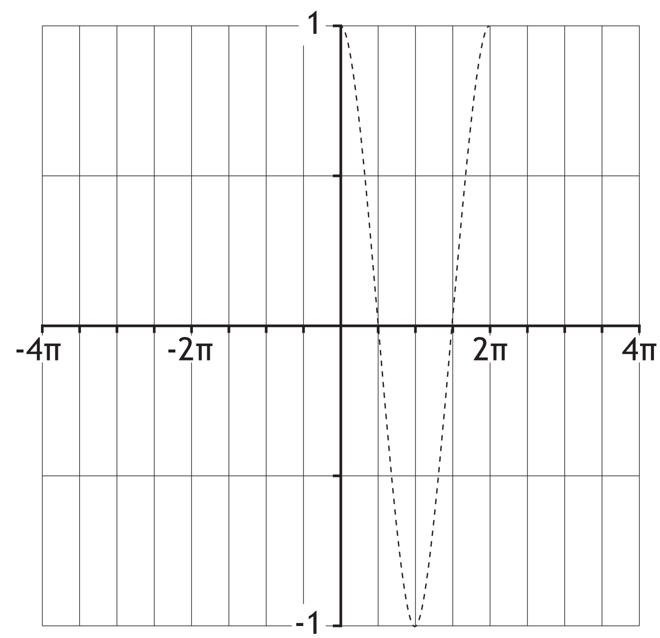
Graph each function over the stated domain.
The base graph is provided as a convenience.

The ***c*** Parameter

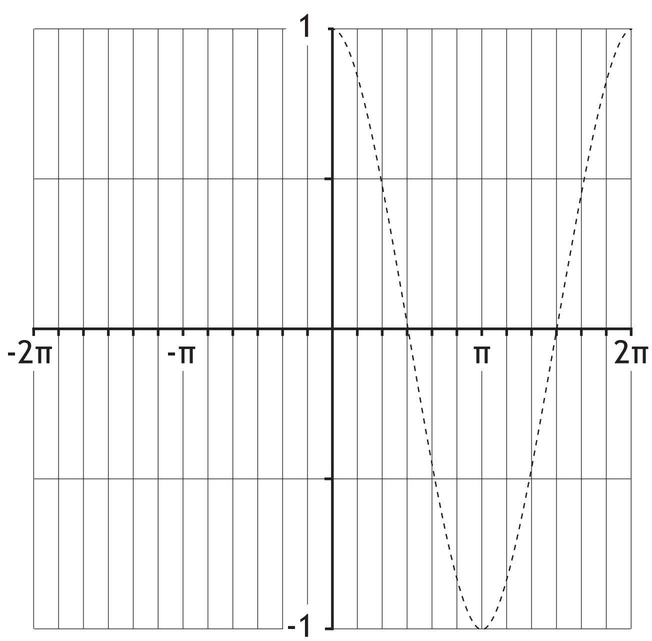
a) $y = \sin\left(\theta - \frac{\pi}{2}\right)$ *($-4\pi \le \theta \le 4\pi$)*



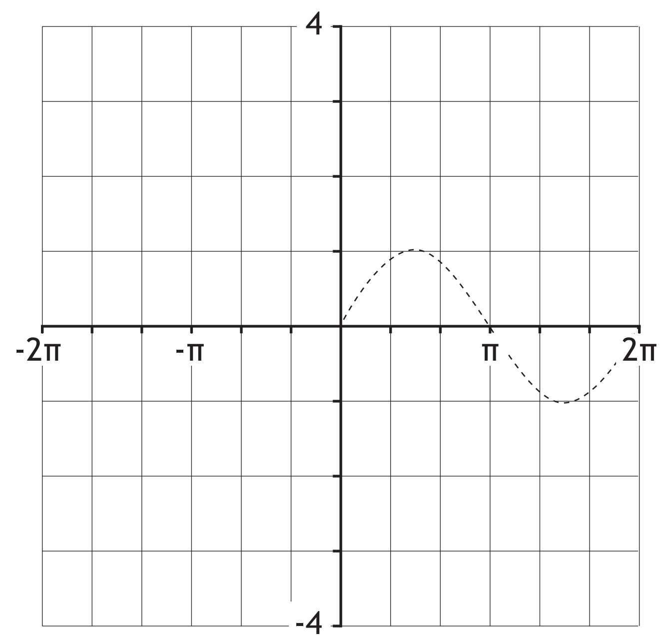
b) $y = \cos\left(\theta + \pi\right)$ *($-4\pi \le \theta \le 4\pi$)*



c) $y = \cos\left(\theta - \frac{\pi}{6}\right)$ *($-2\pi \le \theta \le 2\pi$)*



d) $y = 3\sin\left(\theta + \frac{2\pi}{3}\right)$ *($-2\pi \le \theta \le 2\pi$)*

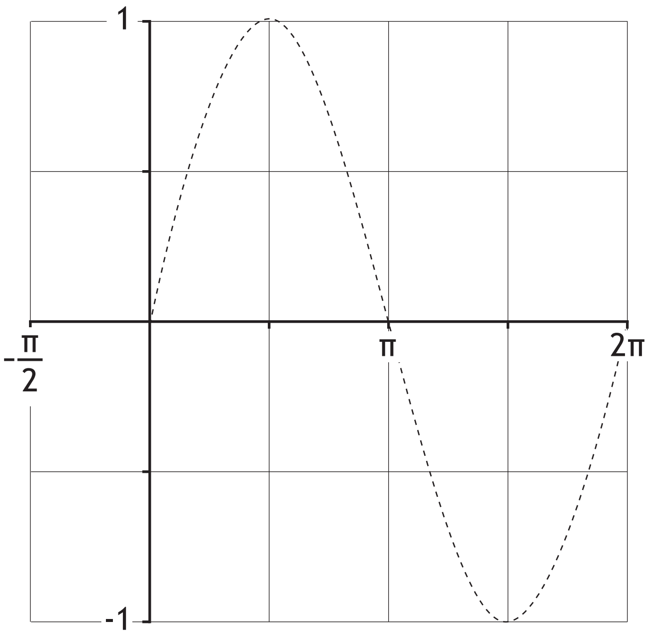


Example 13

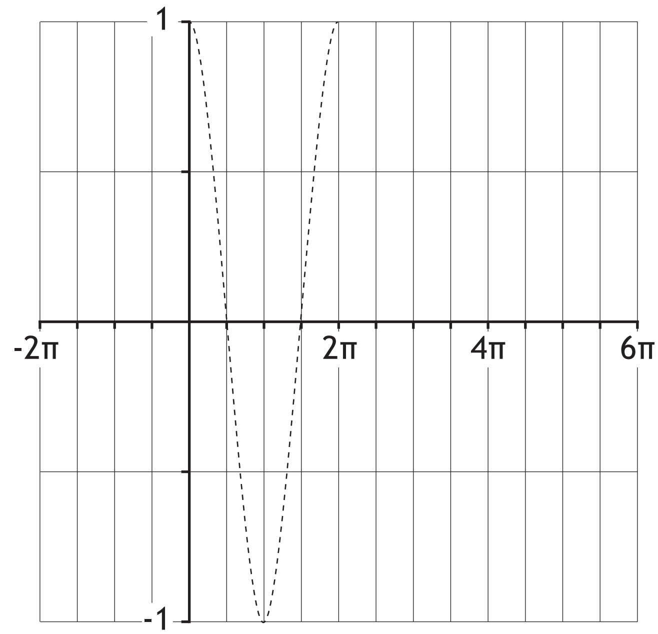
Graph each function over the stated domain.
The base graph is provided as a convenience.

The ***c*** Parameter

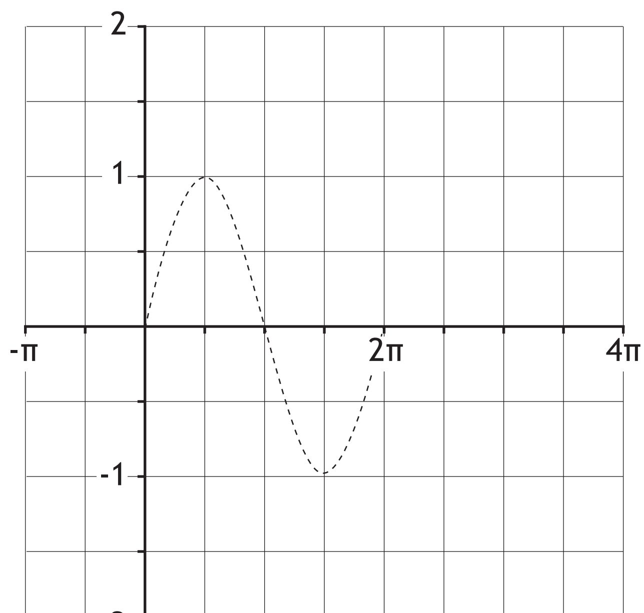
a) $y = \sin\left(2\theta + \pi\right)$ $\left(-\frac{\pi}{2} \leq \theta \leq 2\pi\right)$



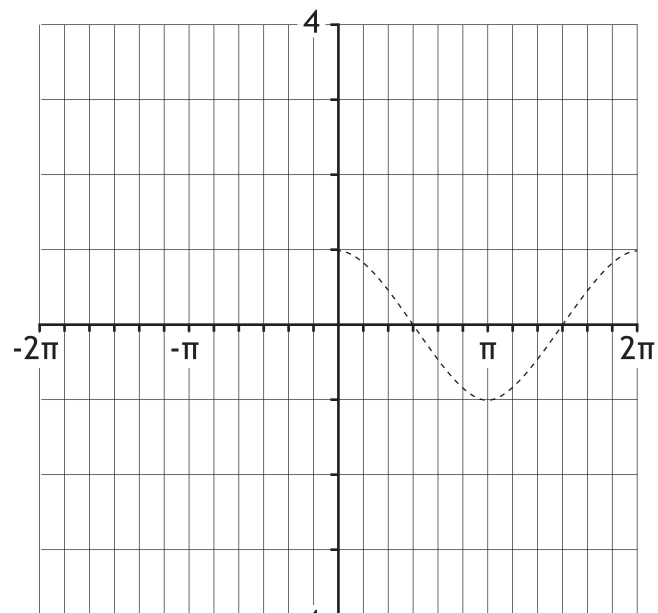
b) $y = \cos\left(\frac{1}{2}\theta + \pi\right)$ $(-2\pi \leq \theta \leq 6\pi)$



c) $y = -\frac{1}{2}\sin\left(2\theta - 3\pi\right) + 1$ $(-\pi \leq \theta \leq 4\pi)$



d) $y = -2\cos\left(4\theta + \frac{4\pi}{3}\right) - 2$ $(-2\pi \leq \theta \leq 2\pi)$

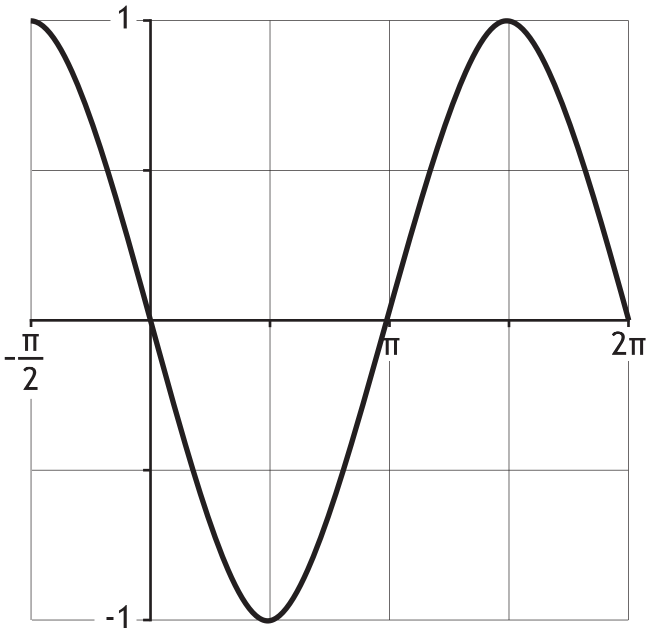


$y = a\sin b(\theta - c) + d$

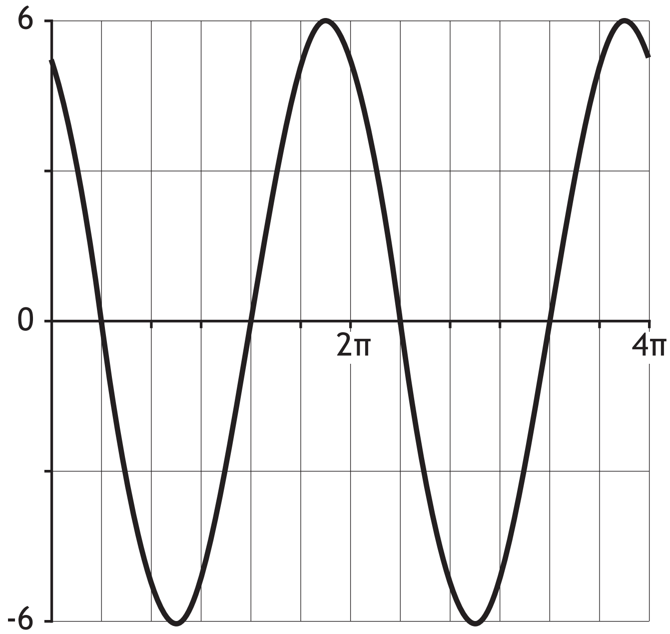
Example 14 Determine the trigonometric function corresponding to each graph.

The *c* Parameter

a) write a **cosine** function.

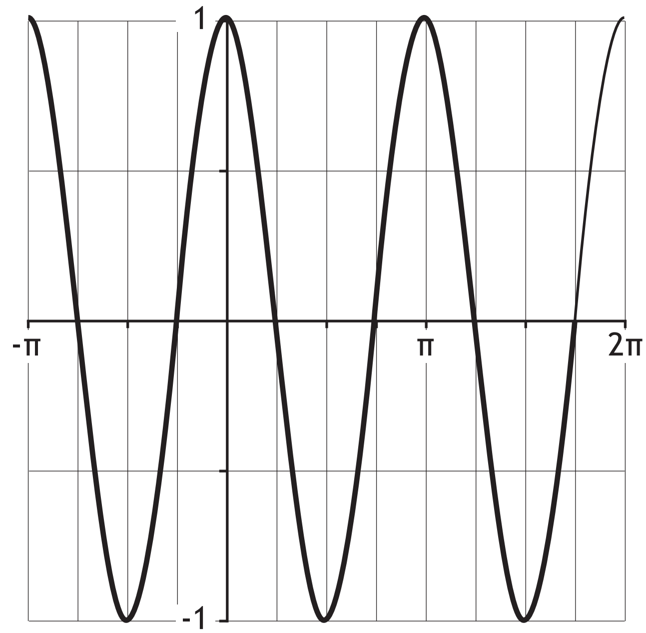


b) write a **sine** function.

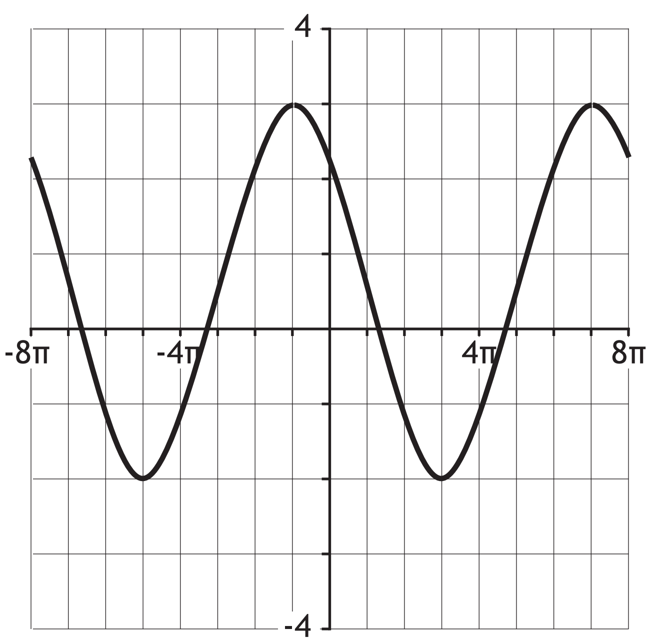


The *c* Parameter

c) write a **sine** function.



d) write a **cosine** function.



Trigonometry

LESSON THREE - *Trigonometric Functions I*

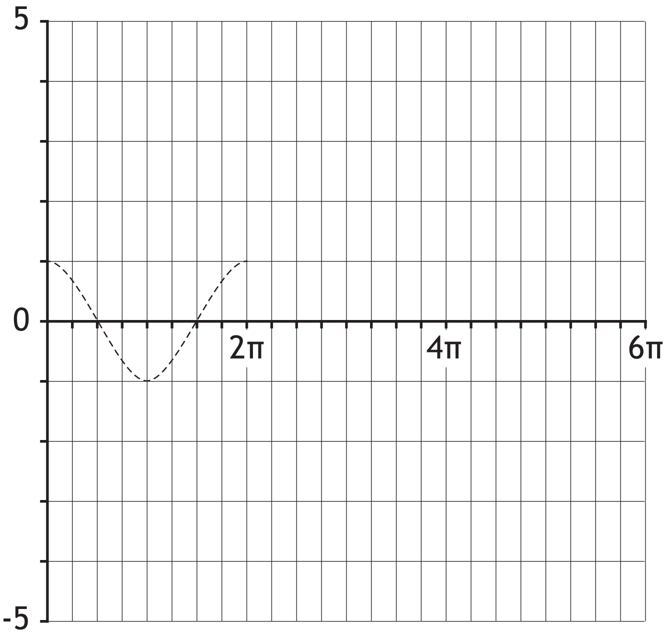
Lesson Notes

$$y = a\sin b(\theta - c) + d$$

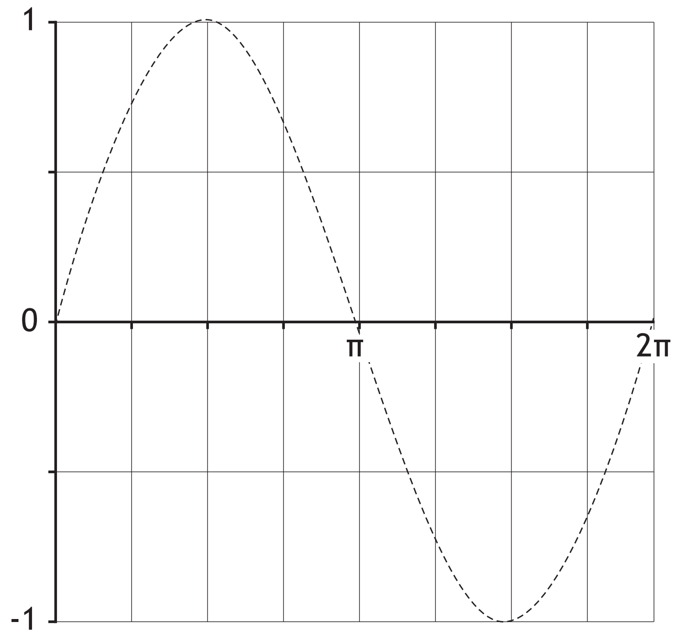
Example 15 Graph each function over the stated domain. The base graph is provided as a convenience.

a, b, c, & d

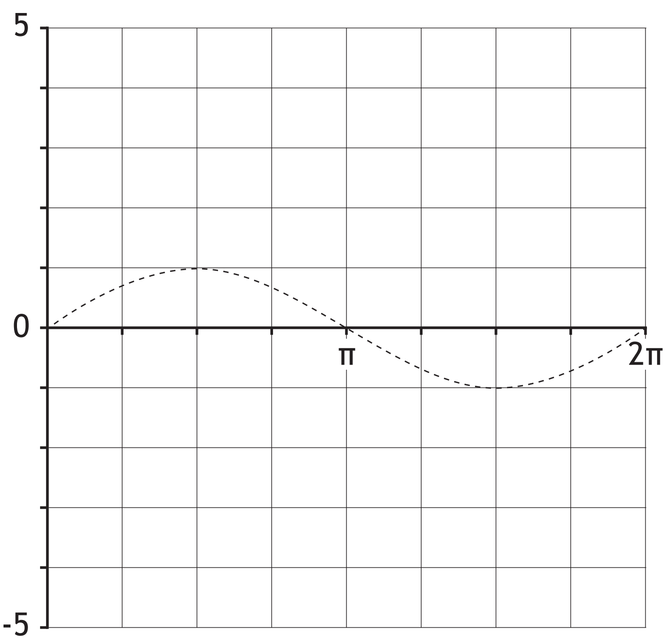
a) $y = 2\cos\left(\frac{1}{2}\theta - \frac{\pi}{8}\right) + 3$ *($0 \le \theta \le 6\pi$)*



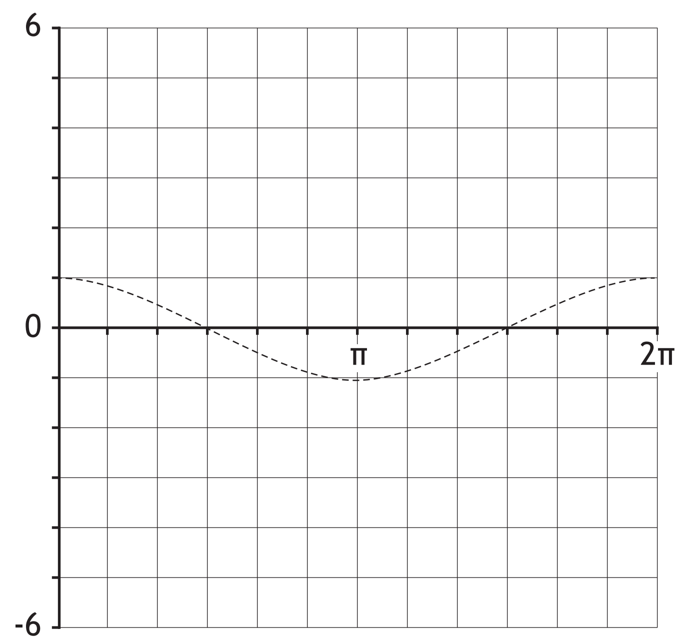
b) $y = \frac{1}{2}\sin\left(2\theta - \frac{\pi}{2}\right)$ *($0 \le \theta \le 2\pi$)*



c) $y = -2\sin\left(4\theta - \pi\right) - 3$ *($0 \le \theta \le 2\pi$)*

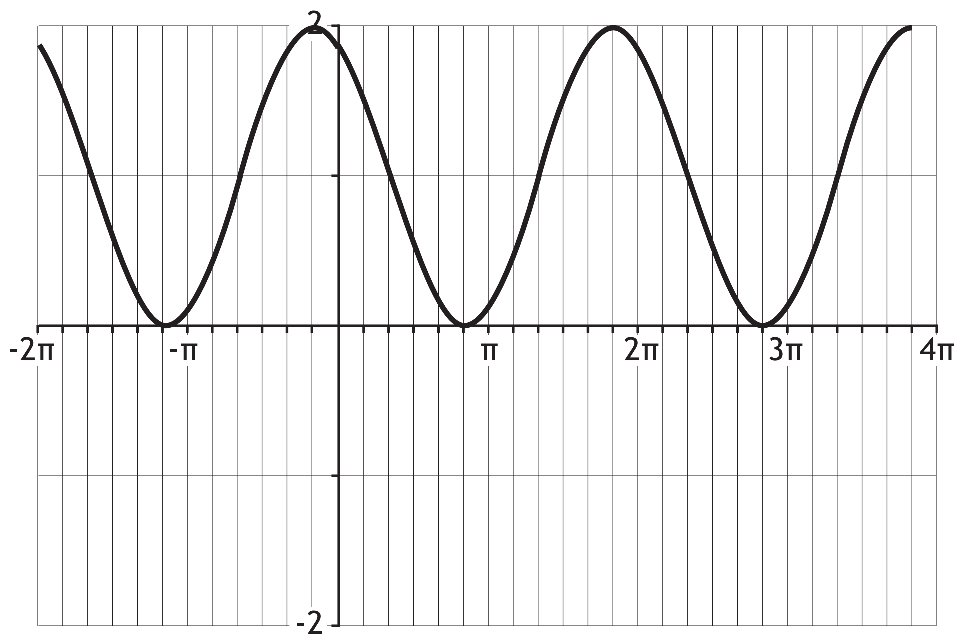


d) $y = -5\cos\left(2\theta - \frac{\pi}{3}\right) + 1$ *($0 \le \theta \le 2\pi$)*

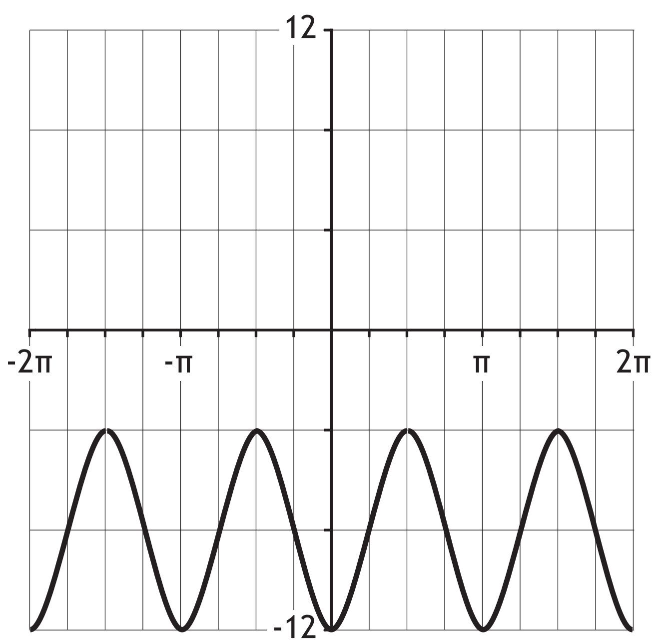


Example 16 Write a trigonometric function for each graph. ***a, b, c, & d***

a)



b)



Trigonometry

LESSON THREE - *Trigonometric Functions I*

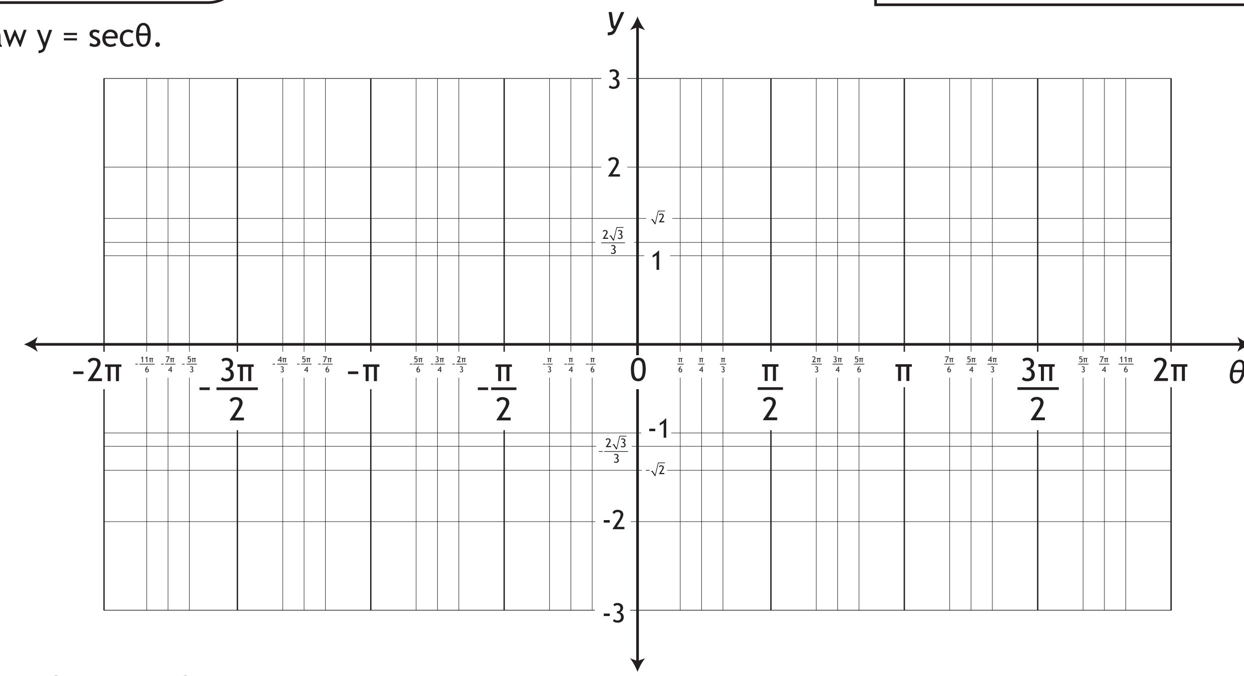
Lesson Notes

$y = a\sin b(\theta - c) + d$

Example 17 Exploring the graph of y = secθ.

Graphing Reciprocal Functions

a) Draw y = secθ.

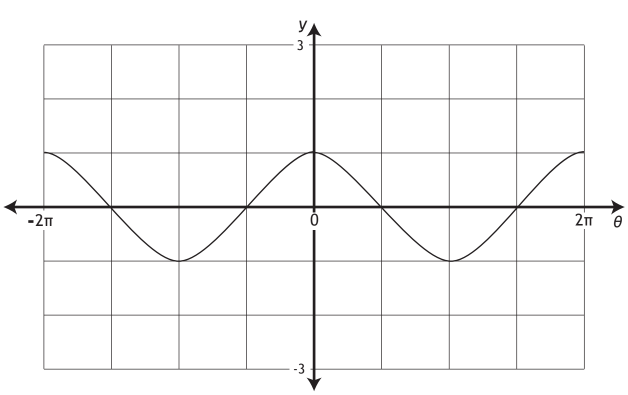


b) State the period.

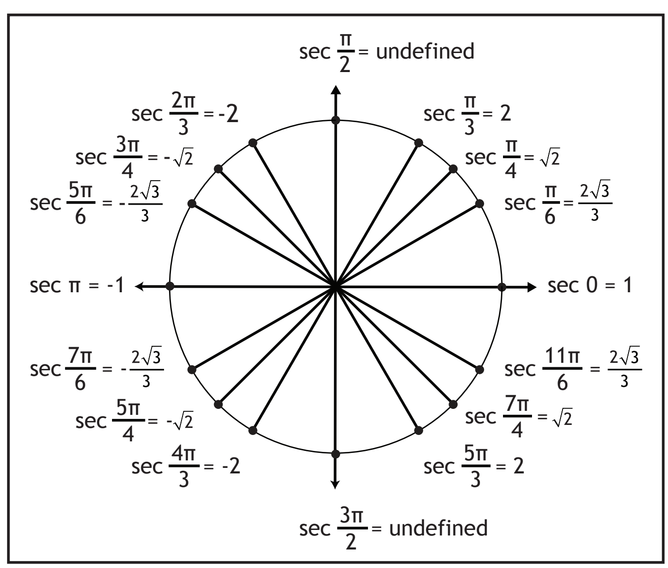
c) State the domain and range.

d) Write the general equation of the asymptotes.

e) Given the graph of f(θ) = cosθ, draw $y = \dfrac{1}{f(\theta)}$.



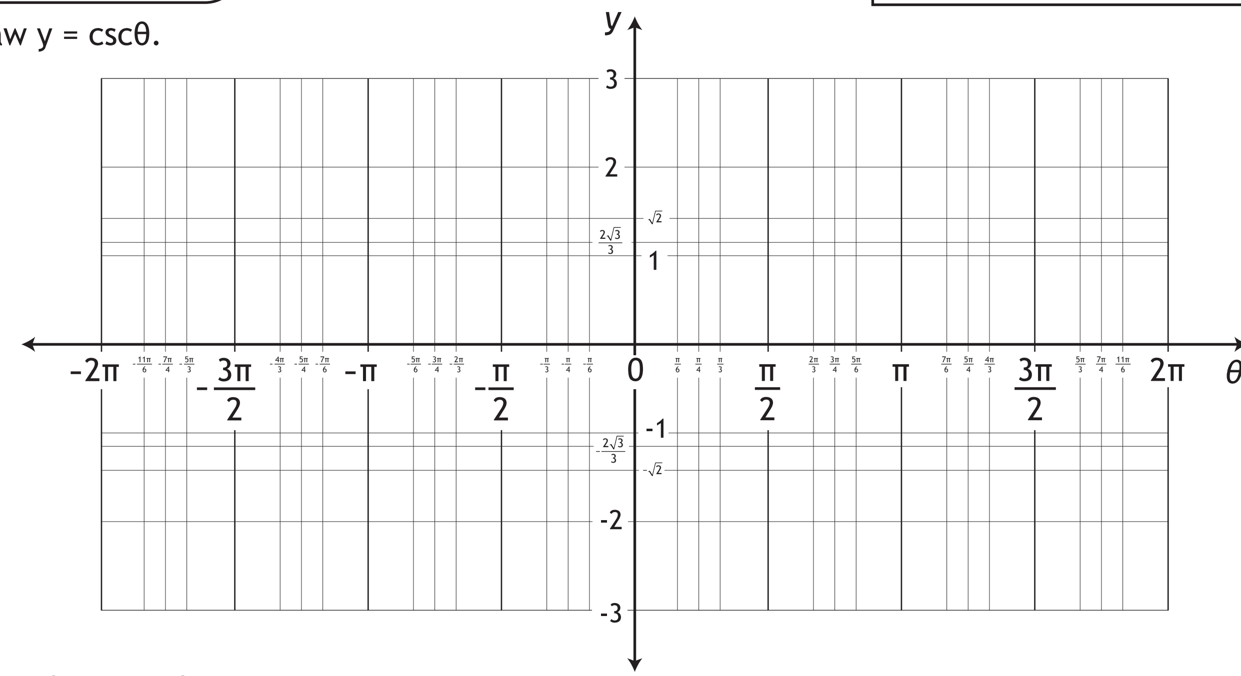
Unit Circle Reference (for secθ)



Example 18 Exploring the graph of $y = \csc\theta$.

Graphing Reciprocal Functions

a) Draw $y = \csc\theta$.

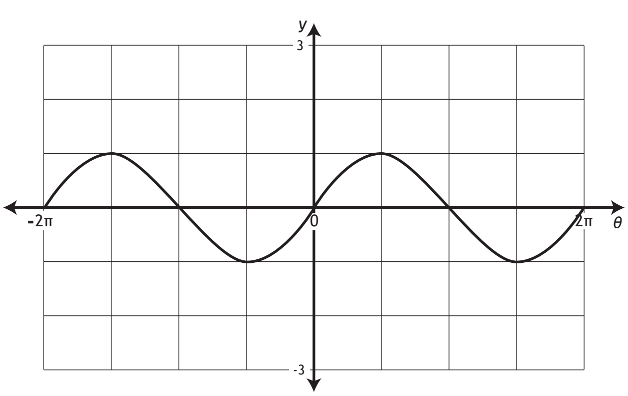


b) State the period.

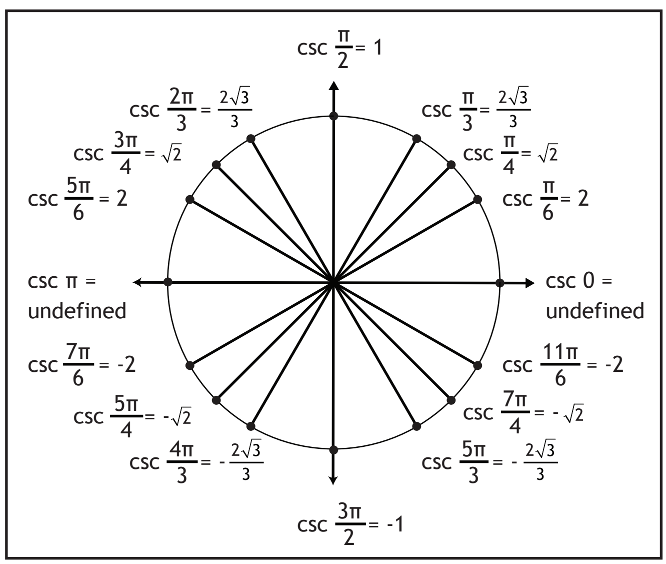
c) State the domain and range.

d) Write the general equation of the asymptotes.

e) Given the graph of $f(\theta) = \sin\theta$, draw $y = \dfrac{1}{f(\theta)}$.



Unit Circle Reference (for cscθ)



Trigonometry

LESSON THREE - *Trigonometric Functions I*

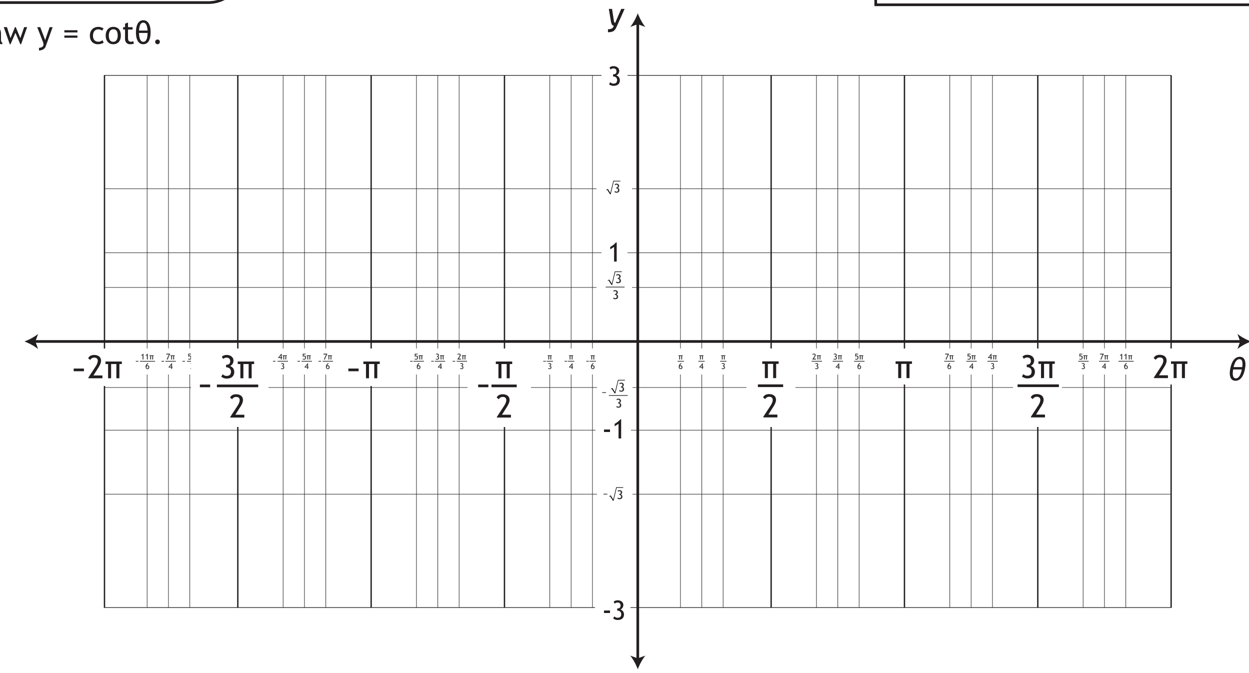
Lesson Notes

$$y = a\sin b(\theta - c) + d$$

Example 19 Exploring the graph of y = cotθ.

Graphing Reciprocal Functions

a) Draw y = cotθ.

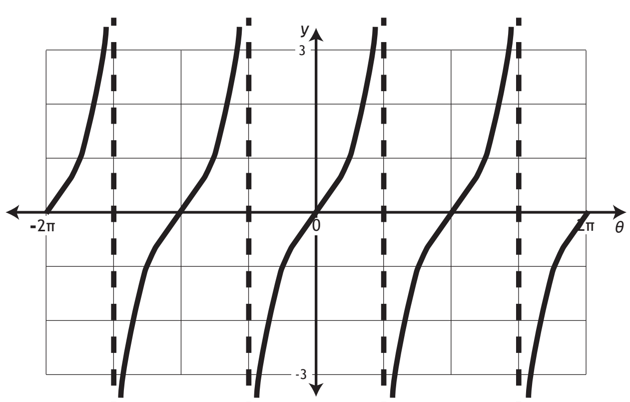


b) State the period.

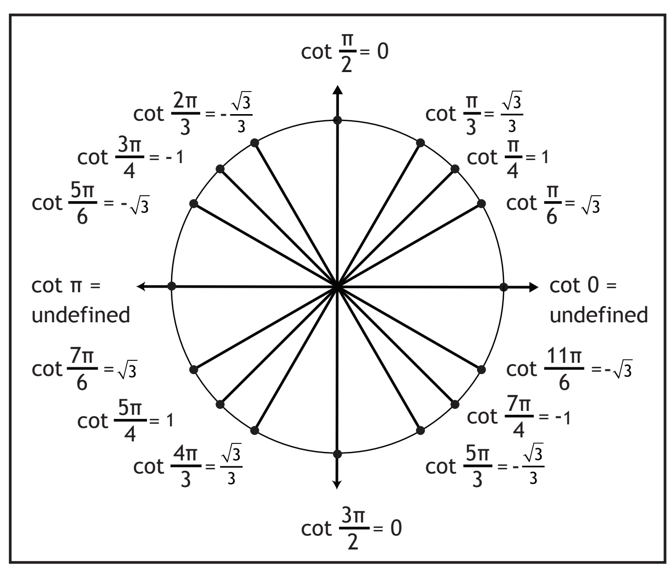
c) State the domain and range.

d) Write the general equation of the asymptotes.

e) Given the graph of f(θ) = tanθ, draw $y = \dfrac{1}{f(\theta)}$.



Unit Circle Reference (for cotθ)

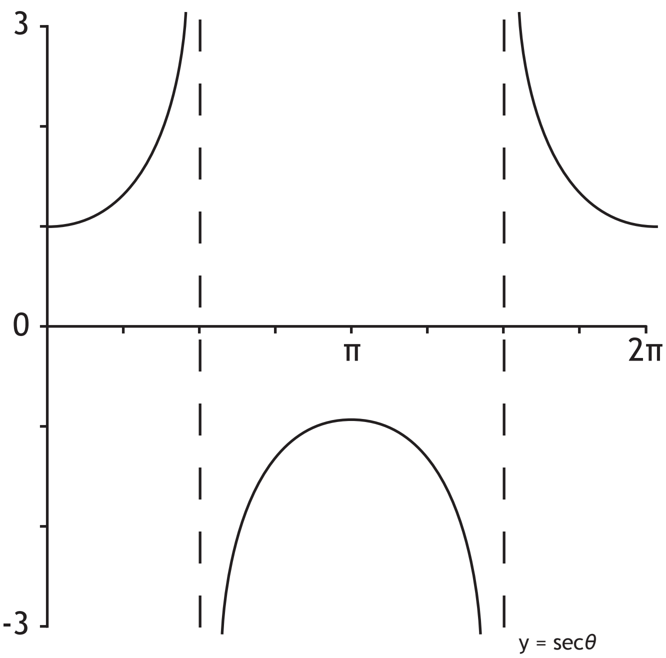


Example 20

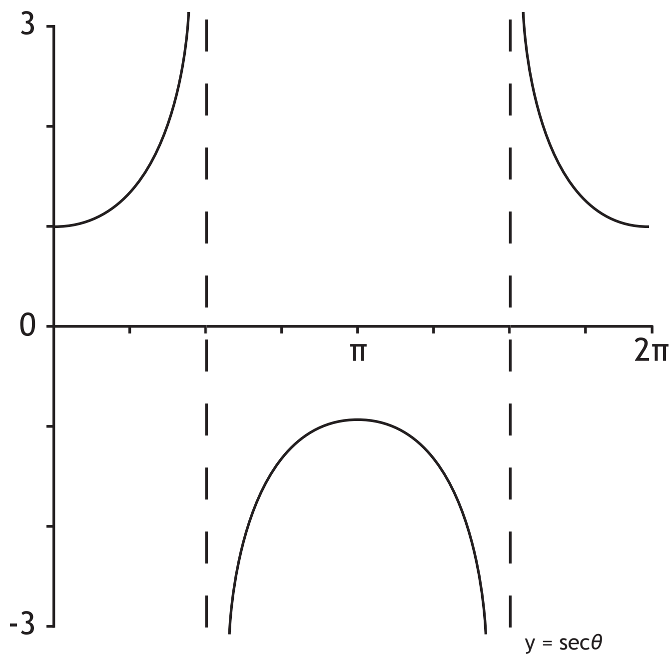
Graph each function over the domain $0 \le \theta \le 2\pi$.
The base graph is provided as a convenience.
State the new domain and range.

Transformations of Reciprocal Functions

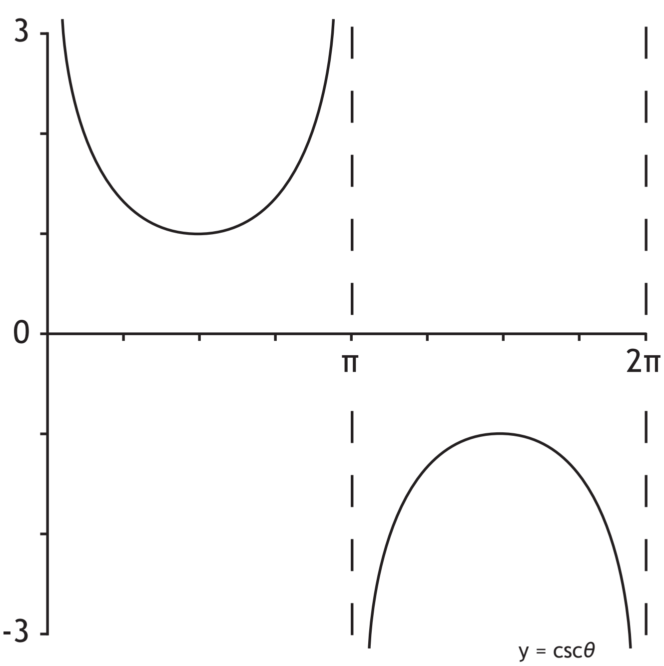
a) $y = \frac{1}{2}\sec\theta$



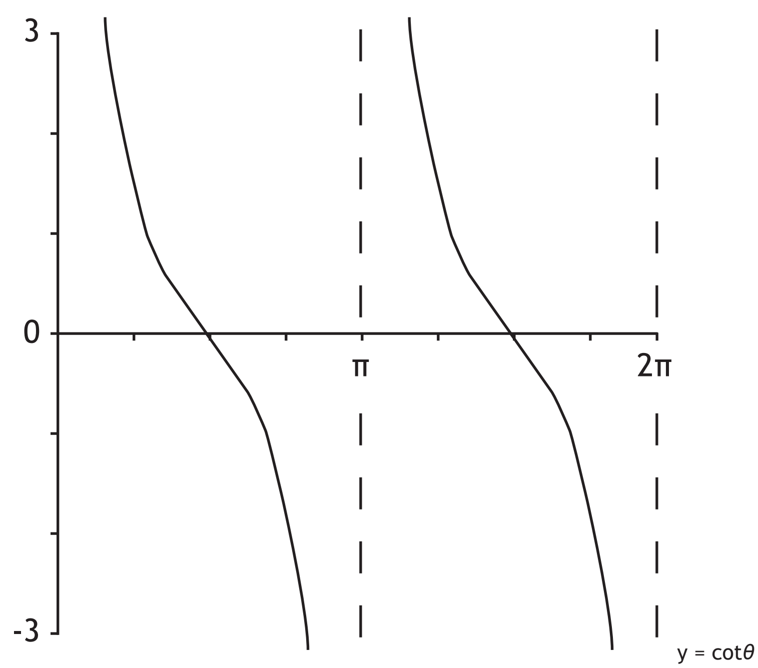
b) $y = \sec 2\theta$



c) $y = \csc\left(\theta - \frac{\pi}{4}\right)$



d) $y = \cot\frac{1}{2}\theta$



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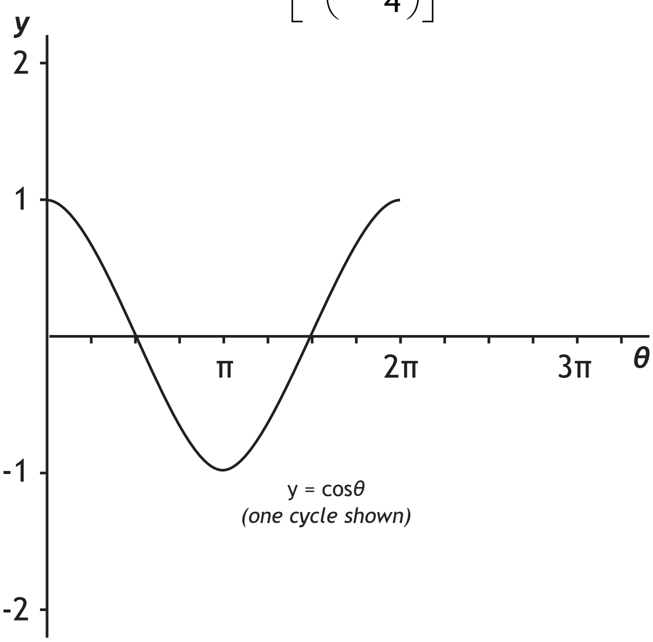
Example 1 Trigonometric Functions of Angles

Trigonometric Functions of Angles

a)

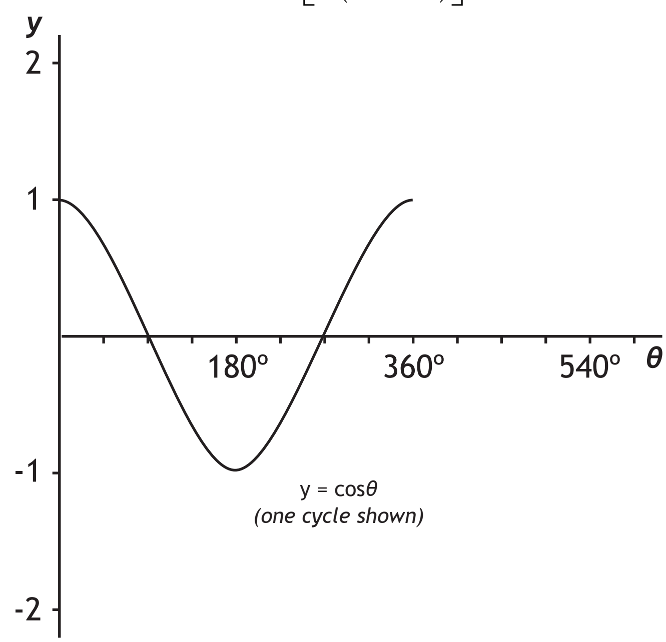
i) Graph: $f\left(\theta\right) = \cos\left[2\left(\theta - \frac{\pi}{4}\right)\right]$ $(0 \leq \theta < 3\pi)$



ii) Graph this function using technology.

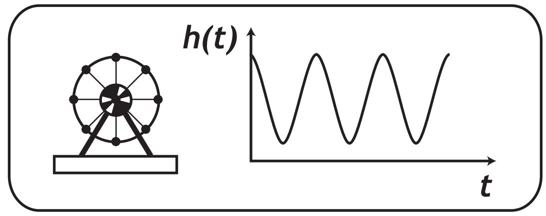
b)

i) Graph: $f\left(\theta\right) = \cos\left[2\left(\theta - 45^{\circ}\right)\right]$ $(0^{\circ} \leq \theta < 540^{\circ})$



ii) Graph this function using technology.

Trigonometry
LESSON FOUR - *Trigonometric Functions II*
Lesson Notes

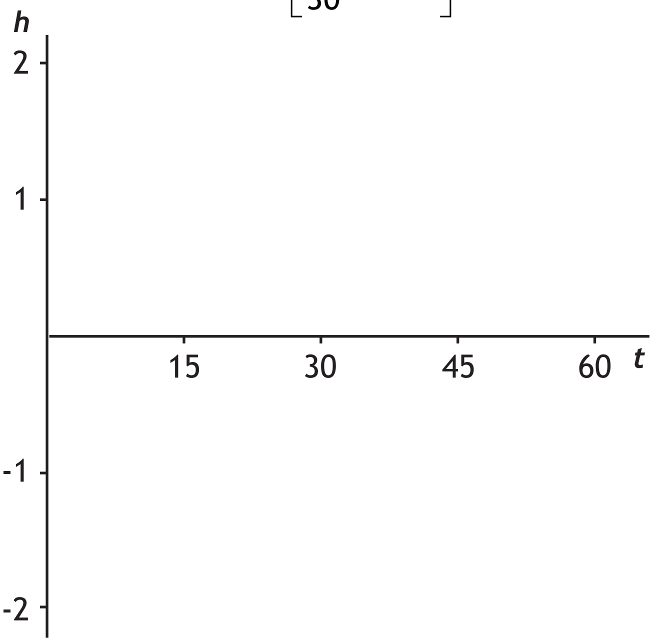


Example 2 Trigonometric Functions of Real Numbers.

Trigonometric Functions of Real Numbers

a)

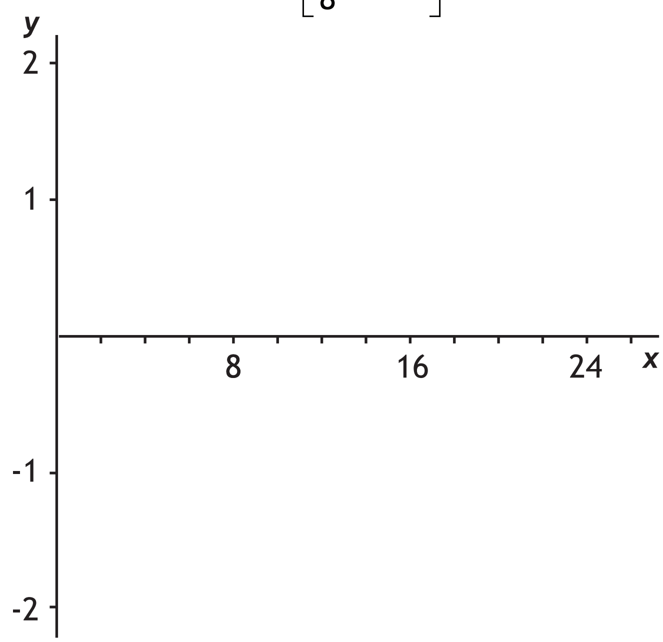
i) Graph: $h(t) = \cos\left[\frac{\pi}{30}(t - 15)\right]$



ii) Graph this function using technology.

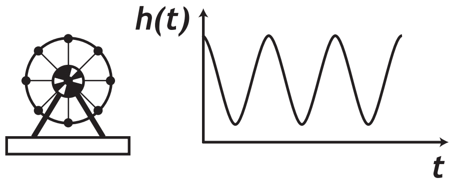
b)

i) Graph: $f(x) = \cos\left[\frac{\pi}{8}(x - 4)\right]$



ii) Graph this function using technology.

c) What are three differences between trigonometric functions of angles and trigonometric functions of real numbers?

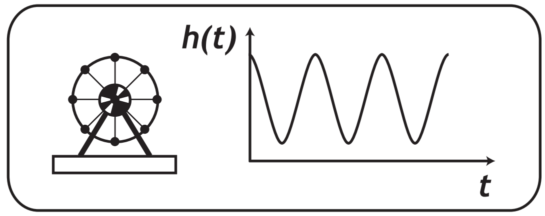


Example 3 Determine the view window for each function and sketch each graph.

Graph Preperation and View Windows

a) $f(x) = 12\sin\left[\frac{\pi}{3}(x - 2)\right] - 14$

b) $f(x) = -25\cos\frac{\pi}{250}(x + 225) + 50$



Example 4 Determine the view window for each function and sketch each graph.

Graph Preperation and View Windows

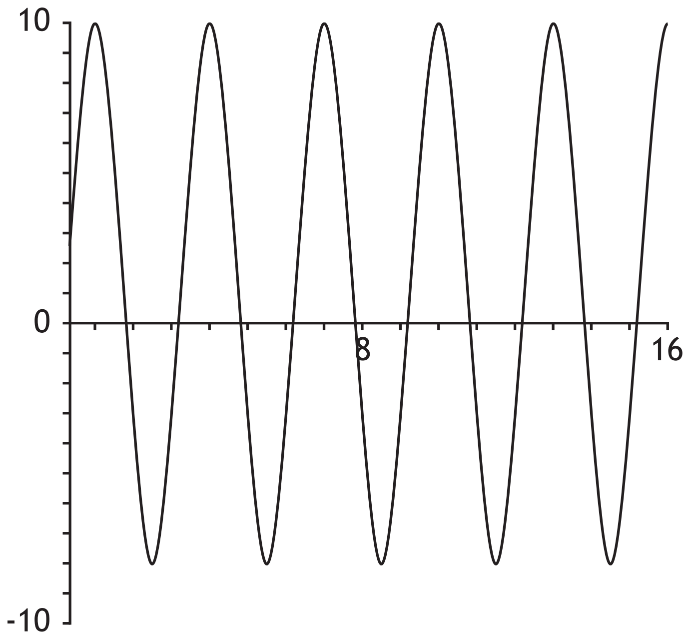
a) $f(x) = 13.5\cos\frac{2\pi}{96}(x - 24) + 6.5$

b) $f(x) = 2.5\sin 0.25\pi(x + 3) + 16$

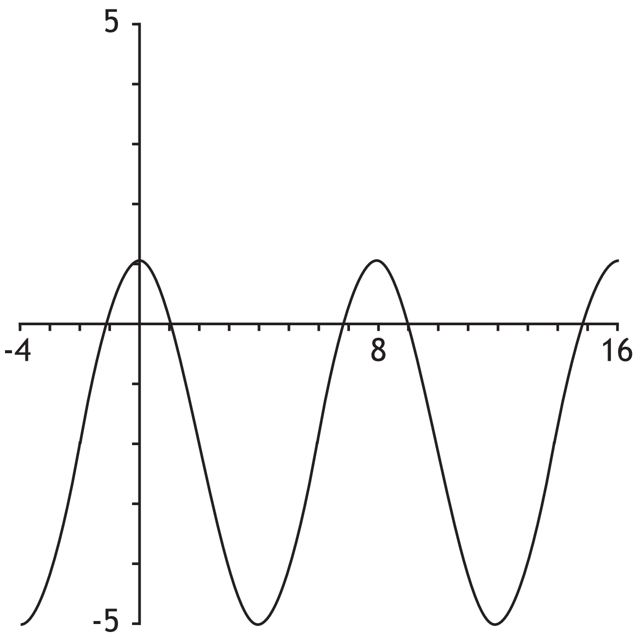
Example 5 Determine the trigonometric function corresponding to each graph.

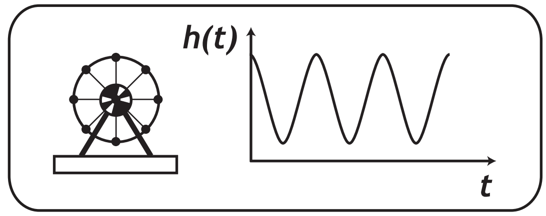
Find the Trigonometric Function of a Graph

a) write a **cosine** function.

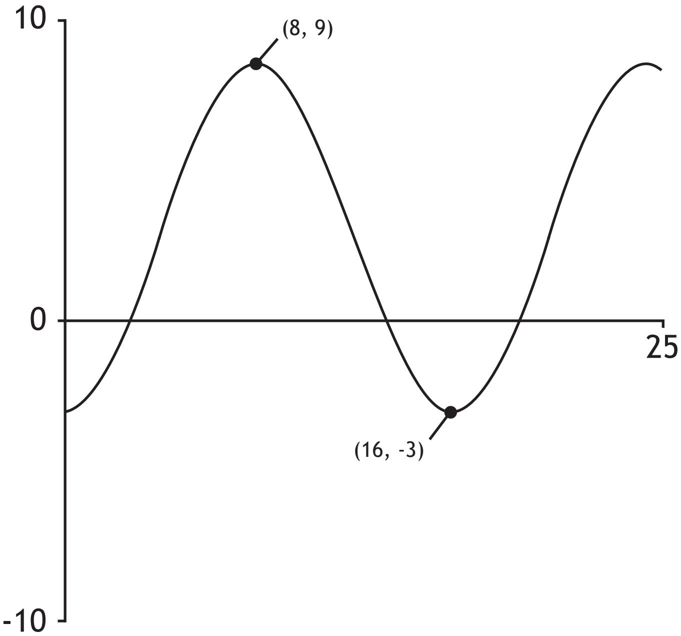


b) write a **sine** function.

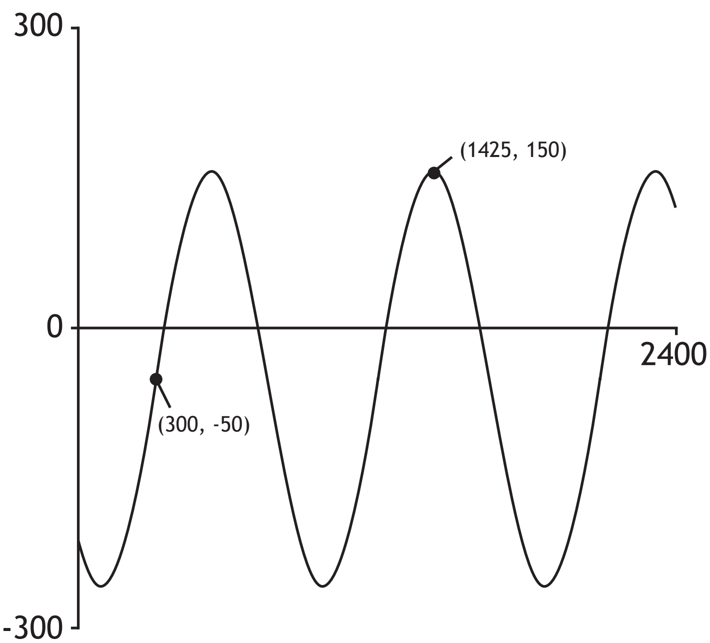


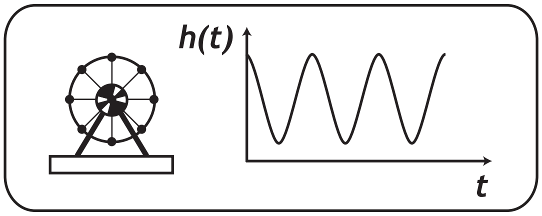


c) write a **cosine** function.



d) write a **sine** function.





Example 6 Answer the following questions:

Assorted Questions

a) If the transformation $g(\theta) - 3 = f(2\theta)$ is applied to the graph of $f(\theta) = \sin\theta$, find the new range.

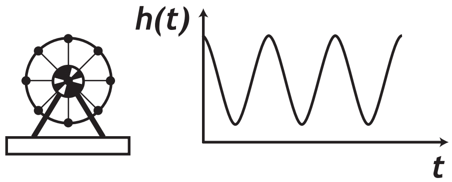
b) Find the range of $f(\theta) = k\sin\left(\theta - \frac{\pi}{4}\right) - 3$.

c) If the range of $y = 3\cos\theta + d$ is $[-4, k]$, determine the values of d and k.

d) State the range of $f(\theta) - 2 = m\sin(2\theta) + n$.

e) The graphs of $f(\theta)$ and $g(\theta)$ intersect at the points $\left(\frac{\pi}{8}, \frac{\sqrt{2}}{2}\right)$ and $\left(\frac{5\pi}{8}, \frac{-\sqrt{2}}{2}\right)$

If the amplitude of each graph is quadrupled, determine the new points of intersection.

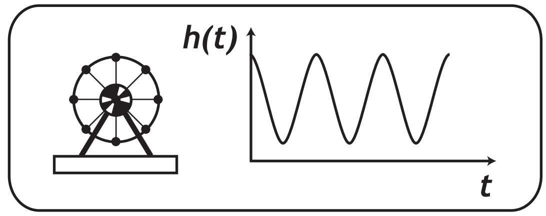


Example 7 Answer the following questions:

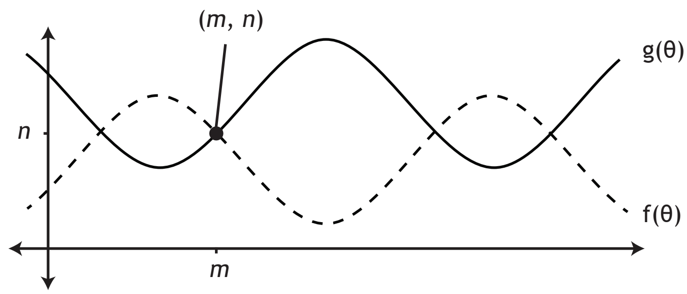
Assorted Questions

a) If the point $\left(\frac{\pi}{2}, -2\right)$ lies on the graph of $f(\theta) = a\cos\left(\theta - \frac{\pi}{4}\right) - 4$, find the value of a.

b) Find the y-intercept of $f(\theta) = -3\cos\left(k\theta + \frac{\pi}{2}\right) - b$.



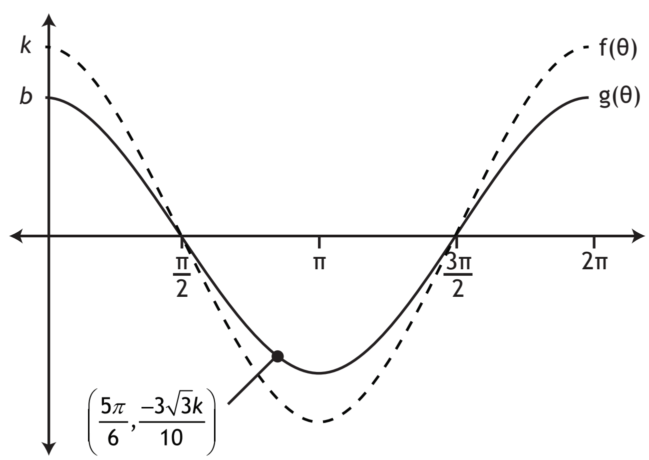
c) The graphs of f(θ) and g(θ) intersect at the point (m, n). Find the value of f(m) + g(m).



d) The graph of f(θ) = kcosθ is transformed to the graph of g(θ) = bcosθ by a vertical stretch about the x-axis.

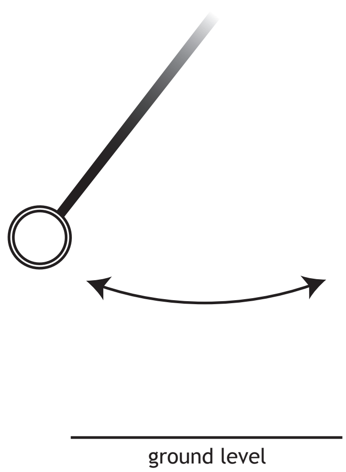
If the point $\left(\frac{5\pi}{6}, \frac{-3\sqrt{3}k}{10}\right)$ exists on the graph of g(θ),

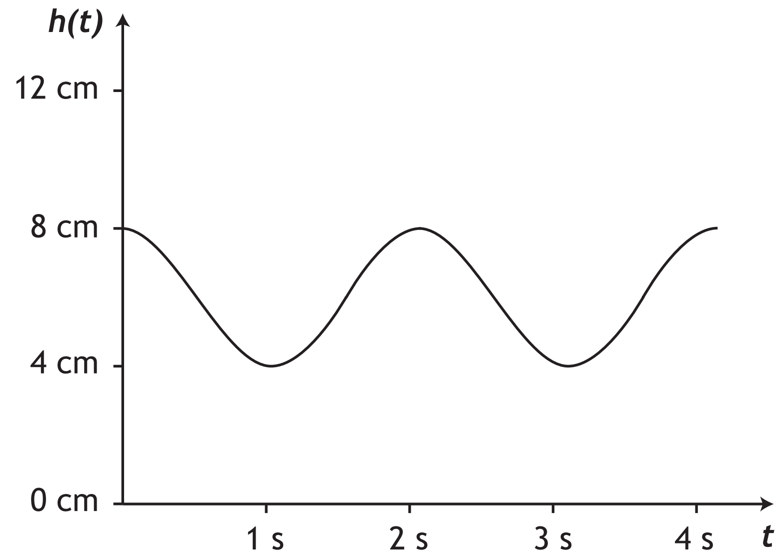
state the vertical stretch factor.



Example 8

The graph shows the height of a pendulum bob as a function of time. One cycle of a pendulum consists of two swings - a right swing and a left swing.

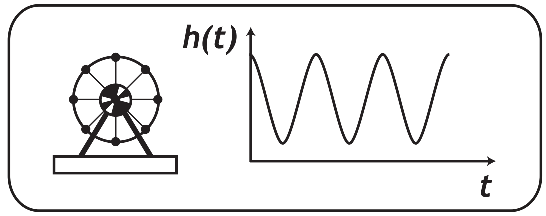




a) Write a function that describes the height of the pendulum bob as a function of time.

b) If the period of the pendulum is halved, how will this change the parameters in the function you wrote in part (a)?

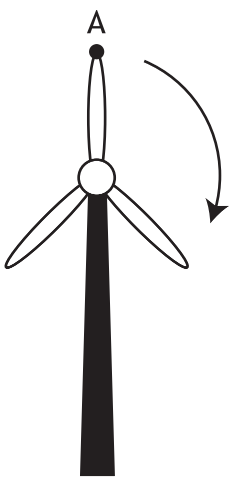
c) If the pendulum is lowered so its lowest point is 2 cm above the ground, how will this change the parameters in the function you wrote in part (a)?



Example 9

A wind turbine has blades that are 30 m long. An observer notes that one blade makes 12 complete rotations (clockwise) every minute. The highest point of the blade during the rotation is 105 m.

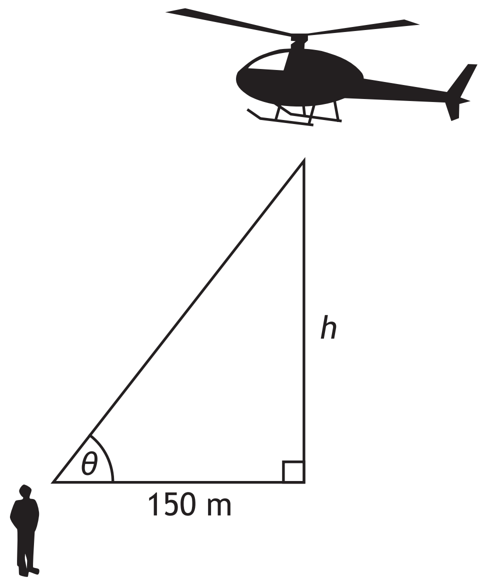
a) Using Point A as the starting point of the graph, draw the height of the blade over two rotations.



b) Write a function that corresponds to the graph.

c) Do we get a different graph if the wind turbine rotates counterclockwise?

Example 10

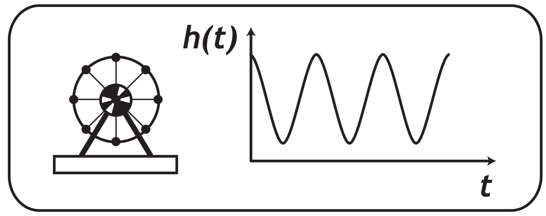


A person is watching a helicopter ascend from a distance 150 m away from the takeoff point.

a) Write a function, h(θ), that expresses the height as a function of the angle of elevation. Assume the height of the person is negligible.

b) Draw the graph, using an appropriate domain.

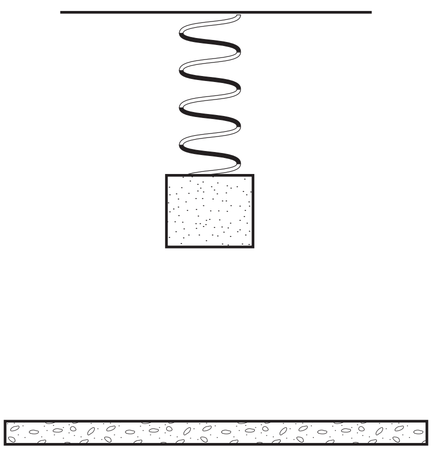
c) Explain how the shape of the graph relates to the motion of the helicopter.



Example 11

A mass is attached to a spring 4 m above the ground and allowed to oscillate from its equilibrium position. The lowest position of the mass is 2.8 m above the ground, and it takes 1 s for one complete oscillation.

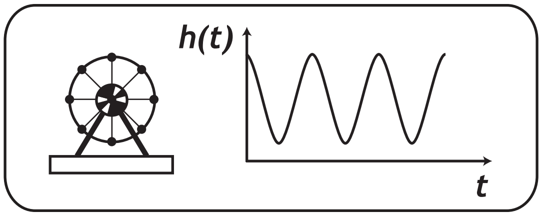
a) Draw the graph for two full oscillations of the mass.



b) Write a sine function that gives the height of the mass above the ground as a function of time.

c) Calculate the height of the mass after 1.2 seconds. Round your answer to the nearest hundredth.

d) In one oscillation, how many seconds is the mass lower than 3.2 m? Round your answer to the nearest hundredth.



Example 12

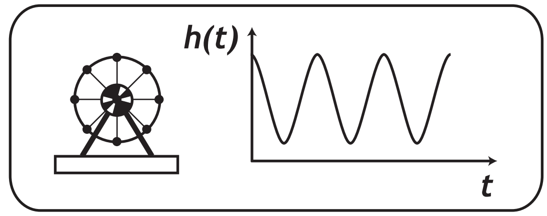
A Ferris wheel with a radius of 15 m rotates once every 100 seconds. Riders board the Ferris wheel using a platform 1 m above the ground.

a) Draw the graph for two full rotations of the Ferris wheel.

b) Write a cosine function that gives the height of the rider as a function of time.

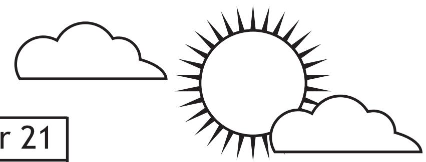
c) Calculate the height of the rider after 1.6 rotations of the Ferris wheel.
Round your answer to the nearest hundredth.

d) In one rotation, how many seconds is the rider higher than 26 m?
Round your answer to the nearest hundredth.



Example 13

The following table shows the number of daylight hours in Grande Prairie.

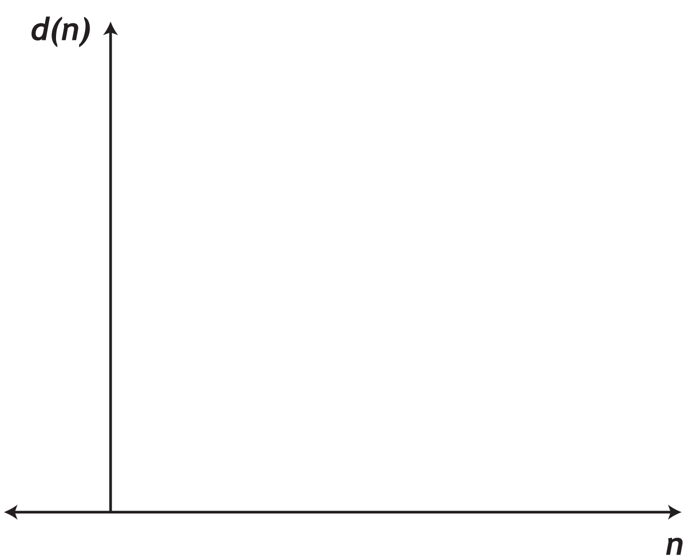


December 21	March 21	June 21	September 21	December 21
6h, 46m	12h, 17m	17h, 49m	12h, 17m	6h, 46m

a) Convert each date and time to a number that can be used for graphing.

Day Number	December 21 =	March 21 =	June 21 =	September 21 =	December 21 =
Daylight Hours	6h, 46m =	12h, 17m =	17h, 49m =	12h, 17m =	12h, 46m =

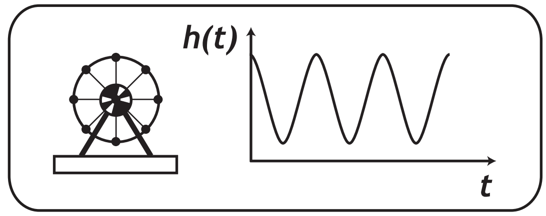
b) Draw the graph for one complete cycle *(winter solstice to winter solstice)*.



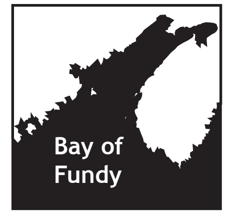
c) Write a cosine function that relates the number of daylight hours, d, to the day number, n.

d) How many daylight hours are there on May 2? Round your answer to the nearest hundredth.

e) In one year, approximately how many days have more than 17 daylight hours?
Round your answer to the nearest day.



Example 14



The highest tides in the world occur between New Brunswick and Nova Scotia, in the Bay of Fundy. Each day, there are two low tides and two high tides. The chart below contains tidal height data that was collected over a 24-hour period.

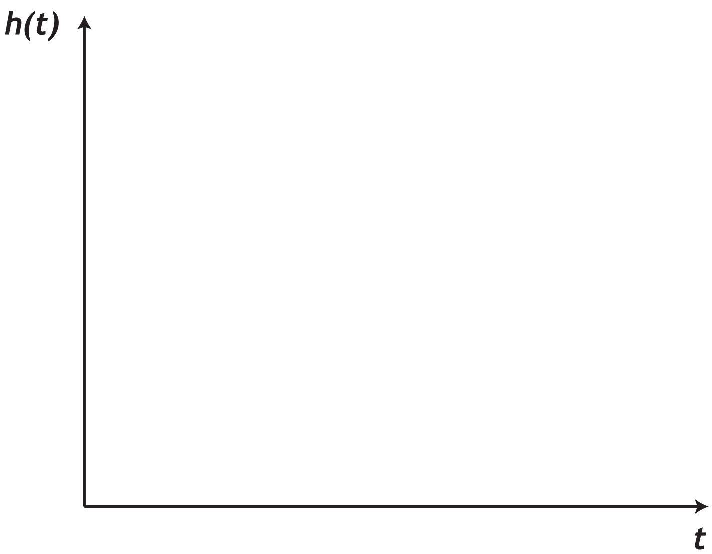
	Time	Decimal Hour	Height of Water (m)
Low Tide	2:12 AM		3.48
High Tide	8:12 AM		13.32
Low Tide	2:12 PM		3.48
High Tide	8:12 PM		13.32

Note: Actual tides at the Bay of Fundy are 6 hours and 13 minutes apart due to daily changes in the position of the moon.

In this example, we will use 6 hours for simplicity.

a) Convert each time to a decimal hour.

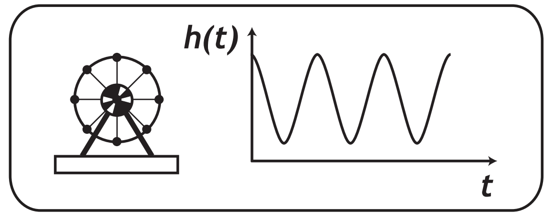
b) Graph the height of the tide for one full cycle *(low tide to low tide)*.



c) Write a cosine function that relates the height of the water to the elapsed time.

d) What is the height of the water at 6:09 AM? Round your answer to the nearest hundredth.

e) For what percentage of the day is the height of the water greater than 11 m?
Round your answer to the nearest tenth.



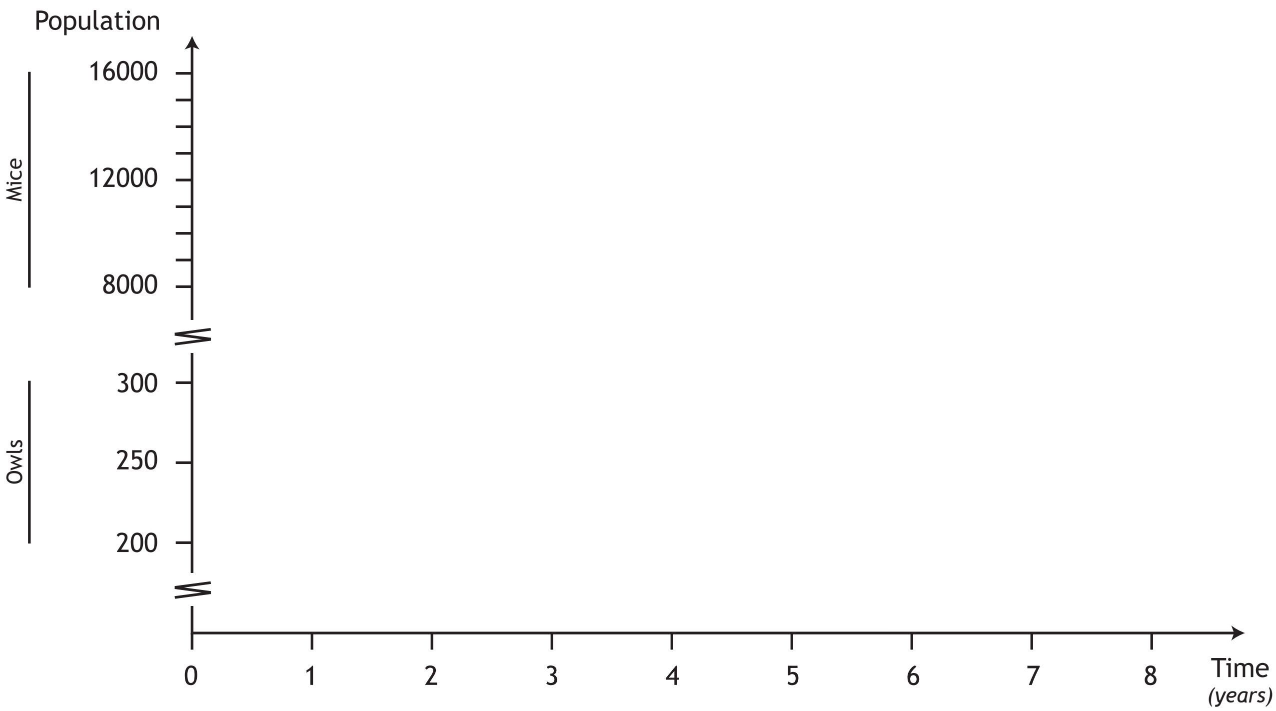
Example 15

A wooded region has an ecosystem that supports both owls and mice. Owl and mice populations vary over time according to the equations:

Owl population:	$O(t) = 50\sin\left[\frac{\pi}{3}(t - 1.5)\right] + 250$
Mouse population:	$M(t) = 4000\sin\left(\frac{\pi}{3}t\right) + 12000$

where O is the population of owls, M is the population of mice, and t is the time in years.

a) Graph the population of owls and mice over six years.



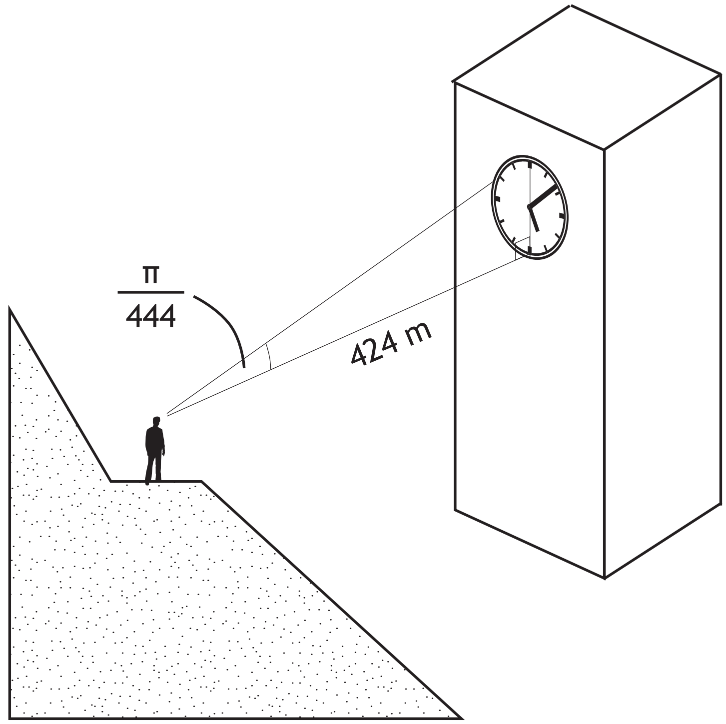
b) Describe how the graph shows the relationship between owl and mouse populations.

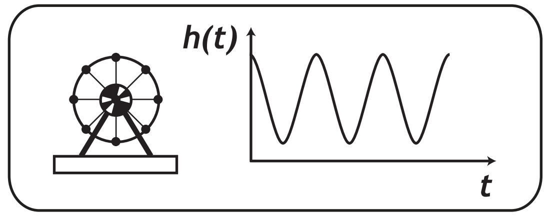
Example 16

The angle of elevation between the 6:00 position and the 12:00 position of a historical building's clock, as measured from an observer standing on a hill, is $\frac{\pi}{444}$.

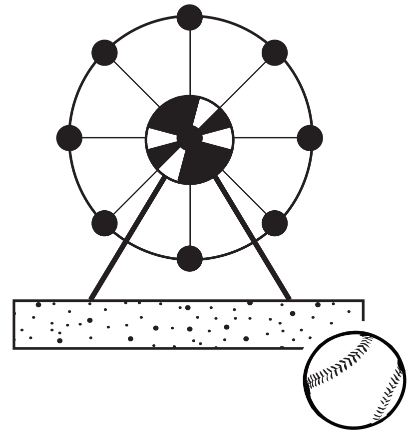
The observer also knows that he is standing 424 m away from the clock, and his eyes are at the same height as the base of the clock. The radius of the clock is the same as the length of the minute hand.

If the height of the minute hand's tip is measured relative to the bottom of the clock, what is the height of the tip at 5:08, to the nearest tenth of a metre?





Example 17



Shane is on a Ferris wheel, and his height can be described by the equation $h_{wheel}\left(t\right) = -9\cos\frac{\pi}{30}t + 10$.

Tim, a baseball player, can throw a baseball with a speed of 20 m/s. If Tim throws a ball directly upwards, the height can be determined by the equation $h_{ball}(t) = -4.905t^2 + 20t + 1$

If Tim throws the baseball 15 seconds after the ride begins, when are Shane and the ball at the same height?

Answer Key

Trigonometry Lesson One: Degrees and Radians

Note: For illustrative purposes, all diagram angles will be in degrees.

Example 1:

a) The rotation angle between the initial arm and the terminal arm is called the standard position angle.

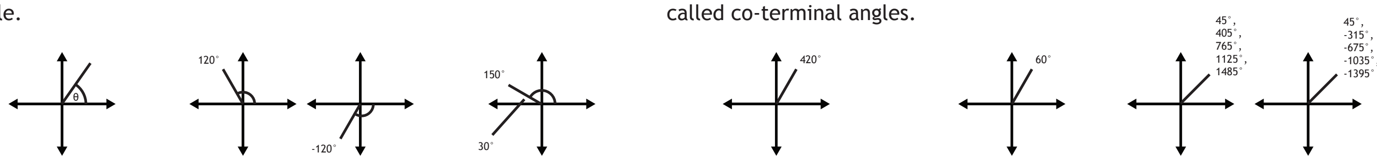
b) An angle is positive if we rotate the terminal arm counter-clockwise, and negative if rotated clockwise.

c) The angle formed between the terminal arm and the x-axis is called the reference angle.

d) If the terminal arm is rotated by a multiple of 360° in either direction, it will return to its original position. These angles are called co-terminal angles.

e) A principal angle is an angle that exists between 0° and 360°.

f) The general form of co-terminal angles is $\theta_c = \theta_p + n(360°)$ using degrees, or $\theta_c = \theta_p + n(2\pi)$ using radians.

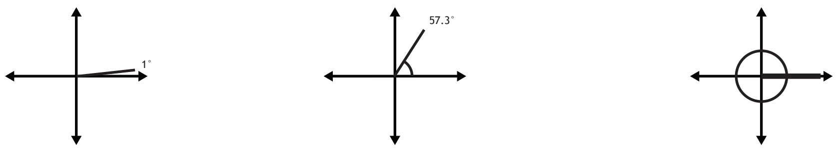


Example 2:

a) i. One degree is defined as 1/360[th] of a full rotation.

ii. One radian is the angle formed when the terminal arm swipes out an arc that has the same length as the terminal arm. One radian is approximately 57.3°.

iii. One revolution is defined as 360º, or 2pi. It is one complete rotation around a circle.



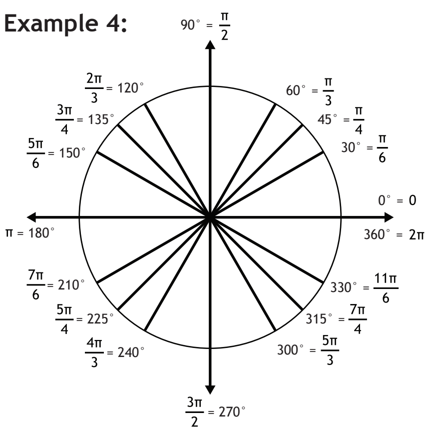
Conversion Multiplier Reference Chart

	degree	radian	revolution
degree		$\frac{\pi}{180°}$	$\frac{1\text{ rev}}{360°}$
radian	$\frac{180°}{\pi}$		$\frac{1\text{ rev}}{2\pi}$
revolution	$\frac{360°}{1\text{ rev}}$	$\frac{2\pi}{1\text{ rev}}$	

b) i. 0.40 rad ii. 0.06 rev iii. 148.97° iv. 0.41 rev v. 270° vi. 4.71 rad **c)** i. 0.79 rad ii. π/4 rad

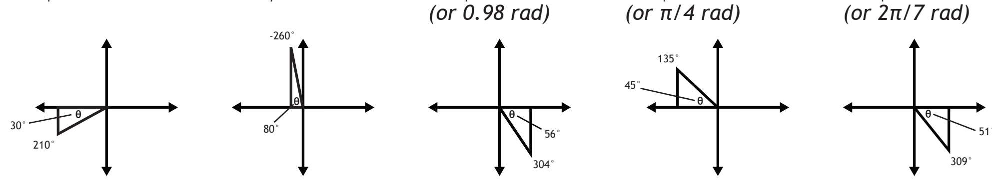
Example 3: **a)** 3.05 rad **b)** 7π/6 rad **c)** 1/3 rev **d)** 143.24° **e)** 270° **f)** 4.71 rad **g)** 1/4 rev **h)** 180° **i)** 6π rad

Example 4:



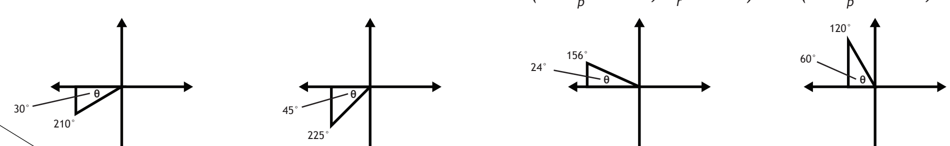
Example 5:

a) $\theta_r = 30°$ **b)** $\theta_r = 80°$ **c)** $\theta_r = 56°$ *(or 0.98 rad)* **d)** $\theta_r = 45°$ *(or π/4 rad)* **e)** $\theta_r = 51°$ *(or 2π/7 rad)*



Example 6:

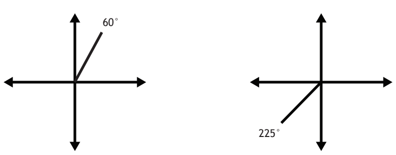
a) $\theta_p = 210°$, $\theta_r = 30°$ **b)** $\theta_p = 225°$, $\theta_r = 45°$ **c)** $\theta_p = 156°$, $\theta_r = 24°$ *(or $\theta_p = 2.72$, $\theta_r = 0.42$)* **d)** $\theta_p = 120°$, $\theta_r = 60°$ *(or $\theta_p = 2\pi/3$, $\theta_r = \pi/3$)*



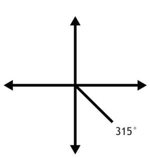
Example 7:

a) $\theta = 60°$, $\theta_p = 60°$
$\theta_c = -300°, 420°, 780°$

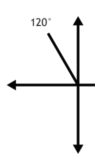
b) $\theta = -495°$, $\theta_p = 225°$
$\theta_c = -855°, -135°, 225°, 585°$



c) $\theta = 675°$, $\theta_p = 315°$
$\theta_c = -45°, 315°$
(or $\theta_c = -0.785, 5.50$)



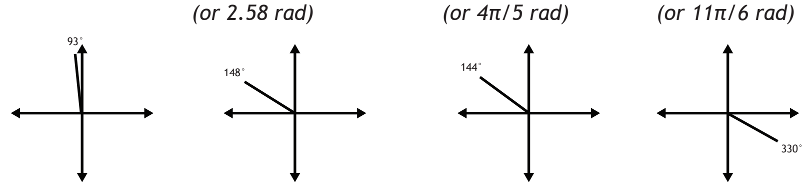
d) $\theta = 480°$, $\theta_p = 120°$
$\theta_c = -960°, -600°, -240°, 120°, 840°, 1200°$



(or $\theta_c = -16\pi/3, -10\pi/3, -4\pi/3, 2\pi/3, 14\pi/3, 20\pi/3$)

Example 8:

a) $\theta_p = 93°$ **b)** $\theta_p = 148°$ *(or 2.58 rad)* **c)** $\theta_p = 144°$ *(or 4π/5 rad)* **d)** $\theta_p = 330°$ *(or 11π/6 rad)*



Example 9:

a) $\theta_c = 1380°$ **b)** $\theta_c = -138\pi/5$ **c)** $\theta_c = 20°$ **d)** $\theta_c = 2\pi/3$

Answer Key

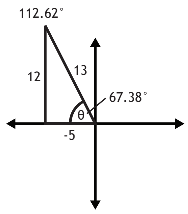
Example 10:

a) $\theta_p = 112.62°$, $\theta_r = 67.38°$

$\sin\theta = \frac{12}{13}$ $\quad \csc\theta = \frac{13}{12}$

$\cos\theta = \frac{-5}{13}$ $\quad \sec\theta = \frac{13}{-5}$

$\tan\theta = \frac{12}{-5}$ $\quad \cot\theta = \frac{12}{-5}$

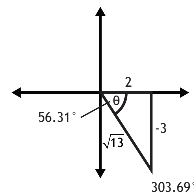


b) $\theta_p = 303.69°$, $\theta_r = 56.31°$

$\sin\theta = \frac{-3\sqrt{13}}{13}$ $\quad \csc\theta = \frac{\sqrt{13}}{-3}$

$\cos\theta = \frac{2\sqrt{13}}{13}$ $\quad \sec\theta = \frac{\sqrt{13}}{2}$

$\tan\theta = \frac{-3}{2}$ $\quad \cot\theta = \frac{2}{-3}$



Example 11:

a) sinθ: QI: +, QII: +, QIII: -, QIV: -
b) cosθ: QI +, QII: -, QIII: -, QIV: +
c) tanθ: QI +, QII: -, QIII: +, QIV: -
d) cscθ: QI: +, QII: +, QIII: -, QIV: -
e) secθ: QI +, QII: -, QIII: -, QIV: +
f) cotθ: QI +, QII: -, QIII: +, QIV: -
g) sinθ & cscθ share the same quadrant signs.
cosθ & secθ share the same quadrant signs.
tanθ & cotθ share the same quadrant signs

Example 12: **a)** i. QIII or QIV ii. QI or QIV iii. QI or QIII **b)** i. QI ii. QIV iii. QIII **c)** i. none ii. QIII iii. QI

Example 13:

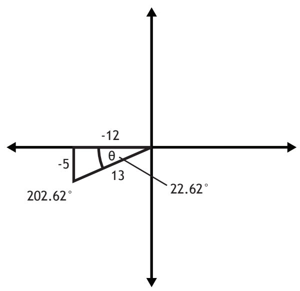
a) $\theta_p = 202.62°$, $\theta_r = 22.62°$

(or $\theta_p = 3.54$ rad, $\theta_r = 0.39$ rad)

$\sin\theta = \frac{-5}{13}$ $\quad \csc\theta = \frac{13}{-5}$

$\cos\theta = \frac{-12}{13}$ $\quad \sec\theta = \frac{13}{-12}$

$\tan\theta = \frac{5}{12}$ $\quad \cot\theta = \frac{12}{5}$



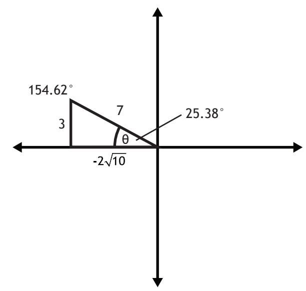
b) $\theta_p = 154.62°$, $\theta_r = 25.38°$

(or $\theta_p = 2.70$ rad, $\theta_r = 0.44$ rad)

$\sin\theta = \frac{3}{7}$ $\quad \csc\theta = \frac{7}{3}$

$\cos\theta = \frac{-2\sqrt{10}}{7}$ $\quad \sec\theta = \frac{7\sqrt{10}}{-20}$

$\tan\theta = \frac{3\sqrt{10}}{-20}$ $\quad \cot\theta = \frac{-2\sqrt{10}}{3}$



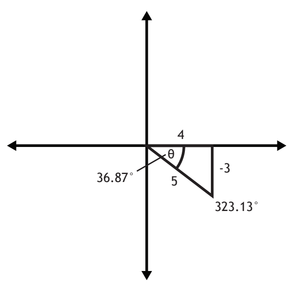
Example 14:

a) $\theta_p = 323.13°$, $\theta_r = 36.87°$

$\sin\theta = \frac{-3}{5}$ $\quad \csc\theta = \frac{5}{-3}$

$\cos\theta = \frac{4}{5}$ $\quad \sec\theta = \frac{5}{4}$

$\tan\theta = \frac{-3}{4}$ $\quad \cot\theta = \frac{4}{-3}$

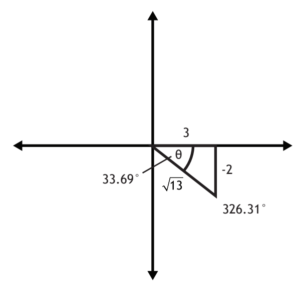


b) $\theta_p = 326.31°$, $\theta_r = 33.69°$

$\sin\theta = \frac{-2\sqrt{13}}{13}$ $\quad \csc\theta = \frac{\sqrt{13}}{-2}$

$\cos\theta = \frac{3\sqrt{13}}{13}$ $\quad \sec\theta = \frac{\sqrt{13}}{3}$

$\tan\theta = \frac{-2}{3}$ $\quad \cot\theta = \frac{3}{-2}$



Example 15:

a)

If the angle θ could exist in either quadrant ___ or ___ ...	The calculator always picks quadrant
I or II	I
I or III	I
I or IV	I
II or III	II
II or IV	IV
III or IV	IV

b) Each answer is different because the calculator is unaware of which quadrant the triangle is in. The calculator assumes Mark's triangle is in QI, Jordan's triangle is in QII, and Dylan's triangle is in QIV.

Example 16:

a) The arc length can be found by multiplying the circumference by the sector percentage.
This gives us:
$a = 2\pi r \times \theta/2\pi = r\theta$.
b) 13.35 cm
c) 114.59°
d) 2.46 cm
e) $n = 7\pi/6$

Example 17:

a) The area of a sector can be found by multiplying the area of the full circle by the sector percentage to get the area of the sector.
This gives us:
$a = \pi r^2 \times \theta/2\pi = r^2\theta/2$.
b) $28\pi/3$ cm^2
c) 3π cm^2
d) $81\pi/2$ cm^2
e) 15π cm^2

Example 18:

a) 600°/s
b) 0.07 rad/s
c) 1.04 km
d) 70 rev/s
e) 2.60 rev/s

Example 19:

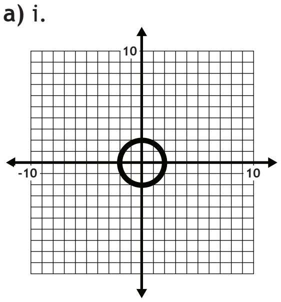
a) $\pi/2700$ rad/s
b) 468.45 km

Answer Key

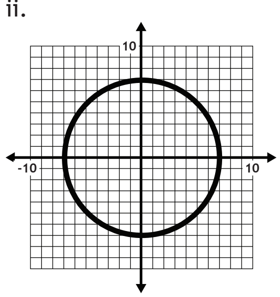
Trigonometry Lesson Two: The Unit Circle

Example 1:

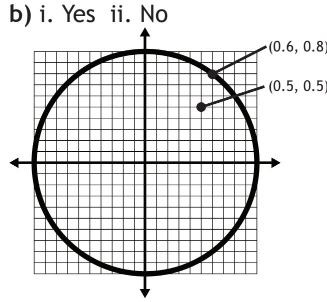
a) i.



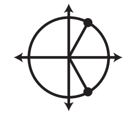
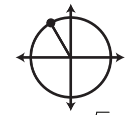
ii.



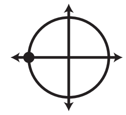
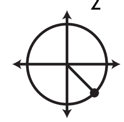
b) i. Yes ii. No



c) i. $y = \pm\frac{\sqrt{3}}{2}$ ii. $x = -\frac{1}{2}$

iii. $y = 0$ iv. $x = \frac{\sqrt{2}}{2}$

Example 2: See Video.

Example 3: **a)** $\frac{1}{2}$ **b)** -1 **c)** $-\frac{\sqrt{2}}{2}$ **d)** $-\frac{1}{2}$ **e)** 0 **f)** 0 **g)** $-\frac{\sqrt{3}}{2}$ **h)** $-\frac{1}{2}$

Example 4: **a)** $\frac{1}{2}$ **b)** 1 **c)** $\frac{1}{2}$ **d)** $-\frac{1}{2}$ **e)** -1 **f)** $-\frac{\sqrt{2}}{2}$ **g)** $\frac{1}{4}$ **h)** $\frac{1}{2}$

Example 5:

a) $\sec 0 = 1$, $\sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3}$, $\sec\frac{\pi}{4} = \sqrt{2}$, $\sec\frac{\pi}{3} = 2$, $\sec\frac{\pi}{2} = \text{undefined}$

b) $\csc 0 = \text{undefined}$, $\csc\frac{\pi}{6} = 2$, $\csc\frac{\pi}{4} = \sqrt{2}$, $\csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3}$, $\sec\frac{\pi}{2} = 1$

Example 6:

a) $\tan 0 = 0$, $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$, $\tan\frac{\pi}{4} = 1$, $\tan\frac{\pi}{3} = \sqrt{3}$, $\tan\frac{\pi}{2} = \text{undefined}$

b) $\cot 0 = \text{undefined}$, $\cot\frac{\pi}{6} = \sqrt{3}$, $\cot\frac{\pi}{4} = 1$, $\cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$, $\cot\frac{\pi}{2} = 0$

Example 7: See Video.

Example 8:

a) -2 **b)** undefined **c)** $\frac{2\sqrt{3}}{3}$ **d)** $-\sqrt{2}$ **e)** $\frac{\sqrt{3}}{3}$ **f)** -1 **g)** 0 **h)** $\sqrt{3}$

Example 9:

a) $\frac{-\sqrt{3}-\sqrt{2}}{2}$ **b)** 1 **c)** $\frac{4}{3}$ **d)** $\frac{1}{2}$

Example 10:

a) 1 **b)** $\frac{\sqrt{6}-\sqrt{2}}{4}$ **c)** $\sqrt{3}$ **d)** $-2-\sqrt{3}$

Example 11:

a) -1 **b)** $-\frac{1}{3}$ **c)** undefined **d)** undefined

Example 12: See Video.

Example 13:

a) P(π/3) means *"point coordinates at π/3"*.

b) $\frac{11\pi}{6}, -\frac{\pi}{6}$ **c)** $P\left(\frac{7\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

d) $P\left(\frac{2\pi}{3}\right) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **e)** P(3) = (-0.9900, 0.1411)

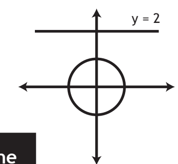
Example 14:

a) C = 2π **b)** The central angle and arc length of the unit circle are equal to each other.

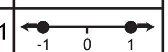
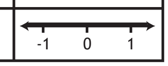
c) a = 2π/3 **d)** a = 7π/6

Example 15:

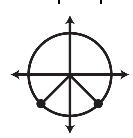
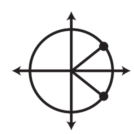
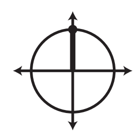
a) The unit circle and the line y = 2 do not intersect, so it's impossible for sinθ to equal 2.



b)

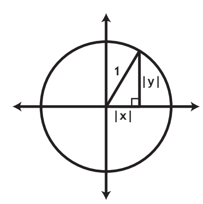
	Range	Number Line
cosθ & sinθ	$-1 \le y \le 1$	
cscθ & secθ	$y \le -1, y \ge 1$	
tanθ & cotθ	$y \in R$	

c) $\frac{5\pi}{4}, \frac{7\pi}{4}$ **d)** 53.13°, 302.70° **e)** $\frac{\pi}{2}$

Example 16:

a) Inscribe a right triangle with side lengths of |x|, |y|, and a hypotenuse of 1 into the unit circle. We use absolute values because technically, a triangle must have positive side lengths. Plug these side lengths into the Pythagorean Theorem to get $x^2 + y^2 = 1$.



b) Use basic trigonometric ratios (SOHCAHTOA) to show that x = cosθ and y = sinθ.

c) $\theta_p = 167.32°$, $\theta_r = 12.68°$

$\sin\theta = \frac{9}{41}$ $\sec\theta = -\frac{41}{40}$

$\cos\theta = -\frac{40}{41}$ $\csc\theta = \frac{41}{9}$

$\tan\theta = -\frac{9}{40}$ $\cot\theta = -\frac{40}{9}$

Example 17:

a) (167, 212) **b)** (-792, 113)

Example 18:

a) See Video **b)** 160 m

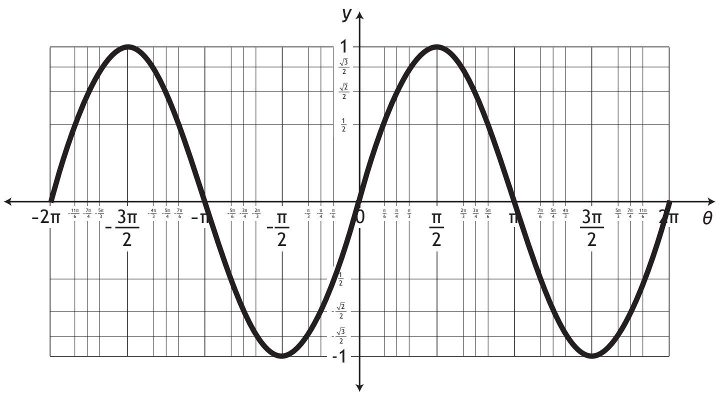
Answer Key

Trigonometry Lesson Three: Trigonometric Functions I

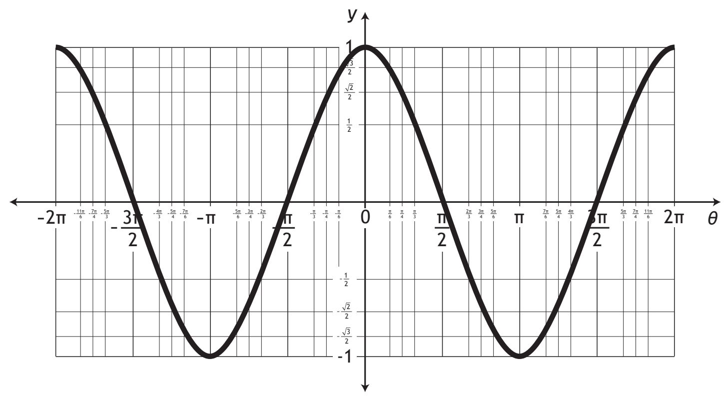
Example 1:

a) (-5π/6, 3), (-π/6, -4), (7π/6, 1) **b)** (-3π/4, -12), (π/4, 16), (7π/4, -8) **c)** (-6π, 8), (-2π, -8), (4π, -4) **d)** (-3π, 10), (3π/2, -30), (5π/2, -20)

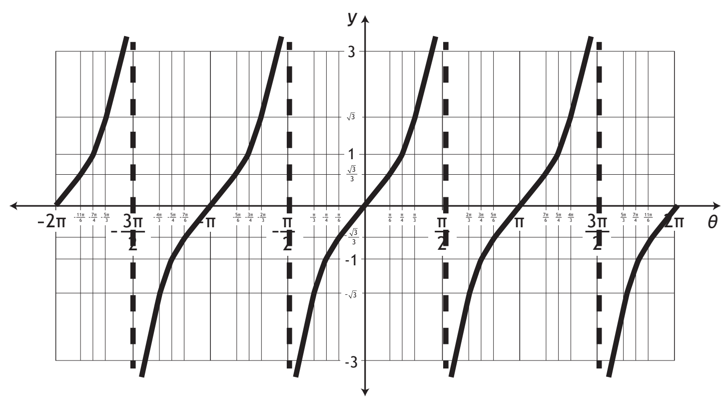
Example 2: a) y = sinθ **b)** a = 1 **c)** P = 2π
d) c = 0 **e)** d = 0 **f)** θ = nπ, nεI **g)** (0, 0)
h) Domain: θ ε R, Range: -1 ≤ y ≤ 1



Example 3: a) y = cosθ **b)** a = 1 **c)** P = 2π
d) c = 0 **e)** d = 0 **f)** θ = π/2 + nπ, nεI **g)** (0, 1)
h) Domain: θ ε R, Range: -1 ≤ y ≤ 1

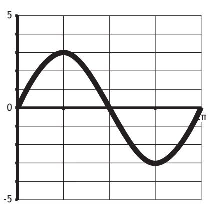


Example 4: a) y = tanθ **b)** Tangent graphs do not have an amplitude. **c)** P = π **d)** c = 0 **e)** d = 0 **f)** θ = nπ, nεI
g) (0, 0) **h)** Domain: θ ε R, θ ≠ π/2 + nπ, nεI, Range: y ε R

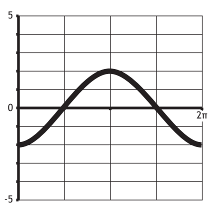


Example 5:

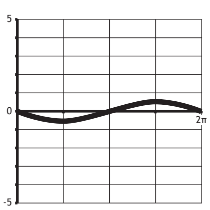
a) $y = 3\sin\theta$



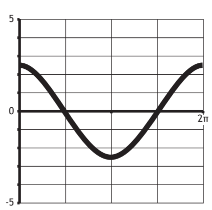
b) $y = -2\cos\theta$



c) $y = -\frac{1}{2}\sin\theta$



d) $y = \frac{5}{2}\cos\theta$



Example 6:

a) $y = 6\sin\theta$

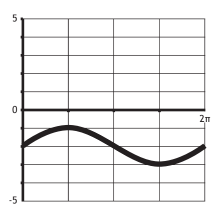
b) $y = -12\sin\theta$

c) $y = \frac{2}{5}\cos\theta$

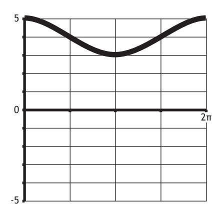
d) $y = -\frac{1}{4}\cos\theta$

Example 7:

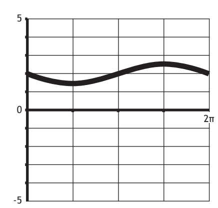
a) $y = \sin\theta - 2$



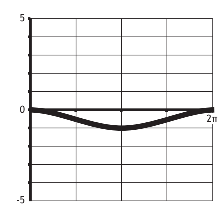
b) $y = \cos\theta + 4$



c) $y = -\frac{1}{2}\sin\theta + 2$



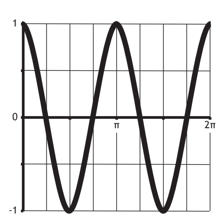
d) $y = \frac{1}{2}\cos\theta - \frac{1}{2}$



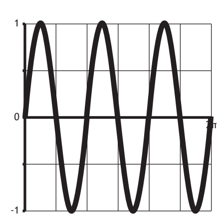
Example 8: **a)** $y = \sin\theta - 1$ **b)** $y = 20\cos\theta + 15$ **c)** $y = -8\cos\theta + 16$ **d)** $y = -\frac{1}{2}\sin\theta + \frac{5}{2}$

Example 9:

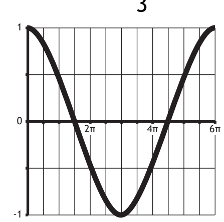
a) $y = \cos 2\theta$



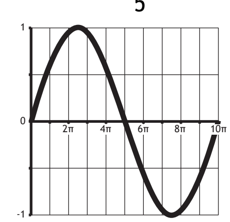
b) $y = \sin 3\theta$



c) $y = \cos\frac{1}{3}\theta$



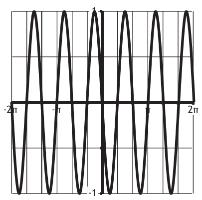
d) $y = \sin\frac{1}{5}\theta$

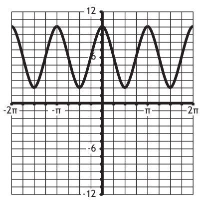


Answer Key

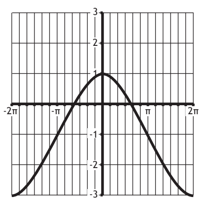
Example 10:

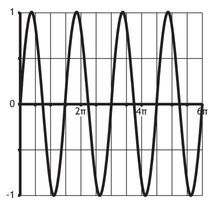
a) $y=-\sin(3\theta)$ b) $y=4\cos(2\theta)+6$





c) $y=2\cos\left(\frac{1}{2}\theta\right)-1$ d) $y=\sin\left(\frac{4}{3}\theta\right)$





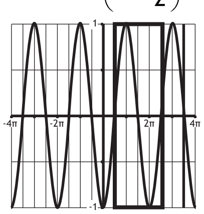
Example 11:

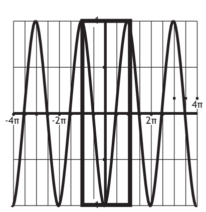
a) $y=\cos(8\theta)$ b) $y=\sin\left(\frac{2}{3}\theta\right)-1$

c) $y=4\cos\left(\frac{1}{6}\theta\right)$ d) $y=\frac{1}{2}\sin\left(\frac{1}{2}\theta\right)+\frac{1}{2}$

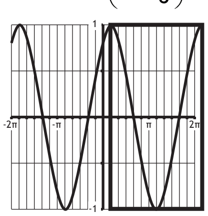
Example 12:

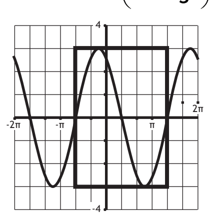
a) $y=\sin\left(\theta-\frac{\pi}{2}\right)$ b) $y=\cos(\theta+\pi)$





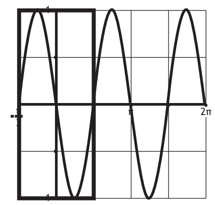
c) $y=\cos\left(\theta-\frac{\pi}{6}\right)$ d) $y=3\sin\left(\theta+\frac{2\pi}{3}\right)$

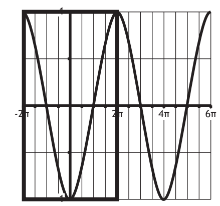




Example 13:

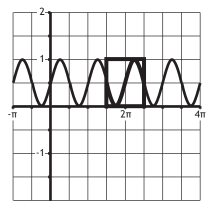
a) $y=\sin(2\theta+\pi)$ b) $y=\cos\left(\frac{1}{2}\theta+\pi\right)$

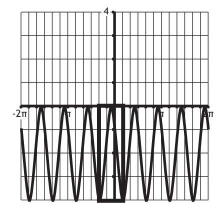




c) $y=-\frac{1}{2}\sin(2\theta-3\pi)+1$

d) $y=-2\cos\left(4\theta+\frac{4\pi}{3}\right)-2$





Example 14:

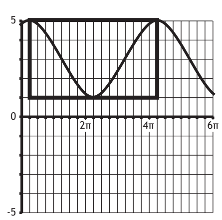
a) $y=\cos\left(\theta+\frac{\pi}{2}\right)$ b) $y=\sin\left[2\left(\theta+\frac{\pi}{4}\right)\right]$

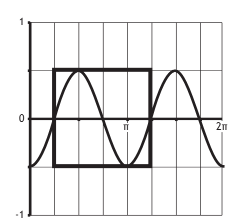
c) $y=6\sin\left(\theta-\frac{4\pi}{3}\right)$ d) $y=\frac{5}{2}\cos\left[\frac{1}{4}(\theta+\pi)\right]+\frac{1}{2}$

Example 15:

a) $y=2\cos\left(\frac{1}{2}\theta-\frac{\pi}{8}\right)+3$

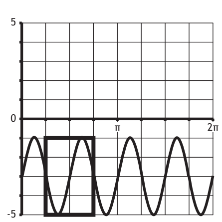
b) $y=\frac{1}{2}\sin\left(2\theta-\frac{\pi}{2}\right)$

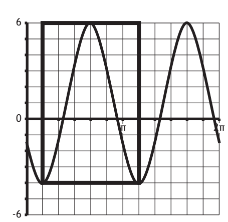




c) $y=-2\sin(4\theta-\pi)-3$

d) $y=-5\cos\left(2\theta-\frac{\pi}{3}\right)+1$



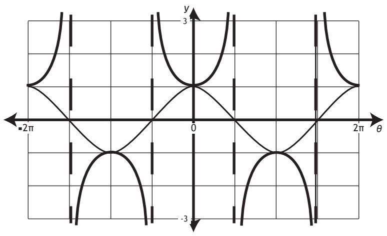


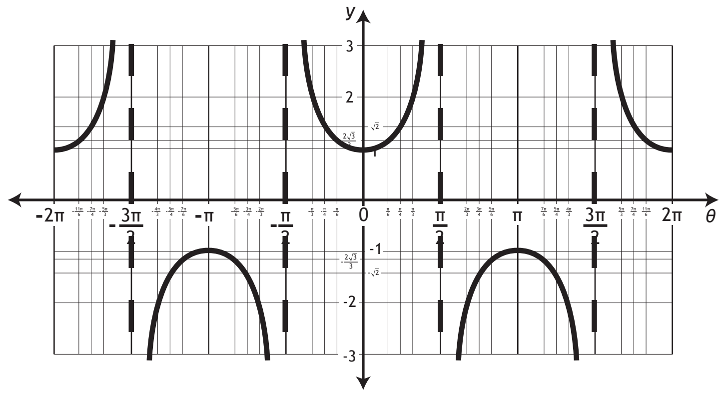
Example 16:

a) $y=\cos\left(\theta+\frac{\pi}{6}\right)+1$ b) $y=-4\cos 2\theta-8$

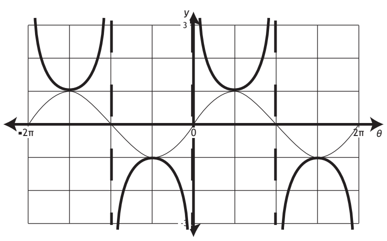
Answer Key

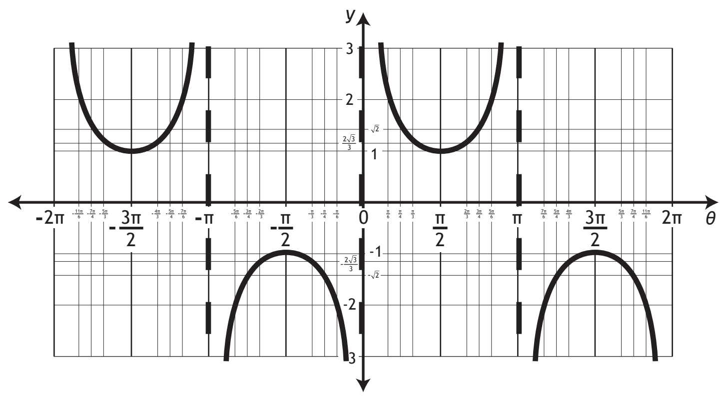
Example 17: **a)** y = secθ **b)** P = 2π
c) Domain: θ ε R, θ ≠ π/2 + nπ, nεI; Range: y ≤ -1, y ≥ 1
d) θ = π/2 + nπ, nεI



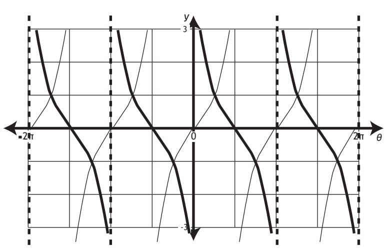


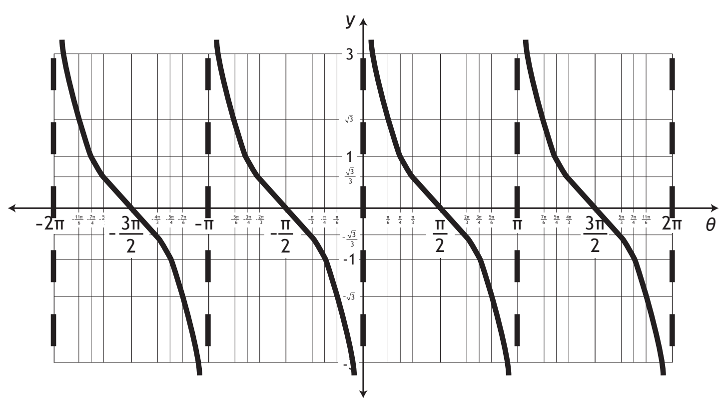
Example 18: **a)** y = cscθ **b)** P = 2π
c) Domain: θ ε R, θ ≠ nπ, nεI; Range: y ≤ -1, y ≥ 1
d) θ = nπ, nεI





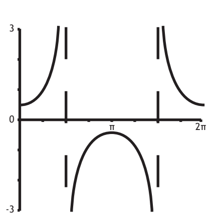
Example 19: **a)** y = cotθ **b)** P = π
c) Domain: θ ε R, θ ≠ nπ, nεI; Range: yεR
d) θ = nπ, nεI





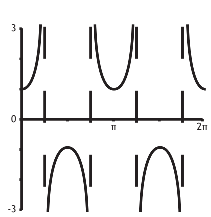
Example 20:

a) $y = \frac{1}{2}\sec\theta$



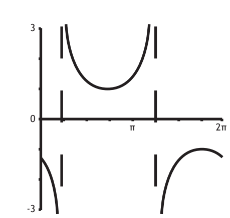
Domain: θ ε R, θ ≠ π/2 + nπ, nεI;
(or: θ ε R, θ ≠ π/2 ± nπ, nεW)
Range: y ≤ -1/2, y ≥ 1/2

b) $y = \sec 2\theta$



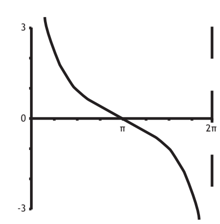
Domain: θ ε R, θ ≠ π/4 + nπ/2, nεI;
(or: θ ε R, θ ≠ π/4 ± nπ/2, nεW)
Range: y ≤ -1, y ≥ 1

c) $y = \csc\left(\theta - \frac{\pi}{4}\right)$



Domain: θ ε R, θ ≠ π/4 + nπ, nεI;
(or: θ ε R, θ ≠ π/4 ± nπ, nεW)
Range: y ≤ -1, y ≥ 1

d) $y = \cot\frac{1}{2}\theta$



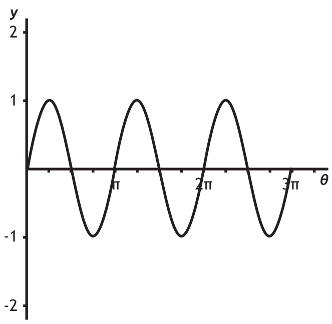
Domain: θ ε R, θ ≠ n(2π), nεI;
(or: θ ε R, θ ≠ ±n(2π), nεW)
Range: y ε R

Answer Key

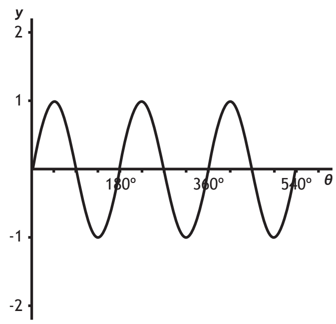
Trigonometry Lesson Four: Trigonometric Functions II

Example 1:

a) $f(\theta) = \cos\left[2\left(\theta - \frac{\pi}{4}\right)\right]$

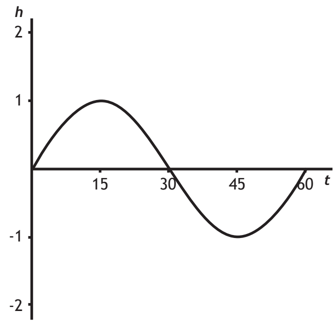


b) $f(\theta) = \cos\left[2\left(\theta - 45°\right)\right]$

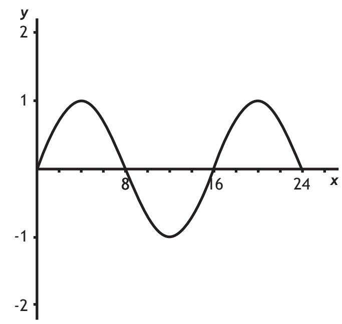


Example 2:

a) $h(t) = \cos\left[\frac{\pi}{30}(t - 15)\right]$

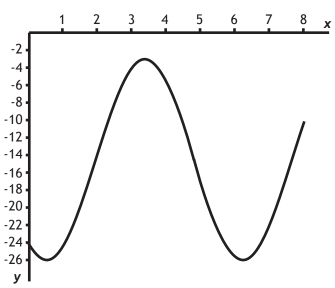


b) $f(x) = \cos\left[\frac{\pi}{8}(x - 4)\right]$

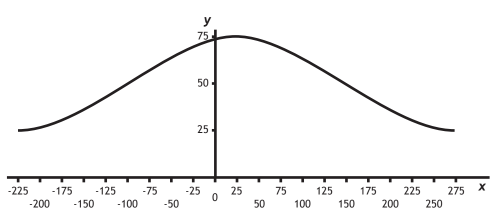


Example 3:

a) $f(x) = 12\sin\left[\frac{\pi}{3}(x - 2)\right] - 14$

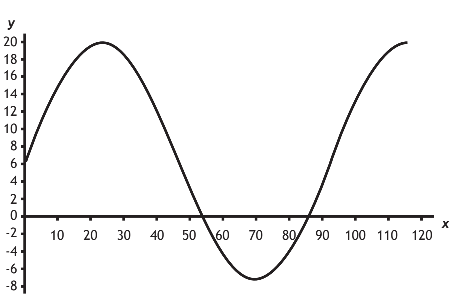


b) $f(x) = -25\cos\frac{\pi}{250}(x + 225) + 50$

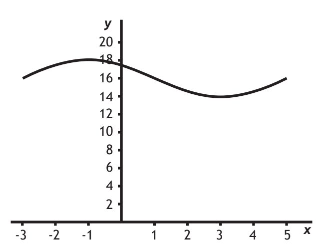


Example 4:

a) $f(x) = 13.5\cos\frac{2\pi}{96}(x - 24) + 6.5$



b) $f(x) = 2.5\sin 0.25\pi(x + 3) + 16$



Example 5:

a) $y = 9\cos\left[\frac{2\pi}{3}(x - 1)\right] + 1$

b) $y = 3\sin\left[\frac{\pi}{4}(x + 2)\right] - 2$

c) $y = 6\cos\left[\frac{\pi}{8}(x - 8)\right] + 3$

d) $y = 200\sin\left[\frac{\pi}{450}(x - 300)\right] - 50$

Example 6:

a) Range: $2 \le y \le 4$

b) Range: $-3 - k \le y \le -3 + k$

c) $d = -1$, $k = 2$

d) Range: $-m + n + 2 \le y \le m + n + 2$

e) $\left(\frac{\pi}{8}, 2\sqrt{2}\right)$, $\left(\frac{5\pi}{8}, -2\sqrt{2}\right)$

Example 7:

a) $a = 2\sqrt{2}$

b) y-intercept: $(0, -b)$

c) $f(m) + g(m) = 2n$

d) Vertical Stretch Factor: $\frac{3}{5}$

Example 8:

a) $h(t) = 2\cos(\pi t) + 6$

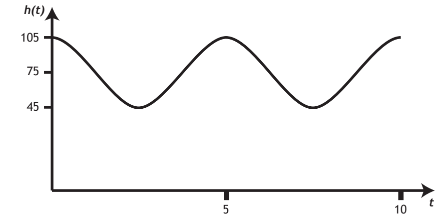
b) The b-parameter is doubled when the period is halved. The a, c, and d parameters remain the same.

c) The d-parameter decreases by 2 units, giving us d = 4. All other parameters remain unchanged.

Answer Key

Example 9:

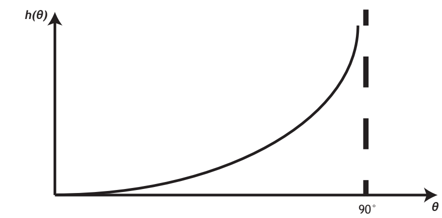
a)



b) $h\left(t\right)=30\cos\left(\frac{2\pi}{5}t\right)+75$ **c)** If the wind turbine rotates counterclockwise, we still get the same graph.

Example 10:

a)

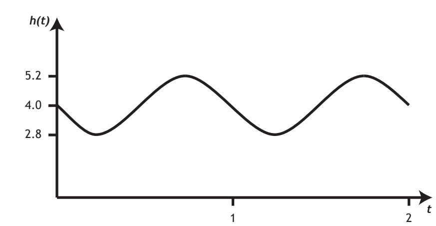


b) $h\left(\theta\right)=150\tan\theta$,
Domain: $0^{\circ}\le\theta<90^{\circ}$

c) The angle of elevation increases quickly at first, but slows down as the helicopter reaches greater heights. The angle never actually reaches 90°.

Example 11:

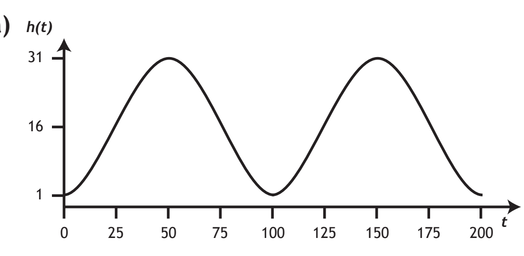
a)



b) $h\left(t\right)=-1.2\sin\left(2\pi t\right)+4$ **c)** 2.86 m **d)** 0.26 s

Example 12:

a)

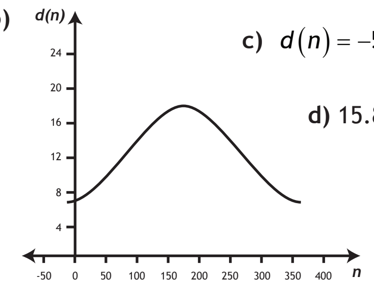


b) $h\left(t\right)=-15\cos\left(\frac{\pi}{50}t\right)+16$ **c)** 28.14 m **d)** 26.78 s

Example 13:

a) Decimal daylight hours: 6.77 h, 12.28 h, 17.82 h, 12.28 h, 6.77 h

b)



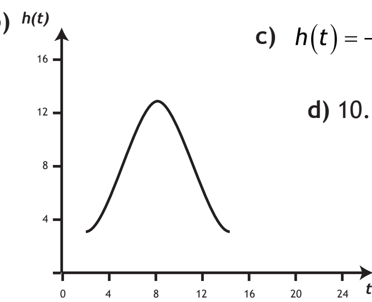
c) $d\left(n\right)=-5.525\cos\left[\frac{2\pi}{365}\left(n+11\right)\right]+12.295$

d) 15.86 h **e)** 64 days

Example 14:

a) Decimal hours past midnight: 2.20 h, 8.20 h, 14.20 h, 20.20 h

b)

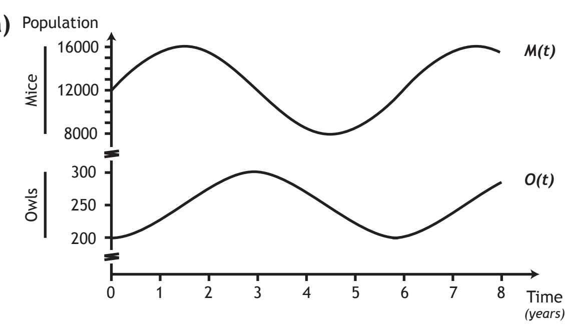


c) $h\left(t\right)=-4.92\cos\left[\frac{\pi}{6}\left(t-2.20\right)\right]+8.40$

d) 10.75 m **e)** 32.3%

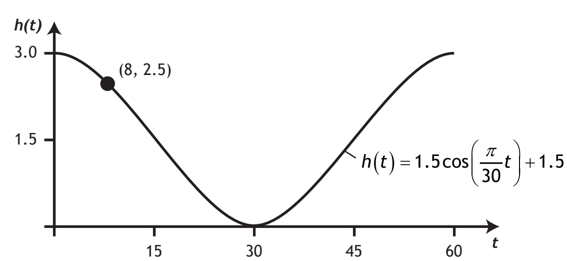
Example 15:

a)



b) See Video.

Example 16:
2.5 m



Example 17:
15.6 s and 18.3 s