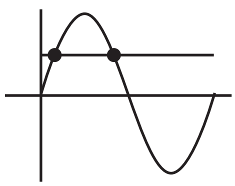
Mathematics 30-1

Student Workbook

Unit
5

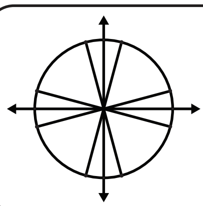
$\sin\theta = \dfrac{1}{2}$



Lesson 1: Trigonometric Equations
Approximate Completion Time: 4 Days

$$
\begin{array}{r|l}
\multicolumn{2}{c}{\cos^3 x + \cos x \sin^2 x = \cos x} \\
\cos x\left(\cos^2 x + \sin^2 x\right) & \\
\cos x(1) & \\
\cos x & \cos x \quad \checkmark
\end{array}
$$

Lesson 2: Trigonometric Identities I
Approximate Completion Time: 4 Days



$\cos 15° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

Lesson 3: Trigonometric Identities II
Approximate Completion Time: 4 Days

UNIT FIVE
Trigonometry II

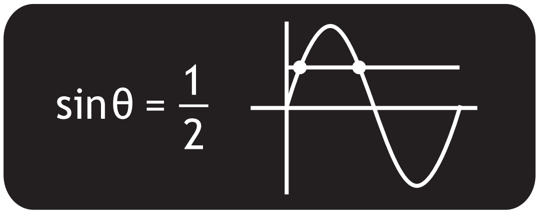
Mathematics 30-1

Student Workbook

Unit
5

Complete this workbook by watching the videos on **www.math30.ca**.
Work neatly and use proper mathematical form in your notes.

UNIT FIVE
Trigonometry II



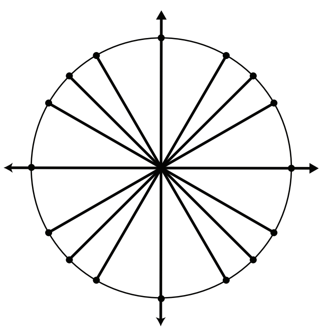
Trigonometry
LESSON FIVE - *Trigonometric Equations*
Lesson Notes

Example 1

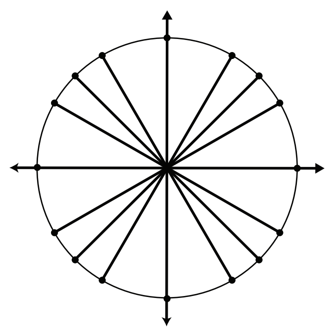
Find all angles in the domain $0 \le \theta \le 2\pi$ that satisfy the given equation.
Write the general solution.

Primary Ratios
Solving equations with the unit circle.

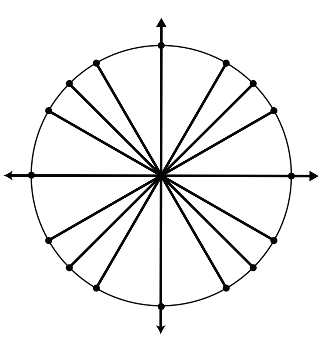
a) $\sin\theta = \frac{\sqrt{3}}{2}$



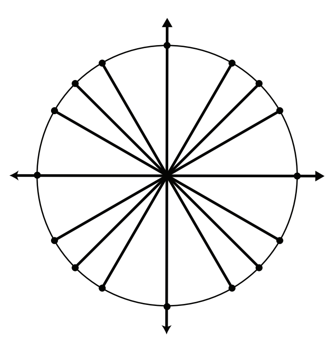
b) $\cos\theta = -\frac{1}{2}$



c) $\tan\theta = 0$



d) $\tan^2\theta = 1$

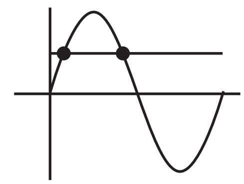


Trigonometry

LESSON FIVE - *Trigonometric Equations*

Lesson Notes

$\sin\theta = \frac{1}{2}$



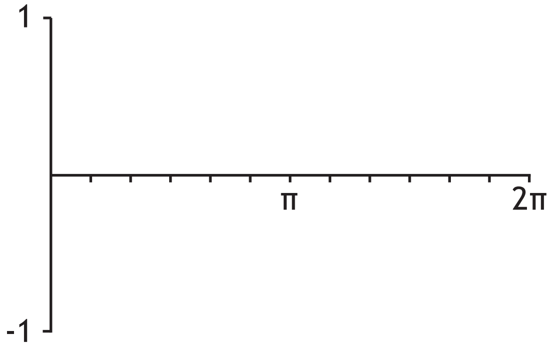
Example 2

Find all angles in the domain $0 \le \theta \le 2\pi$ that satisfy the given equation.
Write the general solution.

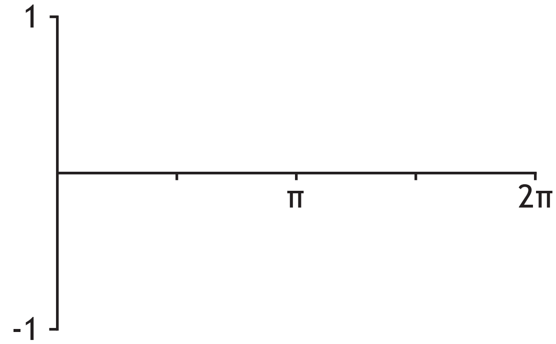
Primary Ratios

Solving equations graphically with intersection points

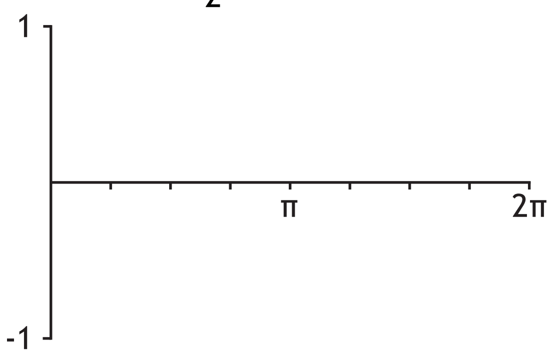
a) $\sin\theta = \frac{1}{2}$



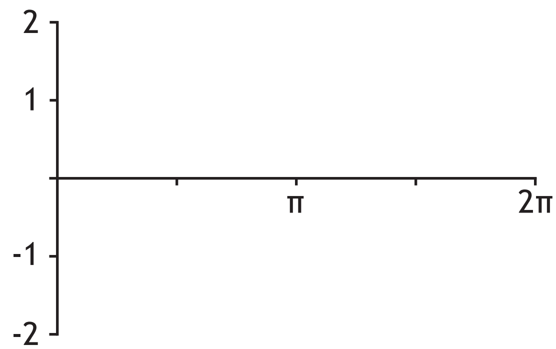
b) $\sin\theta = -1$



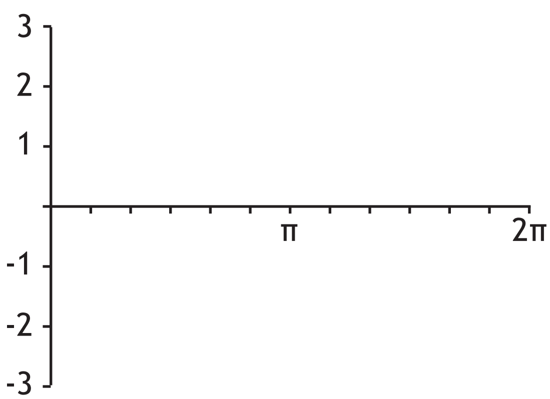
c) $\cos\theta = -\frac{\sqrt{2}}{2}$



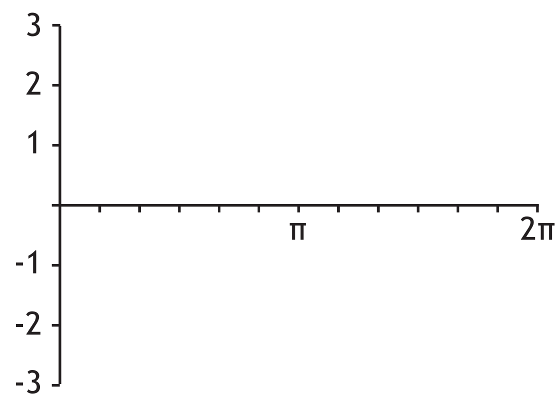
d) $\cos\theta = 2$



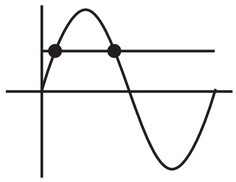
e) $\tan\theta = -\sqrt{3}$



f) $\tan\theta =$ *undefined*



$\sin\theta = \frac{1}{2}$



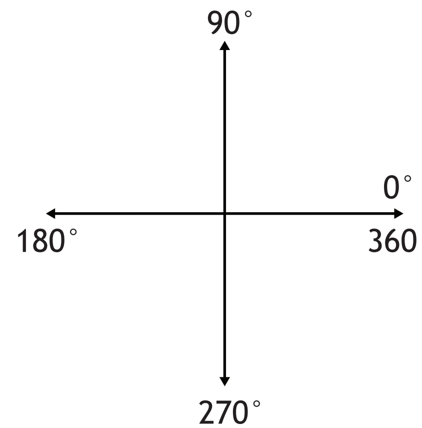
Trigonometry
LESSON FIVE - *Trigonometric Equations*
Lesson Notes

Example 3

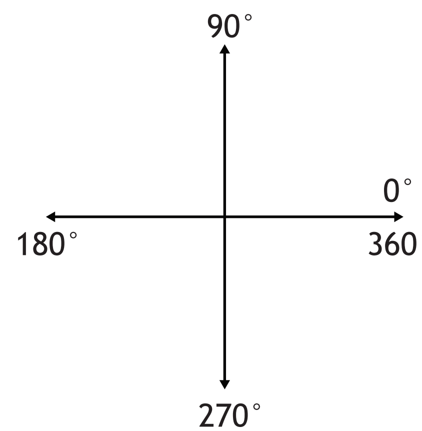
Find all angles in the domain $0^\circ \leq \theta \leq 360^\circ$ that satisfy the given equation.
Write the general solution.

Primary Ratios
Solving equations with a calculator. (degree mode)

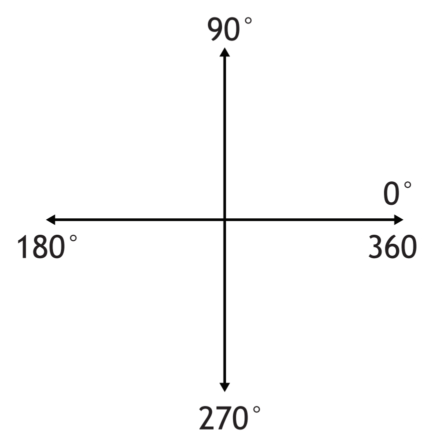
a) $\sin\theta = \frac{1}{2}$



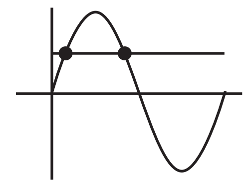
b) $\cos\theta = -\frac{\sqrt{3}}{2}$



c) $\tan\theta = 1$



$\sin\theta = \frac{1}{2}$



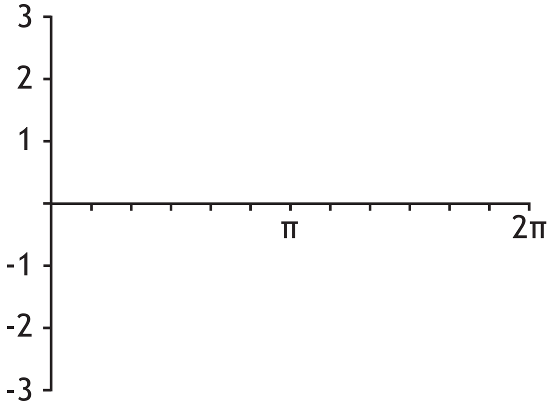
Example 4

Find all angles in the domain $0 \le \theta \le 2\pi$ that satisfy the given equation.

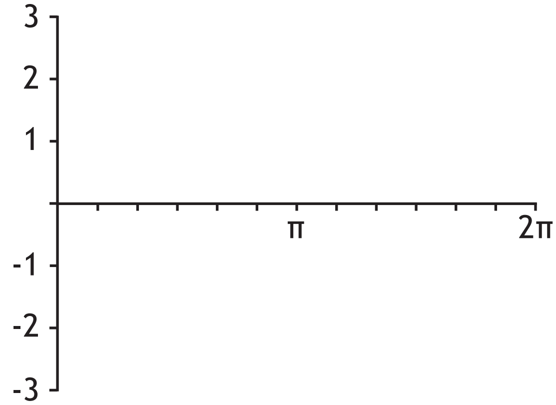
Primary Ratios
Solving equations graphically with θ-intercepts.

a) $\sin\theta = 1$

Intersection Point(s) of Original Equation

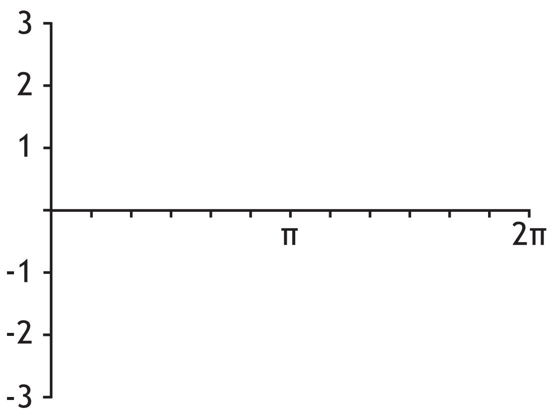


θ-Intercepts

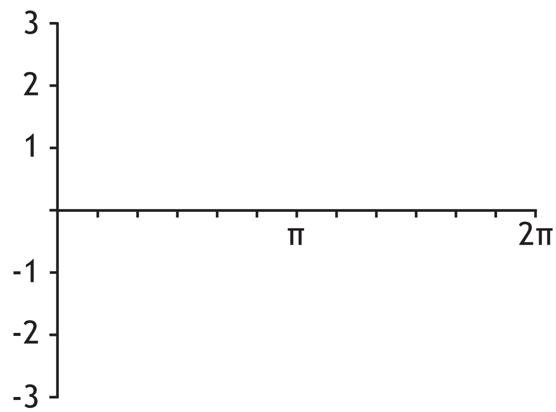


b) $\cos\theta = \frac{1}{2}$

Intersection Point(s) of Original Equation



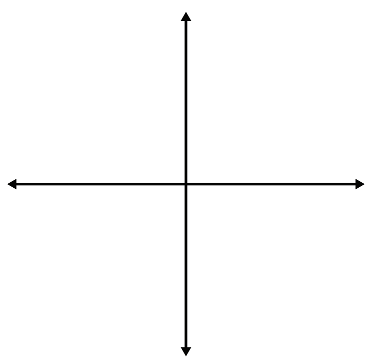
θ-Intercepts



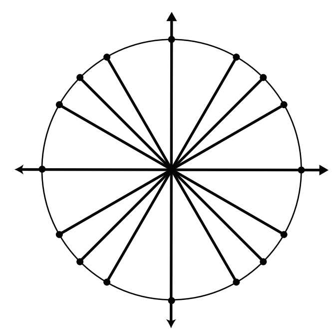
Example 5 Solve $\cos\theta = -\frac{1}{2}$ $\quad 0 \le \theta \le 2\pi$

Primary Ratios
Equations with primary trig ratios

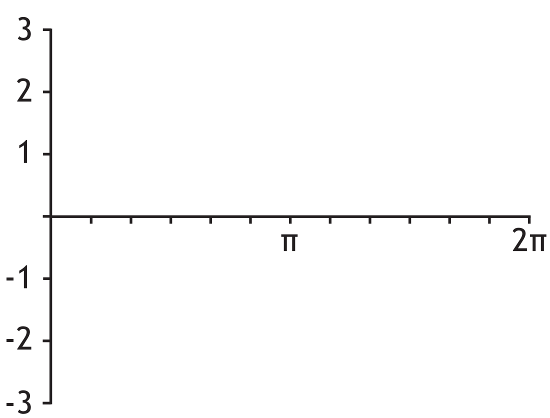
a) non-graphically, using the $\cos^{-1}$ feature of a calculator.



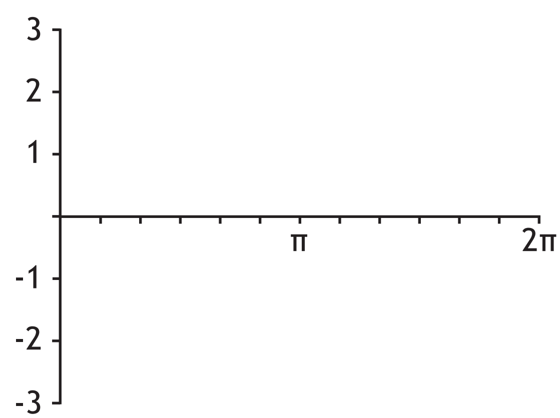
b) non-graphically, using the unit circle.



c) graphically, using the point(s) of intersection.

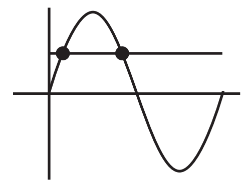


d) graphically, using θ-intercepts.



Trigonometry
LESSON FIVE - *Trigonometric Equations*
Lesson Notes

$\sin\theta = \dfrac{1}{2}$

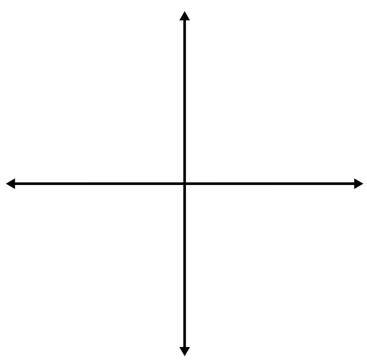


Example 6 Solve $\sin\theta = -0.30 \quad \theta \in R$

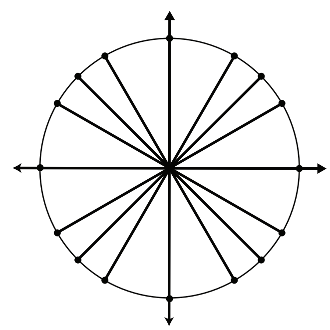
Primary Ratios

Equations with primary trig ratios

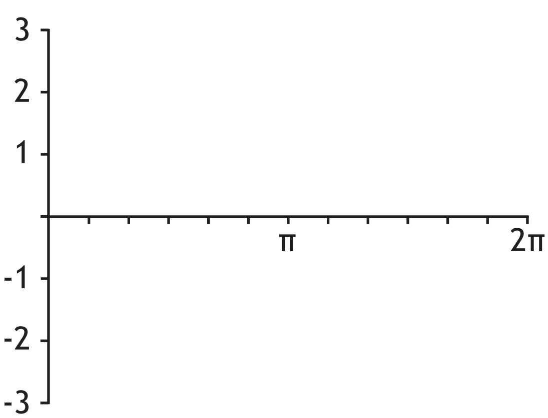
a) non-graphically, using the $\sin^{-1}$ feature of a calculator.



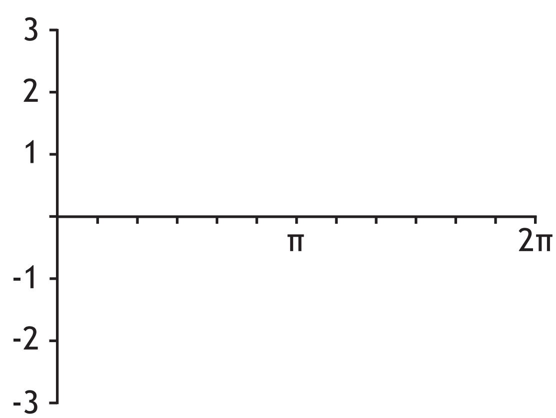
b) non-graphically, using the unit circle.



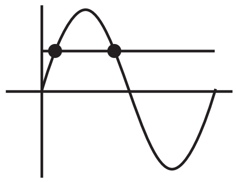
c) graphically, using the point(s) of intersection.



d) graphically, using θ-intercepts.



$\sin\theta = \frac{1}{2}$



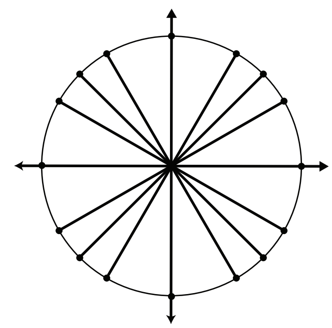
Trigonometry
LESSON FIVE - *Trigonometric Equations*
Lesson Notes

Example 7

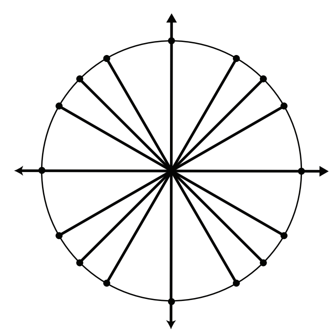
Find all angles in the domain $0 \le \theta \le 2\pi$ that satisfy the given equation.
Write the general solution.

Reciprocal Ratios
Solving equations with the unit circle.

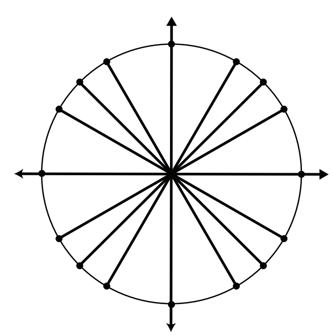
a) $\sec\theta = -2$



b) $\csc\theta$ = undefined

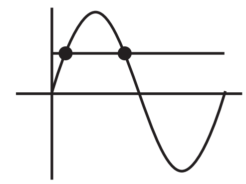


c) $\cot\theta = -1$



Trigonometry
LESSON FIVE - *Trigonometric Equations*
Lesson Notes

$\sin\theta = \frac{1}{2}$



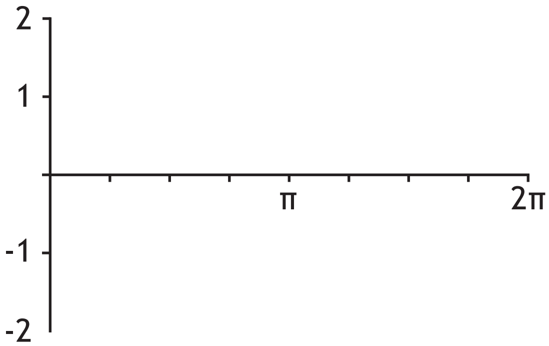
Example 8

Find all angles in the domain $0 \le \theta \le 2\pi$ that satisfy the given equation.
Write the general solution.

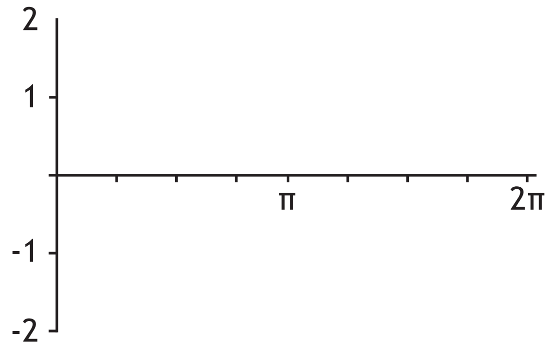
Reciprocal Ratios

Solving equations graphically with intersection points

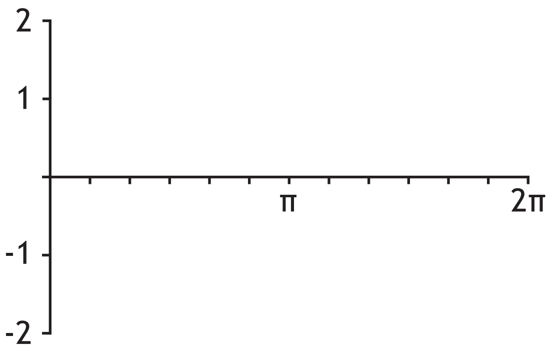
a) $\csc\theta = \frac{1}{2}$



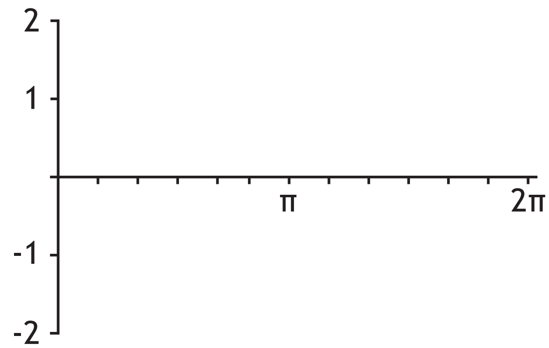
b) $\csc\theta =$ *undefined*



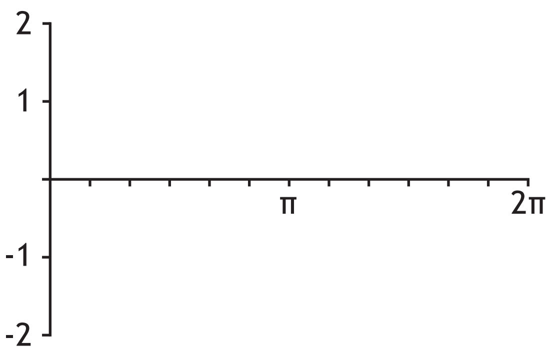
c) $\sec\theta = 2$



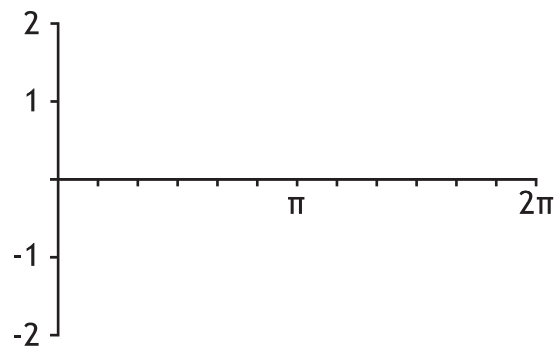
d) $\sec\theta = -1$



e) $\cot\theta = \frac{\sqrt{3}}{3}$



f) $\cot\theta = 0$

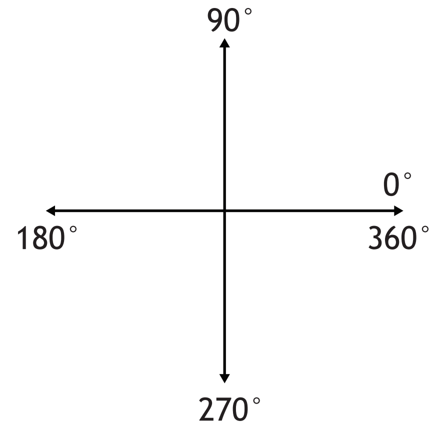


Example 9

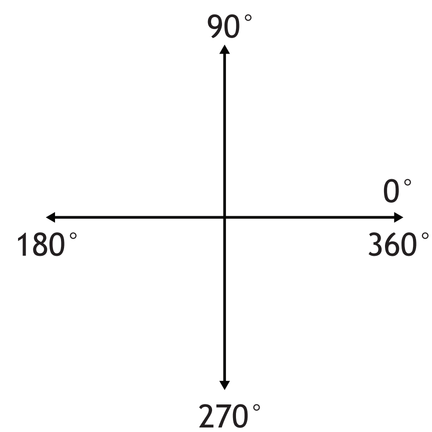
Find all angles in the domain $0^\circ \le \theta \le 360^\circ$ that satisfy the given equation.
Write the general solution

Reciprocal Ratios
Solving equations with a calculator. (degree mode)

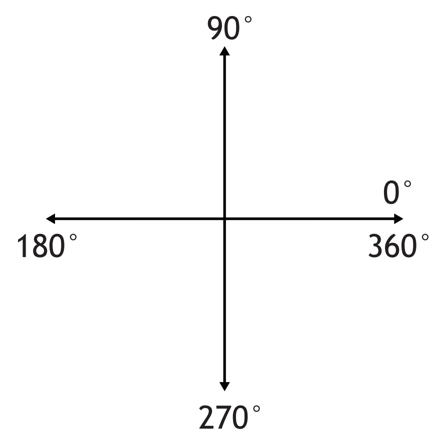
a) $\sec\theta = -2$



b) $\csc\theta = \frac{2\sqrt{3}}{3}$

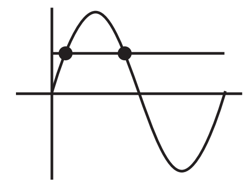


c) $\cot\theta = \frac{\sqrt{3}}{3}$



Trigonometry
LESSON FIVE - *Trigonometric Equations*
Lesson Notes

$\sin\theta = \frac{1}{2}$



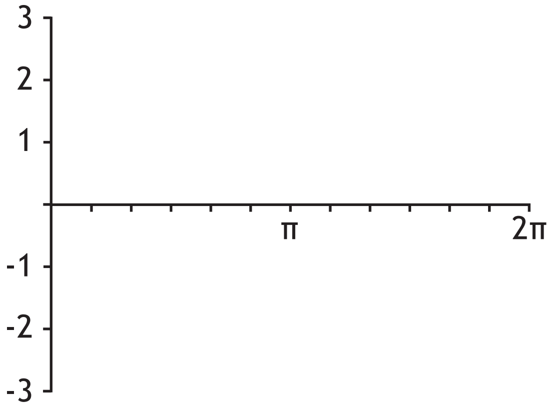
Example 10

Find all angles in the domain $0 \le \theta \le 2\pi$ that satisfy the given equation.
Write the general solution.

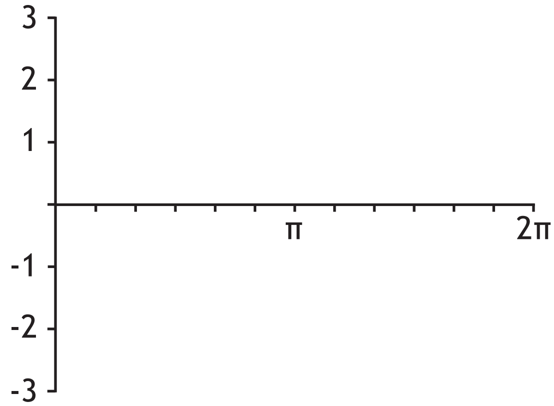
Reciprocal Ratios
Solving equations graphically with θ-intercepts.

a) $\csc\theta = -\frac{1}{2}$

Intersection Point(s) of Original Equation

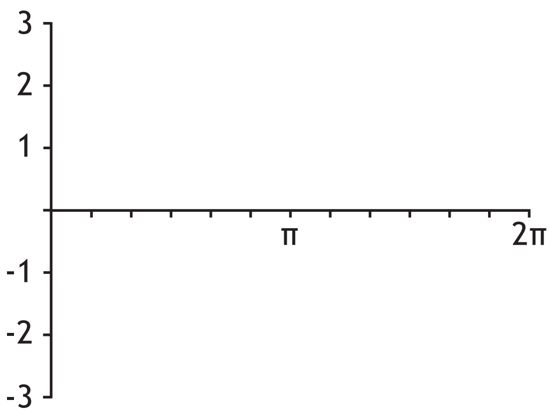


θ-Intercepts

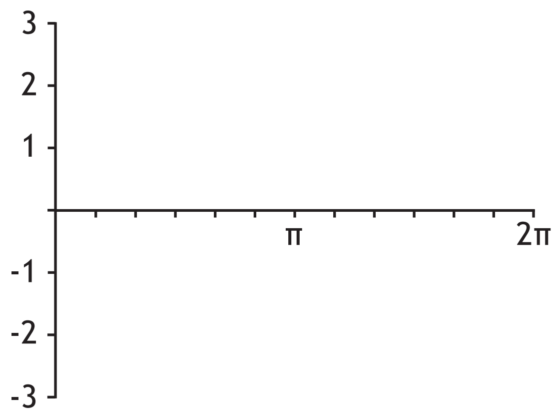


b) $\sec\theta = 1$

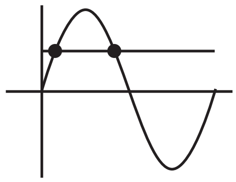
Intersection Point(s) of Original Equation



θ-Intercepts



$\sin\theta = \frac{1}{2}$



Trigonometry
LESSON FIVE - *Trigonometric Equations*
Lesson Notes

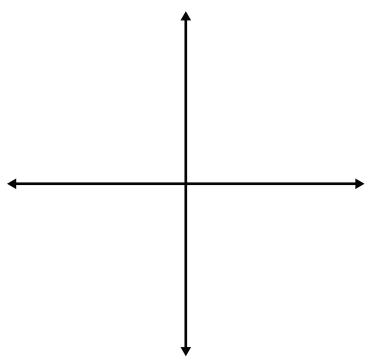
Example 11

Solve $\csc\theta = -2 \quad 0 \le \theta \le 2\pi$

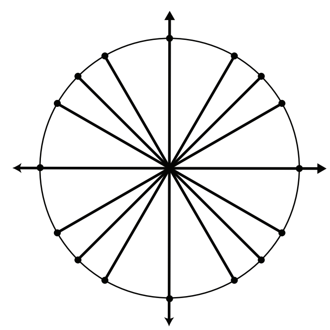
Reciprocal Ratios

Equations with reciprocal trig ratios

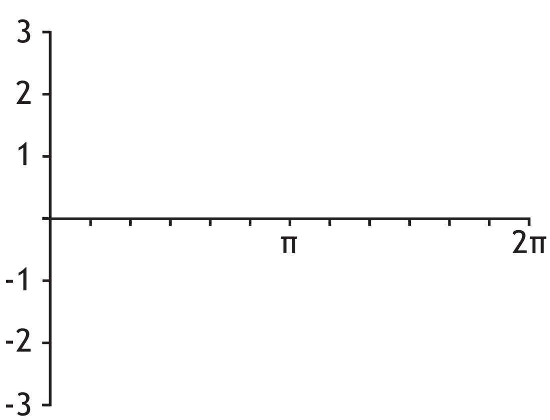
a) non-graphically, using the $\sin^{-1}$ feature of a calculator.



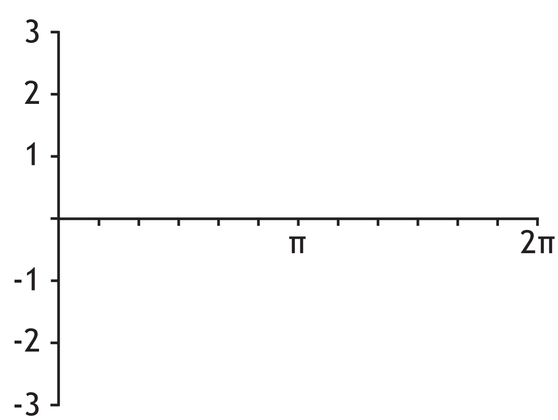
b) non-graphically, using the unit circle.



c) graphically, using the point(s) of intersection.



d) graphically, using θ-intercepts.

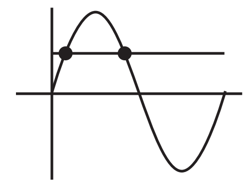


Trigonometry

LESSON FIVE - *Trigonometric Equations*

Lesson Notes

$\sin\theta = \frac{1}{2}$

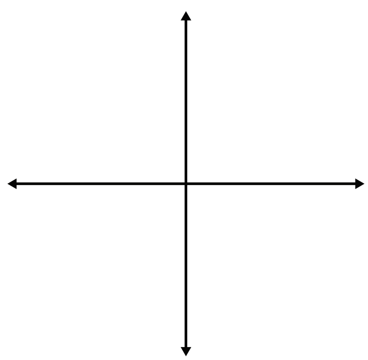


Example 12 Solve $\sec\theta = -2.3662$ $\quad 0^\circ \le \theta \le 360^\circ$

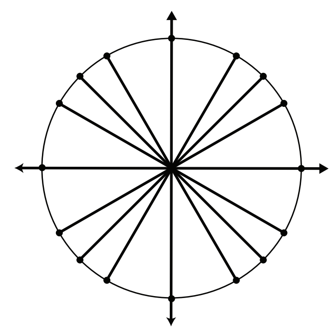
Reciprocal Ratios

Equations with reciprocal trig ratios

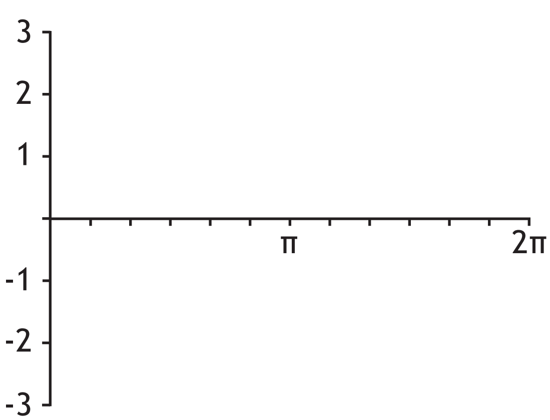
a) non-graphically, using the $\cos^{-1}$ feature of a calculator.



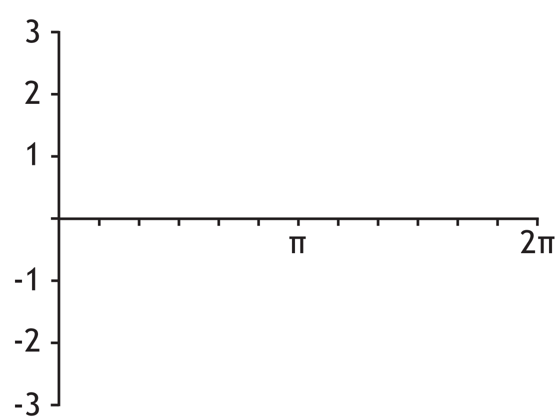
b) non-graphically, using the unit circle.



c) graphically, using the point(s) of intersection.



d) graphically, using θ-intercepts.



Example 13 Find all angles in the domain $0 \le \theta \le 2\pi$ that satisfy the given equation. Write the general solution.

First-Degree Trigonometric Equations

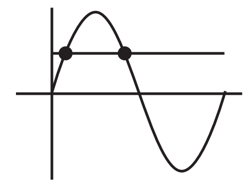
a) $\cos\theta - 1 = 0$

b) $2\sin\theta - \sqrt{3} = 0$

c) $3\tan\theta - 5 = 0$

d) $4\sec\theta + 3 = 3\sec\theta + 1$

$\sin\theta = \dfrac{1}{2}$

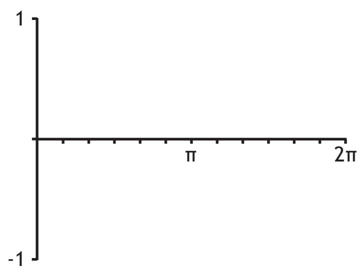


Example 14 Find all angles in the domain $0 \le \theta \le 2\pi$ that satisfy the given equation.

First-Degree Trigonometric Equations

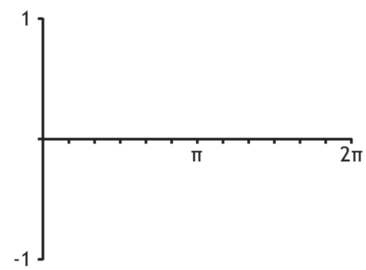
a) $2\sin\theta\cos\theta = \cos\theta$

Check the solution graphically.



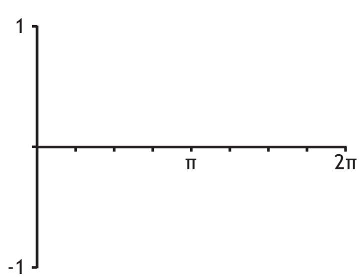
b) $7\sin\theta = 4\sin\theta$

Check the solution graphically.



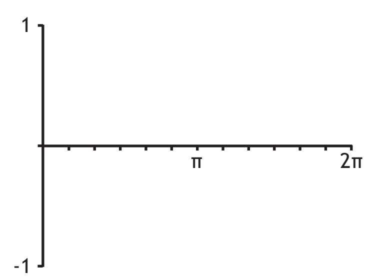
c) $\sin\theta\tan\theta = \sin\theta$

Check the solution graphically.

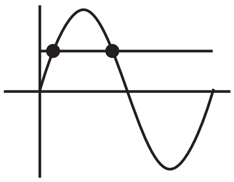


d) $\tan\theta + \cos\theta\tan\theta = 0$

Check the solution graphically.



$\sin\theta = \frac{1}{2}$



Trigonometry
LESSON FIVE - *Trigonometric Equations*
Lesson Notes

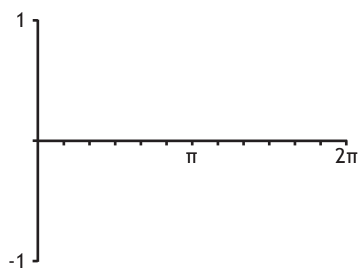
Example 15

Find all angles in the domain $0 \le \theta \le 2\pi$ that satisfy the given equation.

Second-Degree Trigonometric Equations

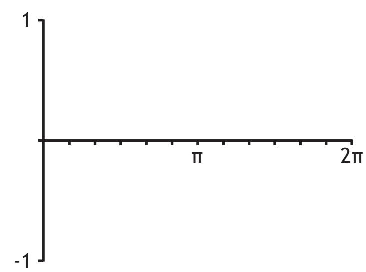
a) $\sin^2\theta = 1$

Check the solution graphically.



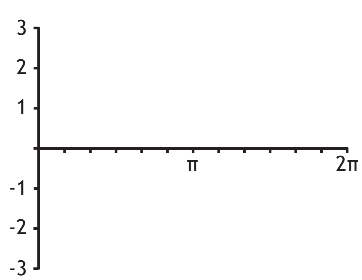
b) $4\cos^2\theta - 3 = 0$

Check the solution graphically.



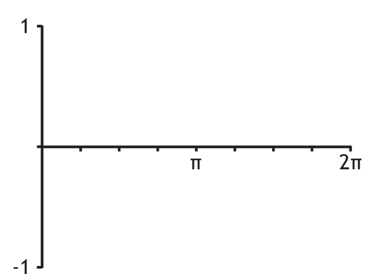
c) $2\cos^2\theta = \cos\theta$

Check the solution graphically.



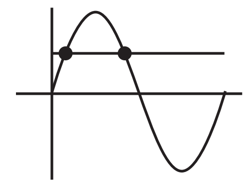
d) $\tan^4\theta - \tan^2\theta = 0$

Check the solution graphically.



Trigonometry
LESSON FIVE - *Trigonometric Equations*
Lesson Notes

$\sin\theta = \frac{1}{2}$

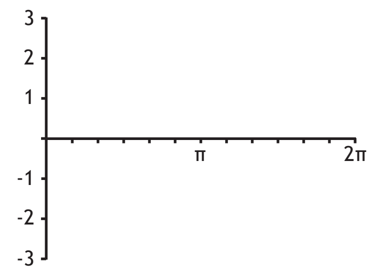


Example 16 Find all angles in the domain $0 \le \theta \le 2\pi$ that satisfy the given equation.

Second-Degree Trigonometric Equations

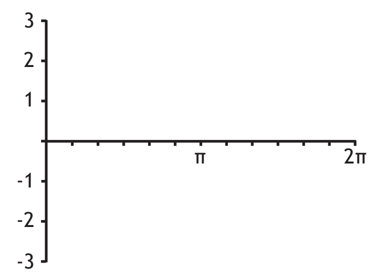
a) $2\sin^2\theta - \sin\theta - 1 = 0$

Check the solution graphically.



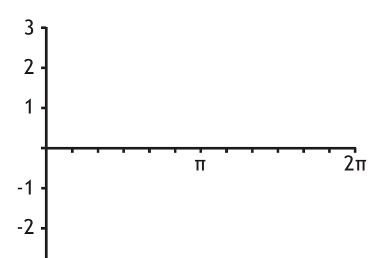
b) $\csc^2\theta - 3\csc\theta + 2 = 0$

Check the solution graphically.



c) $2\sin^3\theta - 5\sin^2\theta + 2\sin\theta = 0$

Check the solution graphically.



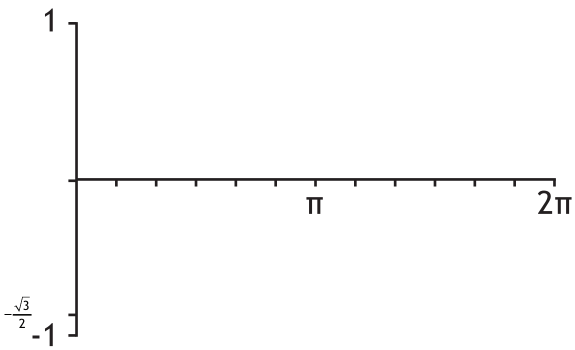
Example 17 Solve each trigonometric equation.

Double and Triple Angles

a) $\sin 2\theta = -\frac{\sqrt{3}}{2}$ $\qquad 0 \le \theta \le 2\pi$

i) graphically:

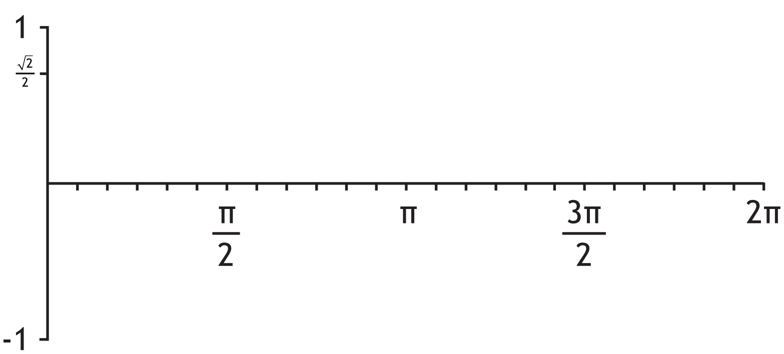
ii) non-graphically:



b) $\cos 3\theta = \frac{\sqrt{2}}{2}$ $\qquad 0 \le \theta \le 2\pi$

i) graphically:

ii) non-graphically:

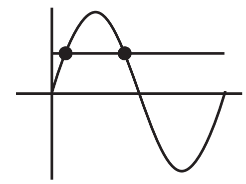


Trigonometry

LESSON FIVE - *Trigonometric Equations*

Lesson Notes

$\sin\theta = \frac{1}{2}$



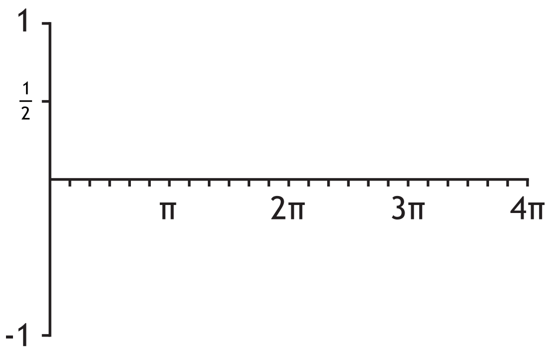
Example 18 Solve each trigonometric equation.

Half and Quarter Angles

a) $\cos\frac{1}{2}\theta = \frac{1}{2}$ $\quad 0 \le \theta \le 4\pi$

i) graphically:

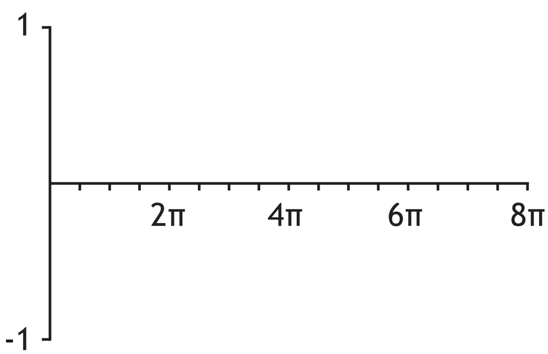
ii) non-graphically:

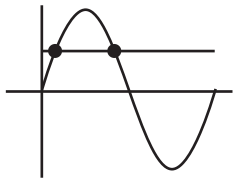


b) $\sin\frac{1}{4}\theta = -1$ $\quad 0 \le \theta \le 8\pi$

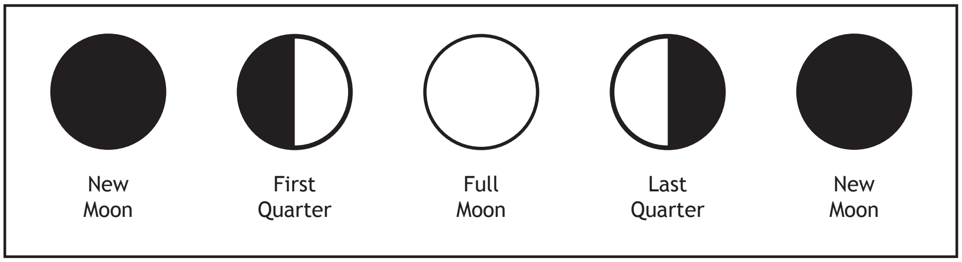
i) graphically:

ii) non-graphically:

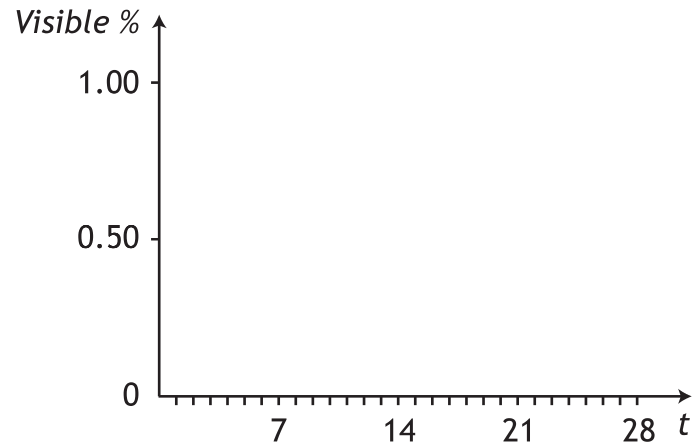




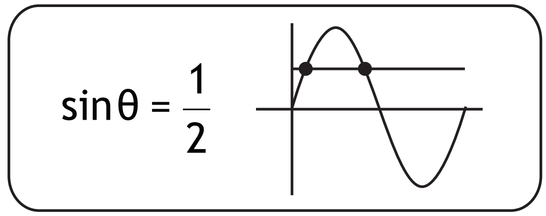
Example 19 It takes the moon approximately 28 days to go through all of its phases.



a) Write a function, P(t), that expresses the visible percentage of the moon as a function of time. Draw the graph.



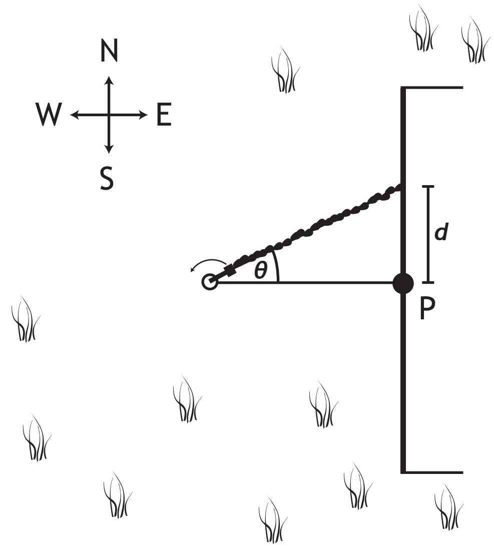
b) In one cycle, for how many days is 60% or more of the moon's surface visible?



Example 20

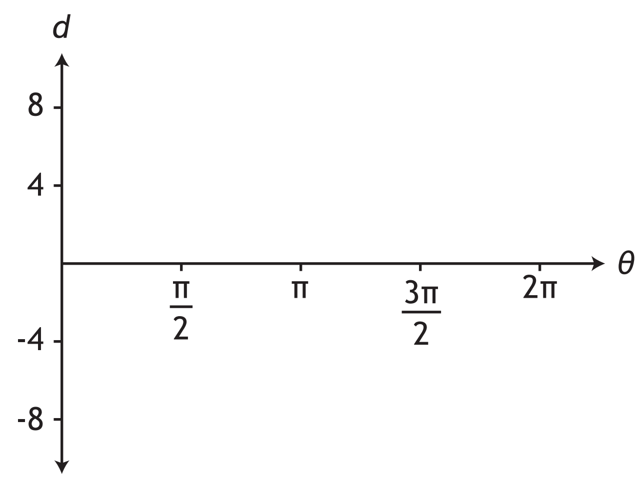
Rotating Sprinkler

A rotating sprinkler is positioned 4 m away from the wall of a house. The wall is 8 m long. As the sprinkler rotates, the stream of water splashes the house *d* meters from point P. *Note: North of point P is a positive distance, and south of point P is a negative distance.*

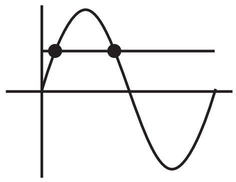


a) Write a tangent function, d(θ), that expresses the distance where the water splashes the wall as a function of the rotation angle θ.

b) Graph the function for one complete rotation of the sprinkler. Draw only the portion of the graph that actually corresponds to the wall being splashed.



c) If the water splashes the wall 2.0 m north of point P, what is the angle of rotation *(in degrees)*?



Example 21

Inverse Trigonometric Functions

Inverse Trigonometric Functions

Enrichment Example

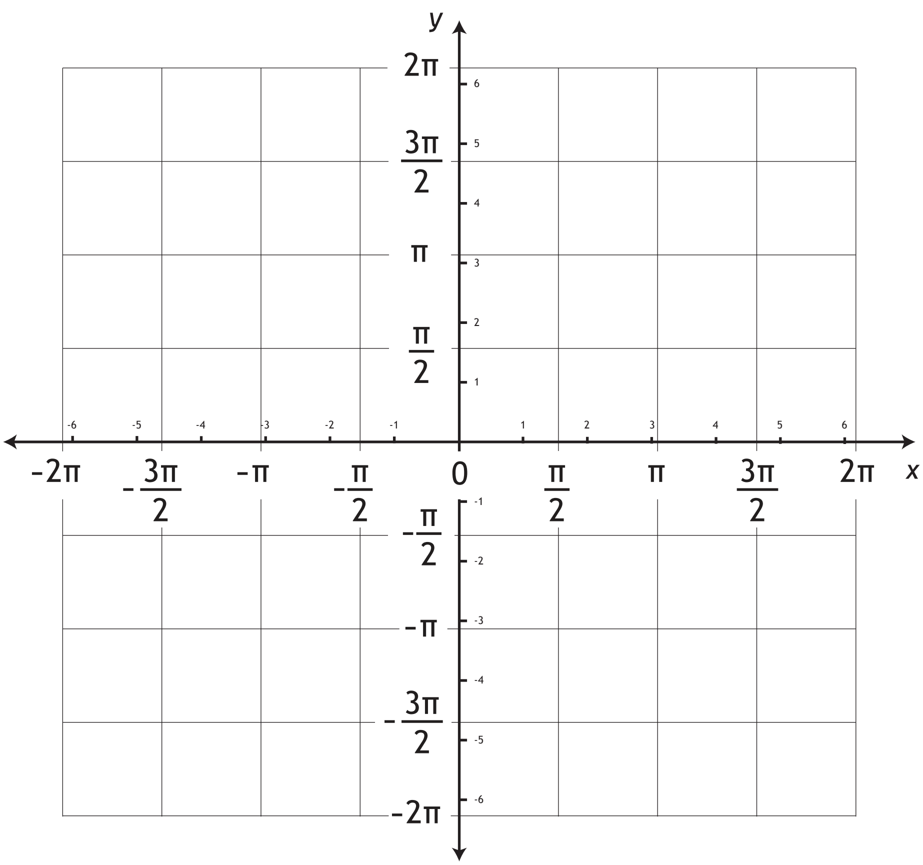
Students who plan on taking university calculus should complete this example.

When we solve a trigonometric equation like cosx = -1, one possible way to write the solution is:

$$\begin{aligned}\cos x &= -1\\ \cos^{-1}(\cos x) &= \cos^{-1}(-1)\\ x &= \pi\end{aligned}$$

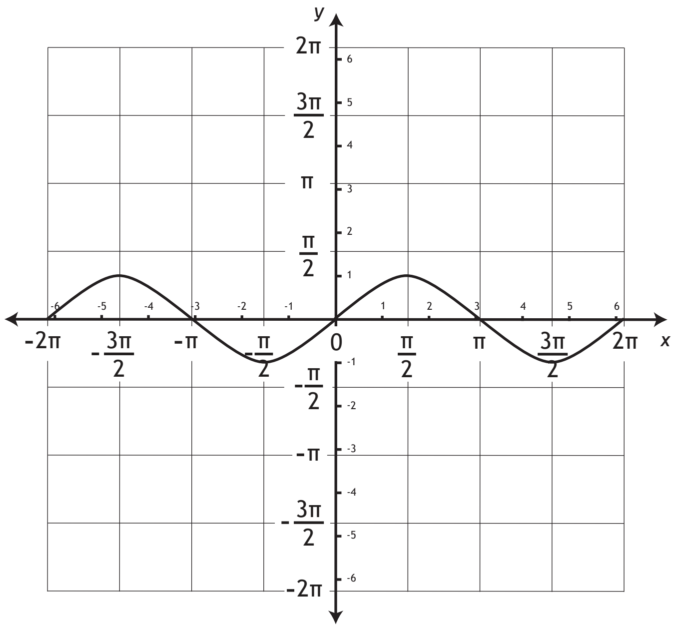
In this example, we will explore the inverse functions of sine and cosine to learn why taking an inverse actually yields the solution.

a) When we draw the inverse of trigonometric graphs, it is helpful to use a grid that is labeled with both radians and integers. Briefly explain how this is helpful.

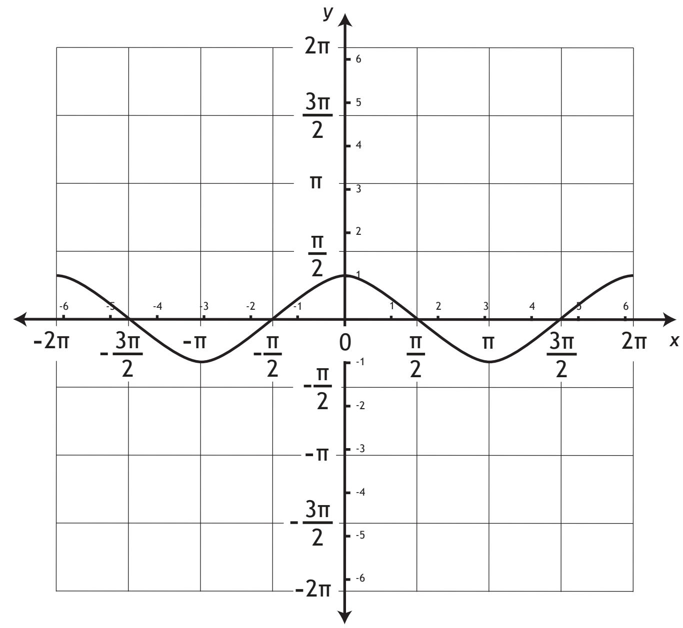


b) Draw the inverse function of each graph. State the domain and range of the original and inverse graphs *(after restricting the domain of the original so the inverse is a function)*.

y = sinx



y = cosx



c) Is there more than one way to restrict the domain of the original graph so the inverse is a function? If there is, generalize the rule in a sentence.

d) Using the inverse graphs from part (b), evaluate each of the following:

i) $\sin^{-1}(1) =$

ii) $\arccos(-1) =$

$$\begin{array}{r|l} \cos^3 x + \cos x \sin^2 x & = \cos x \\ \cos x\left(\cos^2 x + \sin^2 x\right) & \\ \cos x(1) & \\ \cos x & \cos x \ \checkmark \end{array}$$

Trigonometry
LESSON SIX - *Trigonometric Identities I*
Lesson Notes

Example 1

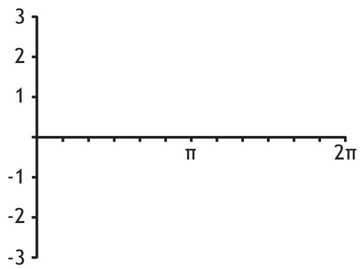
Understanding Trigonometric Identities.

Trigonometric Identities

a) Why are trigonometric identities considered to be a special type of trigonometric equation?

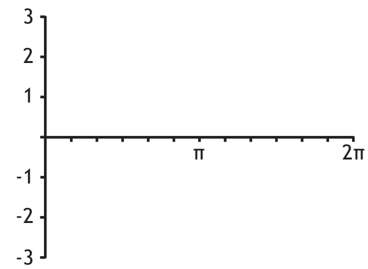
A trigonometric equation that IS an identity:

$$\boxed{\sec x = \frac{1}{\cos x}}$$



A trigonometric equation that is NOT an identity:

$$\boxed{\sec x = 1}$$

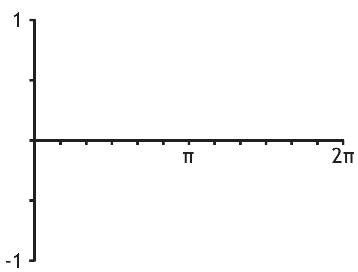


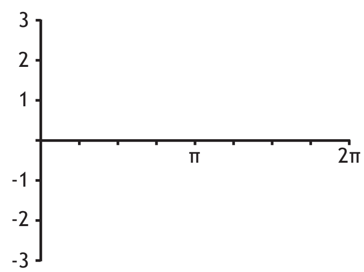
b) Which of the following trigonometric equations are also trigonometric identities?

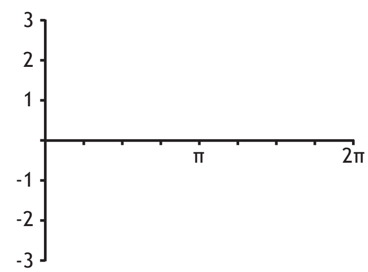
i) $\sin x = -\frac{1}{2}$

ii) $\tan x = 1$

iii) $\tan x = \frac{\sin x}{\cos x}$

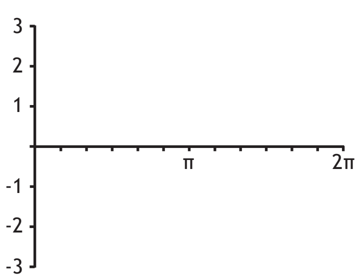


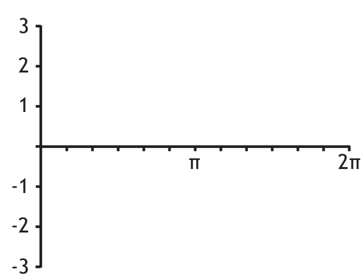




iv) $\csc x = \frac{1}{\sin x}$

v) $\sec x = \textit{undefined}$



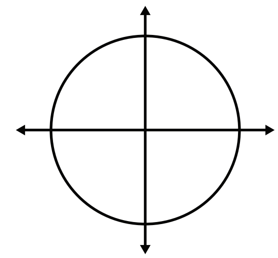


$$\begin{array}{r|l} \cos^3 x + \cos x \sin^2 x & = \cos x \\ \cos x\left(\cos^2 x + \sin^2 x\right) & \\ \cos x(1) & \\ \cos x & \cos x \quad \checkmark \end{array}$$

Example 2 The Pythagorean Identities.

Pythagorean Identities

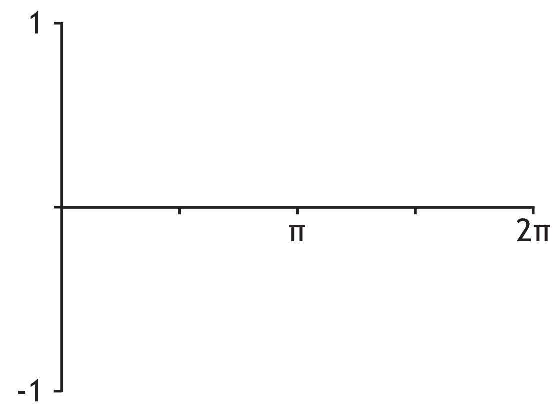
a) Using the definition of the unit circle, derive the identity $\sin^2 x + \cos^2 x = 1$.
Why is $\sin^2 x + \cos^2 x = 1$ called a Pythagorean Identity?



b) Verify that $\sin^2 x + \cos^2 x = 1$ is an identity using i) $x = \frac{\pi}{6}$ and ii) $x = \frac{\pi}{2}$.

c) Verify that $\sin^2 x + \cos^2 x = 1$ is an identity using a graphing calculator to draw the graph.

$\sin^2 x + \cos^2 x = 1$

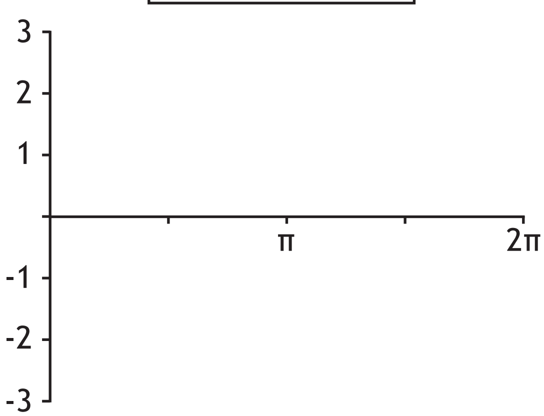


d) Using the identity $\sin^2 x + \cos^2 x = 1$, derive $1 + \cot^2 x = \csc^2 x$ and $\tan^2 x + 1 = \sec^2 x$.

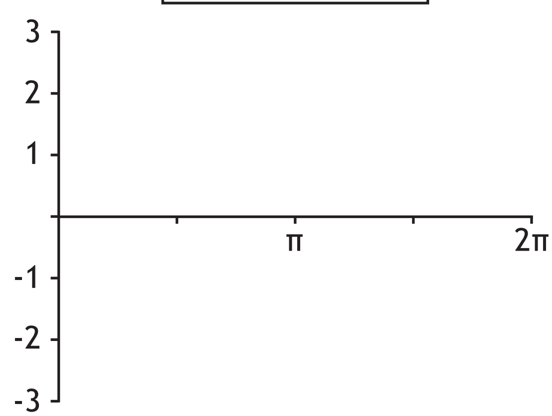
e) Verify that $1 + \cot^2 x = \csc^2 x$ and $\tan^2 x + 1 = \sec^2 x$ are identities for $x = \frac{\pi}{4}$.

f) Verify that $1 + \cot^2 x = \csc^2 x$ and $\tan^2 x + 1 = \sec^2 x$ are identities graphically.

$1 + \cot^2 x = \csc^2 x$



$\tan^2 x + 1 = \sec^2 x$



Trigonometry

LESSON SIX- *Trigonometric Identities I*

Lesson Notes

$$\begin{array}{r|l} \cos^3 x + \cos x \sin^2 x & = \cos x \\ \cos x\left(\cos^2 x + \sin^2 x\right) & \\ \cos x(1) & \\ \cos x & \cos x \quad \checkmark \end{array}$$

Example 3

Prove that each trigonometric statement is an identity. State the non-permissible values of x so the identity is true.

NOTE: You will need to use a graphing calculator to obtain the graphs in this lesson. Make sure the calculator is in RADIAN mode, and use window settings that match the grid provided in each example.

Reciprocal Identities

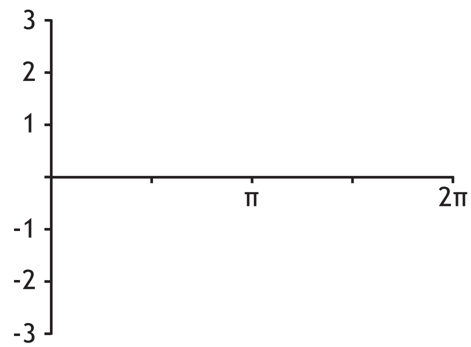
$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

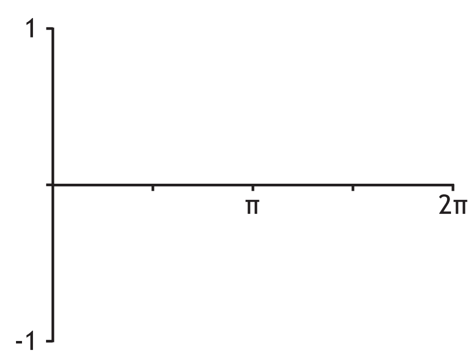
a) $\sin x \sec x = \tan x$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



b) $\cot x \sin x \sec x = 1$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



Example 4

Prove that each trigonometric statement is an identity. State the non-permissible values of x so the identity is true.

Reciprocal Identities

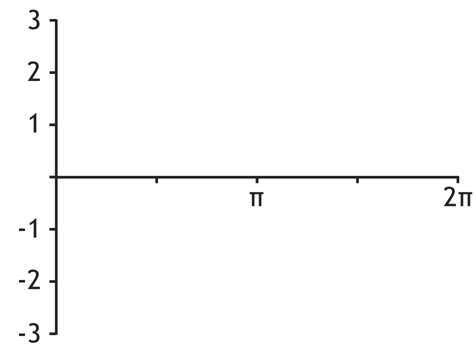
$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

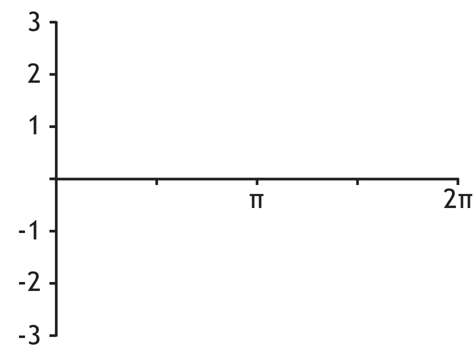
a) $\dfrac{\sin x \sec x}{\cot x} = \tan^2 x$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



b) $\sin 2x \sec 2x = \tan 2x$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



Trigonometry

LESSON SIX- *Trigonometric Identities I*

Lesson Notes

$$\begin{array}{r|l} \cos^3 x + \cos x \sin^2 x & = \cos x \\ \cos x\left(\cos^2 x + \sin^2 x\right) & \\ \cos x(1) & \\ \cos x & \cos x \quad \checkmark \end{array}$$

Example 5

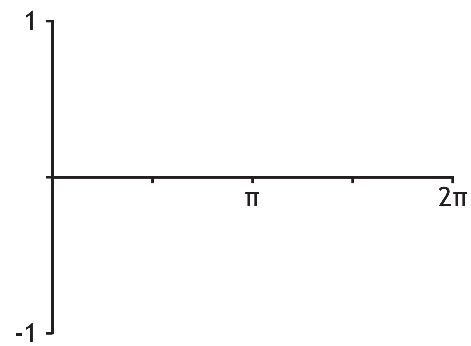
Prove that each trigonometric statement is an identity. State the non-permissible values of x so the identity is true.

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

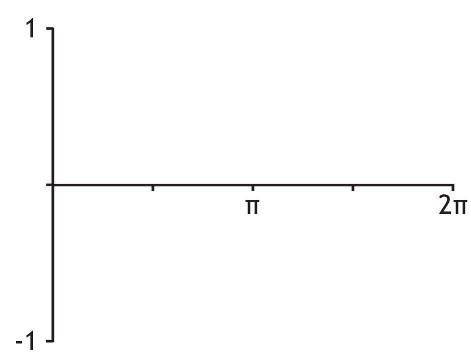
a) $\sin^2 x + \dfrac{1}{\sec^2 x} = 1$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



b) $\cos x - \cos^3 x = \cos x \sin^2 x$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



$$\begin{array}{r|l} \cos^3 x + \cos x \sin^2 x & = \cos x \\ \cos x\left(\cos^2 x + \sin^2 x\right) & \\ \cos x(1) & \\ \cos x & \cos x \quad \checkmark \end{array}$$

c) $\sin^3 x - \sin x = -\sin x \cos^2 x$

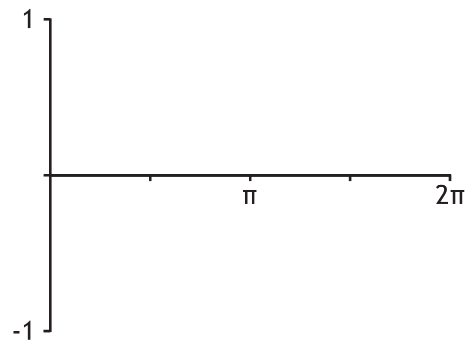
Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

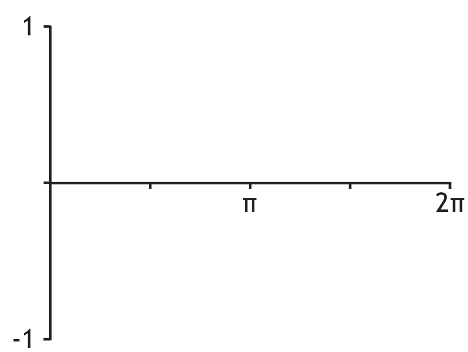
$1 + \cot^2 x = \csc^2 x$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



d) $\sin^2 x + \sin^2 x \cos^2 x = \sin^2 x\left(1 + \cos^2 x\right)$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



Trigonometry

LESSON SIX- *Trigonometric Identities I*

Lesson Notes

$$\cos^3 x + \cos x \sin^2 x = \cos x$$
$$\cos x\left(\cos^2 x + \sin^2 x\right)$$
$$\cos x(1)$$
$$\cos x \quad \Big| \quad \cos x \quad \checkmark$$

Example 6

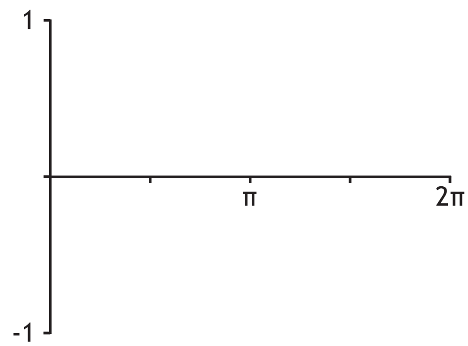
Prove that each trigonometric statement is an identity. State the non-permissible values of x so the identity is true.

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

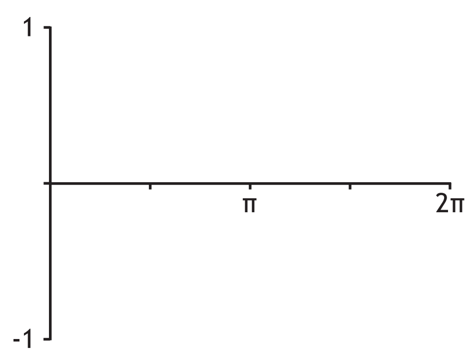
a) $\cos^2 x + \tan^2 x \cos^2 x = 1$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



b) $\dfrac{\sec^2 x - 1}{1 + \tan^2 x} = \sin^2 x$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



$$\cos^3 x + \cos x \sin^2 x = \cos x$$

$$\begin{array}{r|l} \cos x\left(\cos^2 x + \sin^2 x\right) & \\ \cos x(1) & \\ \cos x & \cos x \quad \checkmark \end{array}$$

c) $\dfrac{\sin^2 x}{1 - \cos x} = 1 + \cos x$

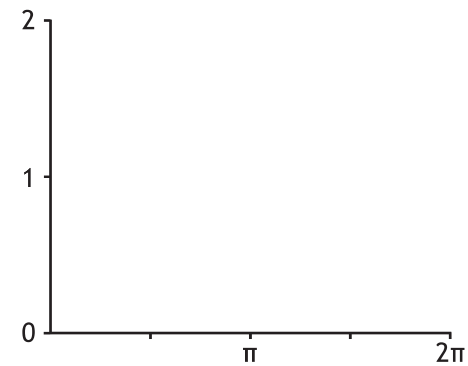
Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

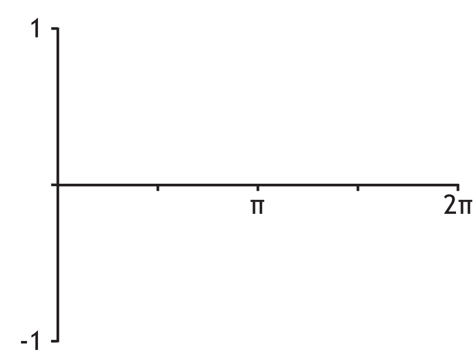
$1 + \cot^2 x = \csc^2 x$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



d) $\left(\dfrac{\sec^2 x}{\csc^2 x}\right)\left(\csc^2 x - 1\right) = 1$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



$$
\begin{array}{r|l}
\cos^3 x + \cos x \sin^2 x & = \cos x \\
\cos x\left(\cos^2 x + \sin^2 x\right) & \\
\cos x\left(1\right) & \\
\cos x & \cos x \quad \checkmark
\end{array}
$$

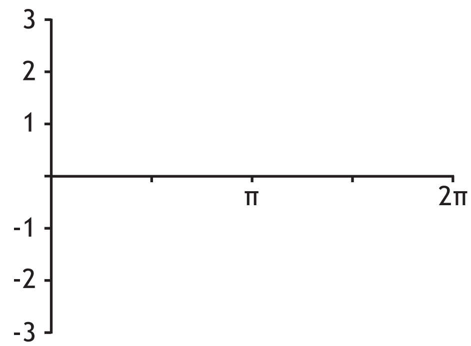
Example 7

Prove that each trigonometric statement is an identity. State the non-permissible values of x so the identity is true.

Common Denominator Proofs

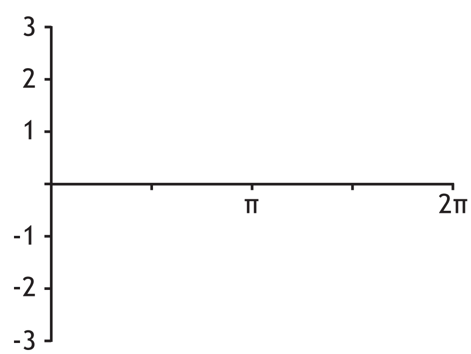
a) $1 + \sec x = \dfrac{\cos x + 1}{\cos x}$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



b) $\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)

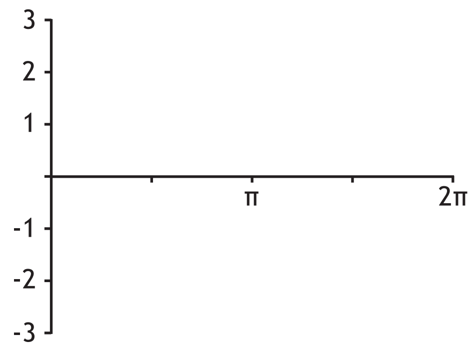


$$\begin{array}{r|l} \cos^3 x + \cos x \sin^2 x & = \cos x \\ \cos x\left(\cos^2 x + \sin^2 x\right) & \\ \cos x(1) & \\ \cos x & \cos x \quad \checkmark \end{array}$$

c) $\cot x + \tan x = \csc x \sec x$

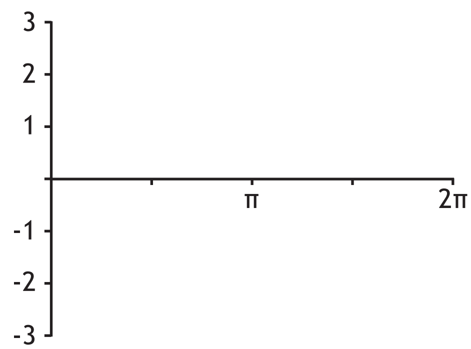
Common Denominator Proofs

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



d) $\dfrac{1+\tan x}{1+\cot x} = \tan x$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



Trigonometry

LESSON SIX- *Trigonometric Identities I*

Lesson Notes

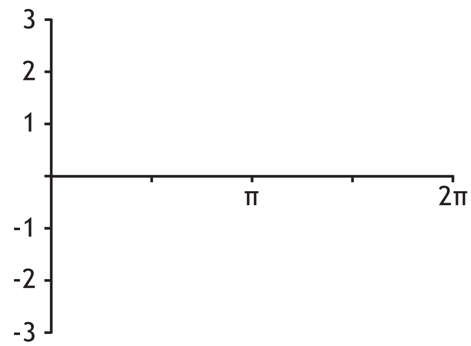
$\cos^3 x + \cos x \sin^2 x$	$= \cos x$
$\cos x\left(\cos^2 x + \sin^2 x\right)$	
$\cos x(1)$	
$\cos x$	$\cos x$ ✓

Example 8 Prove that each trigonometric statement is an identity. State the non-permissible values of x so the identity is true.

Common Denominator Proofs

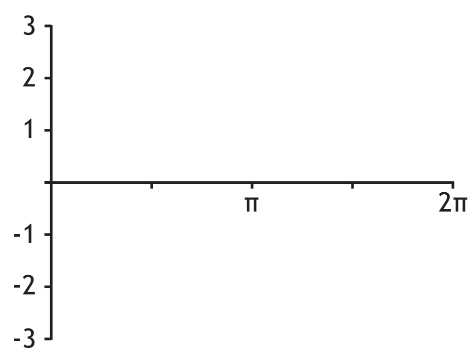
a) $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{1 + \sin x} = \sec x$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



b) $\dfrac{1 + \tan^2 x}{1 + \cot^2 x} = \tan^2 x$

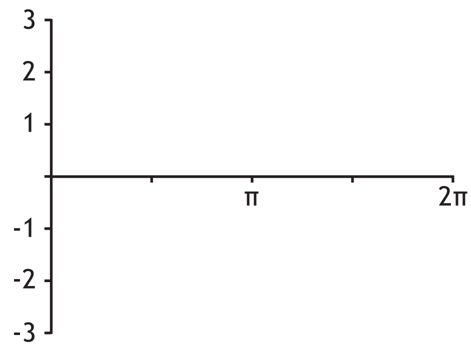
Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



c) $\dfrac{\cos x}{1+\sin x} + \dfrac{\cos x}{1-\sin x} = 2\sec x$

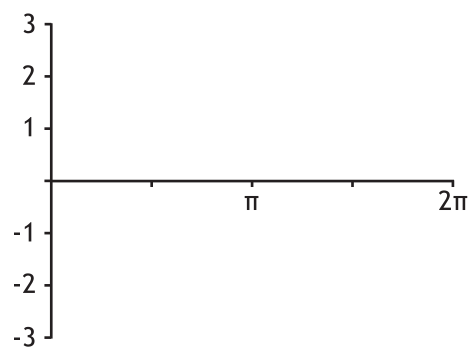
Common Denominator Proofs

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



d) $\dfrac{\cos x}{1-\sin x} = \dfrac{1+\sin x}{\cos x}$

Rewrite the identity so it is **absolutely** true.
(i.e. Include restrictions on the variable)



Trigonometry
LESSON SIX- *Trigonometric Identities I*
Lesson Notes

$$\begin{array}{r|l} \cos^3 x + \cos x \sin^2 x = \cos x & \\ \cos x\left(\cos^2 x + \sin^2 x\right) & \\ \cos x(1) & \\ \cos x & \cos x \quad \checkmark \end{array}$$

Example 9

Prove each identity.
For simplicity, ignore NPV's and graphs.

Assorted Proofs

a) $-\dfrac{4\cot x}{1-\csc^2 x} = 4\tan x$

b) $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

c) $\cot^2 x - \csc^2 x = -1$

d) $\csc x - \sin x = \cos x \cot x$

Example 10 Prove each identity.
For simplicity, ignore NPV's and graphs.

Assorted Proofs

a) $\dfrac{1}{\csc x \sin x \tan x} = \cot x$

b) $\dfrac{\csc^2 x \cos x}{\tan x} = \csc^3 x - \csc x$

c) $\dfrac{1}{5}\sin^2 x + \dfrac{1}{5}\cos^2 x = \dfrac{1}{5}$

d) $\dfrac{\sec x - \cos x}{\sin x} = \tan x$

$$\cos^3 x + \cos x \sin^2 x = \cos x$$
$$\begin{array}{r|l} \cos x\left(\cos^2 x + \sin^2 x\right) & \\ \cos x(1) & \\ \cos x & \cos x \quad \checkmark \end{array}$$

Example 11 Prove each identity.
For simplicity, ignore NPV's and graphs.

Assorted Proofs

a) $\dfrac{\sin x}{1-\cos x} = \dfrac{1+\cos x}{\sin x}$

b) $\dfrac{1-\cos x}{\sin x} - \dfrac{\sin x}{1+\cos x} = 0$

c) $(\tan x - 1)^2 = \dfrac{1-2\sin x \cos x}{\cos^2 x}$

d) $\dfrac{1+\cos x}{1-\cos x} = \left(\dfrac{1+\cos x}{\sin x}\right)^2$

$$\cos^3 x + \cos x \sin^2 x = \cos x$$
$$\begin{array}{r|l} \cos x\left(\cos^2 x + \sin^2 x\right) & \\ \cos x(1) & \\ \cos x & \cos x \ \checkmark \end{array}$$

Example 12 Exploring the proof of $\sin x = \tan x \cos x$

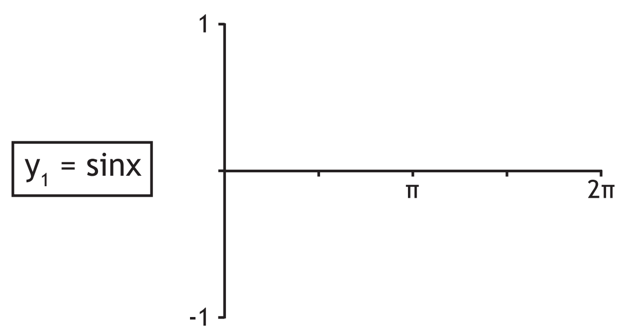
Exploring a Proof

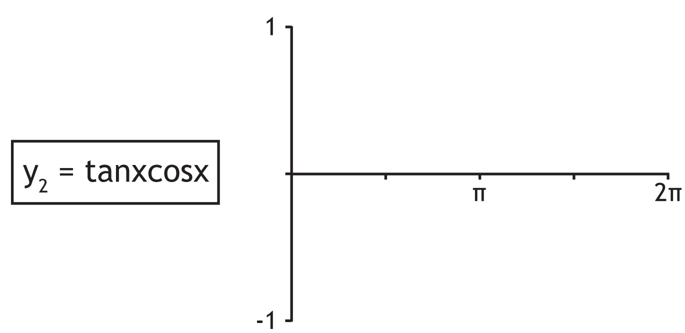
a) Prove algebraically that $\sin x = \tan x \cos x$.

b) Verify that $\sin x = \tan x \cos x$ for $\frac{\pi}{3}$.

c) State the non-permissible values for $\sin x = \tan x \cos x$.

d) Show graphically that $\sin x = \tan x \cos x$
Are the graphs *exactly* the same?





Trigonometry

LESSON SIX- *Trigonometric Identities I*

Lesson Notes

$$\cos^3 x + \cos x \sin^2 x = \cos x$$

$$\begin{array}{r|l} \cos x\left(\cos^2 x + \sin^2 x\right) & \\ \cos x(1) & \\ \cos x & \cos x \quad \checkmark \end{array}$$

Example 13 Exploring the proof of $\csc x + \cot x = \dfrac{1+\cos x}{\sin x}$

Exploring a Proof

a) Prove algebraically that $\csc x + \cot x = \dfrac{1+\cos x}{\sin x}$.

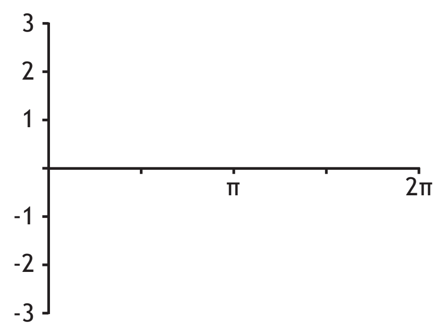
b) Verify that $\csc x + \cot x = \dfrac{1+\cos x}{\sin x}$ for $\dfrac{\pi}{3}$.

c) State the non-permissible values for $\csc x + \cot x = \dfrac{1+\cos x}{\sin x}$.

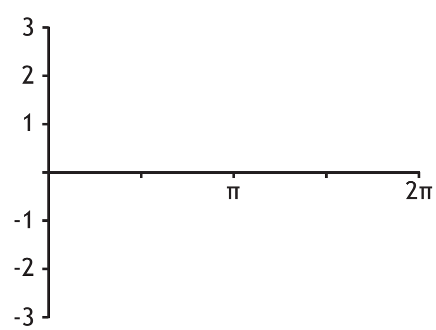
d) Show graphically that $\csc x + \cot x = \dfrac{1+\cos x}{\sin x}$

Are the graphs *exactly* the same?

$y_1 = \csc x + \cot x$



$y_2 = \dfrac{1+\cos x}{\sin x}$



$$\begin{array}{r|l} \cos^3 x + \cos x \sin^2 x & = \cos x \\ \cos x\left(\cos^2 x + \sin^2 x\right) & \\ \cos x(1) & \\ \cos x & \cos x \quad \checkmark \end{array}$$

Example 14

Exploring the proof of $\dfrac{1}{1-\cos x} + \dfrac{1}{1+\cos x} = 2\csc^2 x$

Exploring a Proof

a) Prove algebraically that $\dfrac{1}{1-\cos x} + \dfrac{1}{1+\cos x} = 2\csc^2 x$.

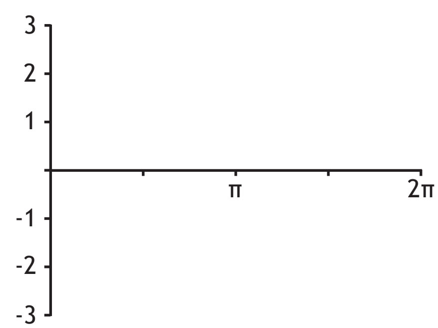
b) Verify that $\dfrac{1}{1-\cos x} + \dfrac{1}{1+\cos x} = 2\csc^2 x$ for $\dfrac{\pi}{2}$.

c) State the the non-permissible values for $\dfrac{1}{1-\cos x} + \dfrac{1}{1+\cos x} = 2\csc^2 x$.

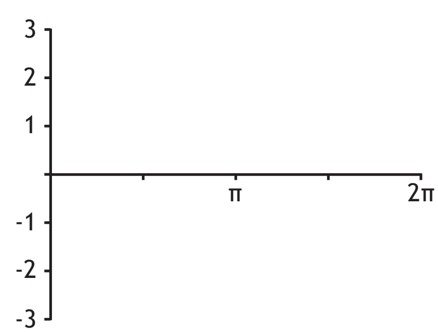
d) Show graphically that $\dfrac{1}{1-\cos x} + \dfrac{1}{1+\cos x} = 2\csc^2 x$

Are the graphs *exactly* the same?

$$y_1 = \frac{1}{1-\cos x} + \frac{1}{1+\cos x}$$



$$y_2 = 2\csc^2 x$$



Trigonometry

LESSON SIX- *Trigonometric Identities I*

Lesson Notes

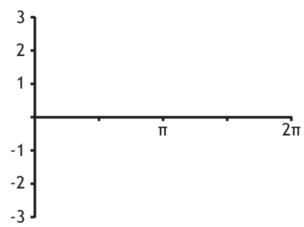
$$\cos^3 x + \cos x \sin^2 x = \cos x$$

$$\begin{array}{r|l} \cos x\left(\cos^2 x + \sin^2 x\right) & \\ \cos x(1) & \\ \cos x & \cos x \quad \checkmark \end{array}$$

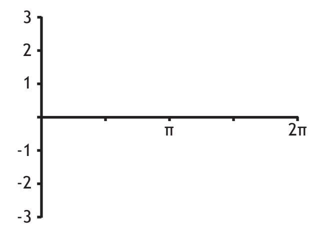
Example 15 Solve each trigonometric equation over the domain $0 \le x \le 2\pi$.

Equations With Identities

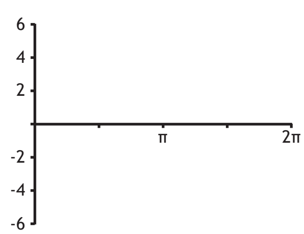
a) $2\sin^2 x - \cos x - 1 = 0$



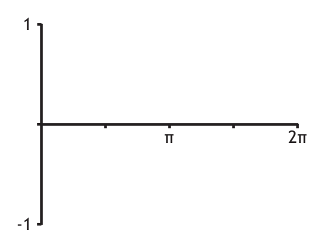
b) $\sin x = \sec x \cot x$



c) $2\tan^2 x = -3\sec x$



d) $\cos^2 x = \sin^2 x$



$$\cos^3 x + \cos x \sin^2 x = \cos x$$

$$\cos x\left(\cos^2 x + \sin^2 x\right) \quad \Big|$$

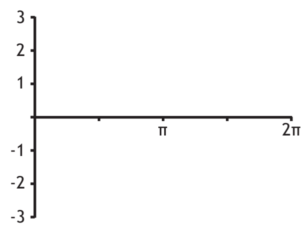
$$\cos x(1) \quad \Big|$$

$$\cos x \quad \Big| \quad \cos x \quad \checkmark$$

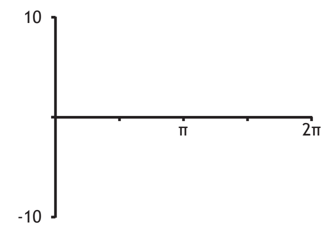
Example 16 Solve each trigonometric equation over the domain $0 \le x \le 2\pi$.

Equations With Identities

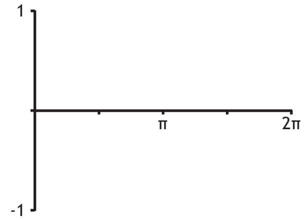
a) $3 - 3\csc x + \cot^2 x = 0$



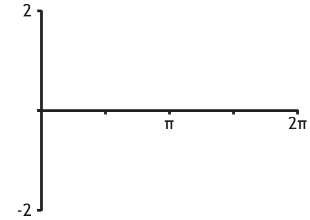
b) $3\sin^2 x + 3\cos x - 4 = \sin^2 x - 2\cos x$



c) $\sin^3 x = \sin x$



d) $2\sin^3 x - 2\cos^2 x - \sin x + 1 = 0$



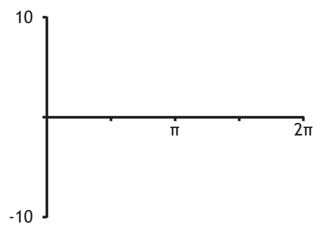
Trigonometry
LESSON SIX- *Trigonometric Identities I*
Lesson Notes

$$\begin{array}{r|l} \cos^3 x + \cos x \sin^2 x & \cos x \\ \cos x\left(\cos^2 x + \sin^2 x\right) & \\ \cos x(1) & \\ \cos x & \cos x \quad \checkmark \end{array}$$

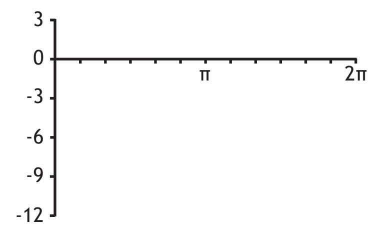
Example 17 Solve each trigonometric equation over the domain 0 ≤ x ≤ 2π.

Equations With Identities

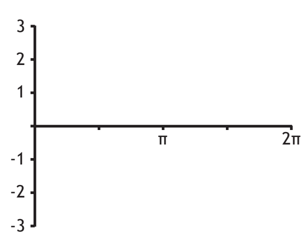
a) $2\sec^2 x - \tan^4 x = -1$



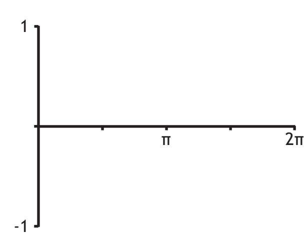
b) $2\cos^3 x + 3\cos x = 7\cos^2 x$



c) $\tan^2 x + 2\sec^2 x - 3 = 0$



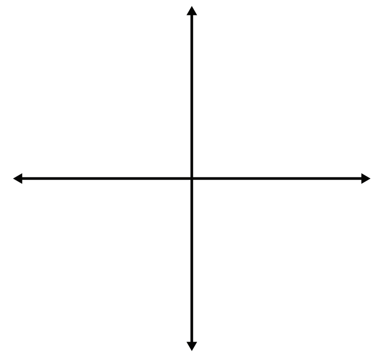
d) $4\sin^2 x + 2\sqrt{2}\sin x + 2\sqrt{3}\sin x + \sqrt{6} = 0$



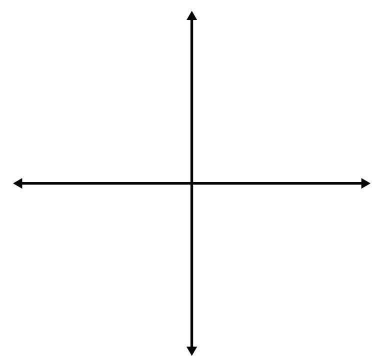
Example 18 Use the Pythagorean identities to find the indicated value and draw the corresponding triangle.

Pythagorean Identities and Finding an Unknown

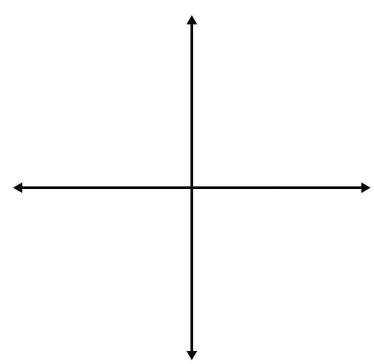
a) If the value of $\sin x = \frac{4}{7}$, $0 \le x \le \frac{\pi}{2}$, find the value of cos*x* within the same domain.



b) If the value of $\tan A = \frac{3}{2}$, $\pi < A < \frac{3\pi}{2}$, find the value of sec*A* within the same domain.



c) If $\cos\theta = \frac{\sqrt{7}}{7}$, and $\cot\theta < 0$, find the exact value of $\sin\theta$.

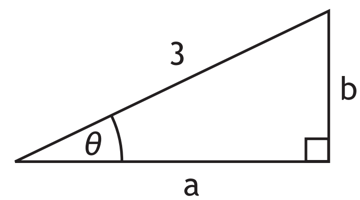


Example 19 Trigonometric Substitution.

Trigonometric Substitution

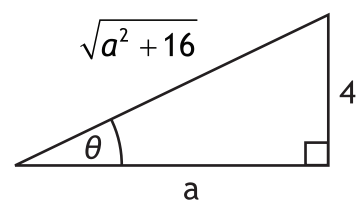
a) Using the triangle to the right, show that $\frac{\sqrt{9-b^2}}{b^2}$ can be expressed as $\frac{\cos\theta}{3\sin^2\theta}$.

Hint: *Use the triangle to find a trigonometric expression equivalent to b.*



b) Using the triangle to the right, show that $\frac{a^2}{\sqrt{a^2+16}}$ can be expressed as $4\cot\theta\cos\theta$.

Hint: *Use the triangle to find a trigonometric expression equivalent to a.*



Example 1

Evaluate each trigonometric sum or difference.

Sum and Difference Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

a) $\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) =$

b) $\sin\left(\frac{\pi}{2} - \frac{\pi}{6}\right) =$

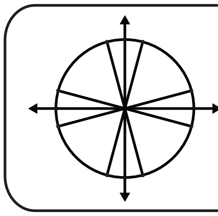
c) $\cos\left(45^\circ - 60^\circ\right) =$

d) $\cos\left(\frac{\pi}{3} + \frac{\pi}{6}\right) =$

e) $\tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right) =$

f) $\tan\left(\frac{\pi}{6} - \frac{\pi}{3}\right) =$

Trigonometry
LESSON SEVEN- *Trigonometric Identities II*
Lesson Notes



$\cos 15° = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15° = \frac{\sqrt{6}-\sqrt{2}}{4}$

Example 2

Write each expression as a single trigonometric ratio.

Sum and Difference Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

a) $\sin\frac{\pi}{6}\cos\frac{\pi}{2} + \cos\frac{\pi}{6}\sin\frac{\pi}{2}$

b) $\dfrac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{6}}$

c) $\cos\frac{\pi}{3}\cos\frac{\pi}{6} + \sin\frac{\pi}{3}\sin\frac{\pi}{6}$

Example 3

Find the exact value of each expression.

Sum and Difference Identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

a) $\cos 15^\circ$

b) $\sin\dfrac{5\pi}{12}$

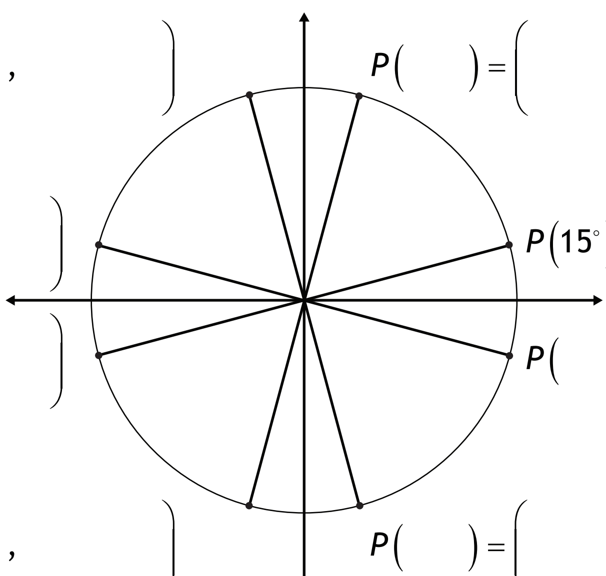
c) $\tan 195^\circ$

d) Given the exact values of cosine and sine for 15°, fill in the blanks for the other angles.

$P(\quad) = (\quad , \quad)$ $P(\quad) = (\quad , \quad)$

$P(\quad) = (\quad , \quad)$ $P(15^\circ) = \left(\dfrac{\sqrt{6}+\sqrt{2}}{4}, \dfrac{\sqrt{6}-\sqrt{2}}{4}\right)$

$P(\quad) = (\quad , \quad)$ $P(\quad) = (\quad , \quad)$



$P(\quad) = (\quad , \quad)$ $P(\quad) = (\quad , \quad)$

Trigonometry

LESSON SEVEN- ***Trigonometric Identities II***

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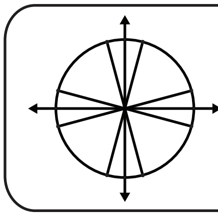


$\cos 15° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

Example 4

Find the exact value of each expression.

For simplicity, do not rationalize the denominator.

Sum and Difference Identities

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

a) $\csc\left(\dfrac{\pi}{3}+\dfrac{\pi}{4}\right)$

b) $\sec\left(\dfrac{\pi}{12}\right)$

c) $\cot\left(\dfrac{\pi}{2}-\dfrac{\pi}{4}\right)$

Example 5 Double-angle identities.

Double-Angle Identities

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

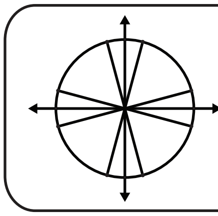
$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

a) Prove the double-angle sine identity, sin2x = 2sinxcosx.

b) Prove the double-angle cosine identity, cos2x = $\cos^2$x - $\sin^2$x.

c) The double-angle cosine identity, cos2x = $\cos^2$x - $\sin^2$x, can be expressed as cos2x = 1 - 2$\sin^2$x or cos2x = 2$\cos^2$x - 1. Derive each identity.

d) Derive the double-angle tan identity, $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$.



$\cos 15° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

Example 6

Double-angle identities.

Double-Angle Identities

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

a) Evaluate each of the following expressions using a double-angle identity.

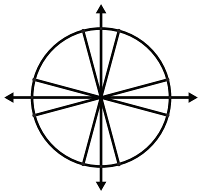
i) $\sin 60^\circ$ *ii*) $\cos\dfrac{\pi}{2}$ *iii*) $\tan 90^\circ$

b) Express each of the following expressions using a double-angle identity.

i) $\sin 8x$ *ii*) $\cos 4x$ *iii*) $\sin x$ *iv*) $\cos\dfrac{1}{2}x$

c) Write each of the following expression as a single trigonometric ratio using a double-angle identity.

i) $\cos^2 30^\circ - \sin^2 30^\circ$ *ii*) $\sin\dfrac{\pi}{8}\cos\dfrac{\pi}{8}$ *iii*) $1-\sin^2\dfrac{1}{2}x$ *iv*) $\dfrac{2\tan\frac{x}{8}}{1-\tan^2\frac{x}{8}}$



$\cos 15° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

Example 7

Prove each trigonometric identity.

Note: Variable restrictions may be ignored for the proofs in this lesson.

Sum and Difference Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

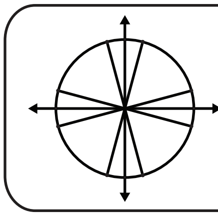
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

a) $\cos\left(x + \dfrac{5\pi}{6}\right) = -\dfrac{\sqrt{3}\cos x + \sin x}{2}$

b) $\sin\left(\dfrac{3\pi}{2} - x\right) = -\cos x$

c) $\tan\left(x - \dfrac{3\pi}{4}\right) = \dfrac{\tan x + 1}{1 - \tan x}$

d) $\cos(x + y) + \cos(x - y) = 2\cos x \cos y$



$\cos 15° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

Example 8 Prove each trigonometric identity.

Sum and Difference Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

a) $\cos\left(x + \dfrac{\pi}{6}\right) - \sin\left(x + \dfrac{2\pi}{3}\right) = 0$

b) $\dfrac{\sin(x - y)}{\cos x \cos y} = \tan x - \tan y$

c) $\cos(x + y)\cos(x - y) = (\cos x \cos y)^2 - (\sin x \sin y)^2$

d) $\cos 2x = \cos^2 x - \sin^2 x$

Example 9

Prove each trigonometric identity.

Double-Angle Identities

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

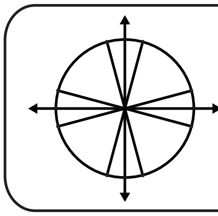
a) $\cos 2x + 2\sin^2 x = 1$

b) $\frac{2}{1+\cos 2x} = \sec^2 x$

c) $\frac{\sin 2x}{\cos 2x + \sin^2 x} = 2\tan x$

d) $\frac{2\sin x \cos x}{\cos^2 x - \sin^2 x} = \tan 2x$

Trigonometry
LESSON SEVEN- *Trigonometric Identities II*
Lesson Notes



$\cos 15° = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15° = \frac{\sqrt{6}-\sqrt{2}}{4}$

Example 10 Prove each trigonometric identity.

Double-Angle Identities

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

a) $\cos^4 x - \sin^4 x = \cos 2x$

b) $1-\left(\sin x + \cos x\right)^2 = -\sin 2x$

c) $\frac{2\left(\tan x - \cot x\right)}{\tan^2 x - \cot^2 x} = \sin 2x$

d) $\frac{1}{1-\tan x} - \frac{1}{1+\tan x} = \tan 2x$

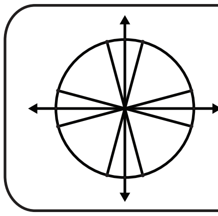
Example 11 Prove each trigonometric identity. **Assorted Proofs**

a) $2\csc 2x = \csc x \sec x$

b) $\dfrac{\sin(x+y)}{\cos x \sin y} = \tan x \cot y + 1$

c) $\sin 88^\circ \cos 58^\circ - \cos 88^\circ \sin 58^\circ = \dfrac{1}{2}$

d) $\tan\left(x + \dfrac{\pi}{4}\right) = \dfrac{\tan x + 1}{1 - \tan x}$



$\cos 15^\circ = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15^\circ = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

Example 12 Prove each trigonometric identity. **Assorted Proofs**

a) $(\sin x + \cos x)^2 - 1 = \sin 2x$

b) $\dfrac{1}{2}\sin\dfrac{2x}{5} = \sin\dfrac{x}{5}\cos\dfrac{x}{5}$

c) $\cos^2\left(x - \dfrac{\pi}{2}\right) = \sin^2 x$

d) $\sin 3x = 3\sin x - 4\sin^3 x$

Example 13

Prove each trigonometric identity.

Assorted Proofs

a) $\dfrac{5\sin x - \cos 2x - 11}{2\sin x - 3} = \sin x + 4$

b) $\cos 3x = 4\cos^3 x - 3\cos x$

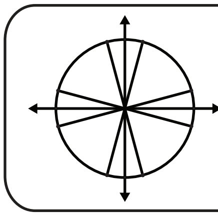
c) $\cos 34^\circ \cos 41^\circ - \sin 34^\circ \sin 41^\circ = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

d) $\dfrac{\tan x + \tan y}{\sec x \sec y} = \sin\left(x + y\right)$

Trigonometry

LESSON SEVEN- *Trigonometric Identities II*

Lesson Notes



$$\cos 15° = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\sin 15° = \frac{\sqrt{6}-\sqrt{2}}{4}$$

Example 14 Solve each trigonometric equation over the domain $0 \le x \le 2\pi$.

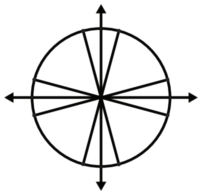
Assorted Equations

a) $\cos 2x = \cos^2 x$

b) $\cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right) = -1$

c) $4\sin^2 x + 4\cos 2x - 1 = 0$

d) $2\cos^2 \frac{1}{2}x - 2\sin^2 \frac{1}{2}x = 1$



$\cos 15^\circ = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15^\circ = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

Example 15 Solve each trigonometric equation over the domain $0 \le x \le 2\pi$.

Assorted Equations

a) $\cos 2x + 7\sin x - 4 = 0$

b) $\sin 2x - \cos x = 0$

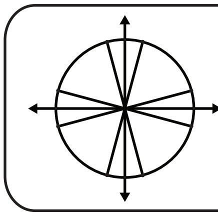
c) $\sin\left(\dfrac{\pi}{3} + x\right) - \sin\left(\dfrac{\pi}{3} - x\right) = 1$

d) $\sin x \cos x = \dfrac{1}{4}$

Trigonometry

LESSON SEVEN- *Trigonometric Identities II*

Lesson Notes



$\cos 15° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

Example 16 Solve each trigonometric equation over the domain $0 \le x \le 2\pi$.

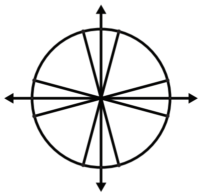
Assorted Equations

a) $\cos 2x - \cos x = 0$

b) $\csc\left(x + \dfrac{\pi}{2}\right) - \csc\left(x - \dfrac{\pi}{2}\right) = 4$

c) $\dfrac{1}{2}\sin 2x + \sin x = 0$

d) $2\cot^2 x - 3\csc x = 0$



$\cos 15^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$

Example 17 Solve each trigonometric equation over the domain $0 \le x \le 2\pi$.

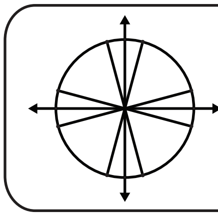
Assorted Equations

a) $8\sin x\cos x = 2$

b) $(\cos x - \sin x)^2 = \sin 2x + 1$

c) $\tan(x - \pi) + \sec x = 0$

d) $\cos(x + \pi) - \cos^2 x = 0$

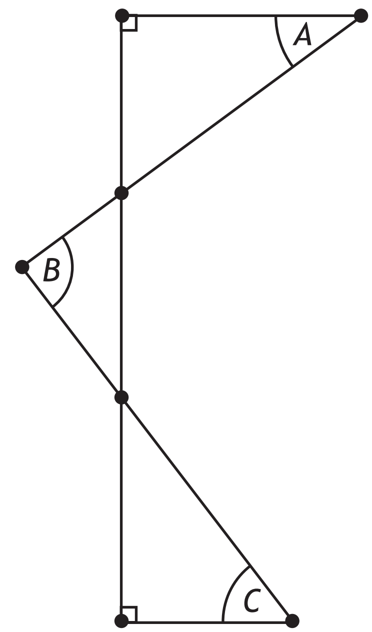


$\cos 15° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

Example 18

Trigonometric identities and geometry.



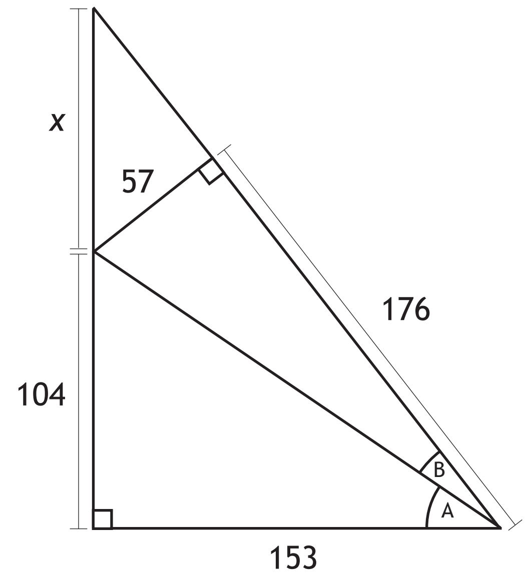
a) Show that $\tan B = \dfrac{\tan A + \tan C}{1 - \tan A \tan C}$

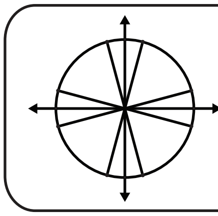
b) If A = 32° and B = 89°, what is the value of C?

Example 19

Trigonometric identities and geometry.

Solve for x. Round your answer to the nearest tenth.

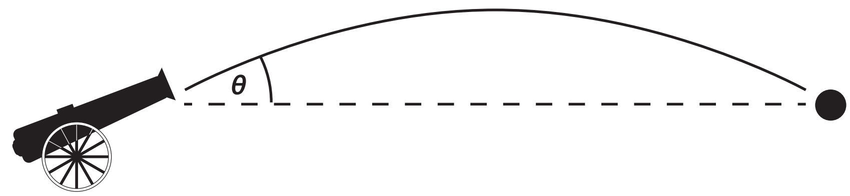




$\cos 15° = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15° = \frac{\sqrt{6}-\sqrt{2}}{4}$

Example 20



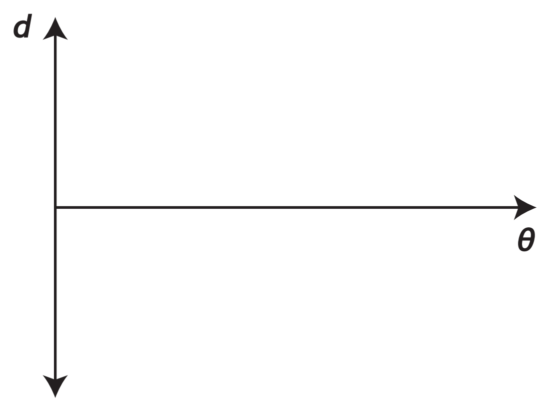
If a cannon shoots a cannonball θ degrees above the horizontal, the horizontal distance traveled by the cannonball before it hits the ground can be found with the function:

$$\boxed{d(\theta) = \frac{v_i^2 \sin\theta\cos\theta}{4.9}}$$

The initial velocity of the cannonball is 36 m/s.

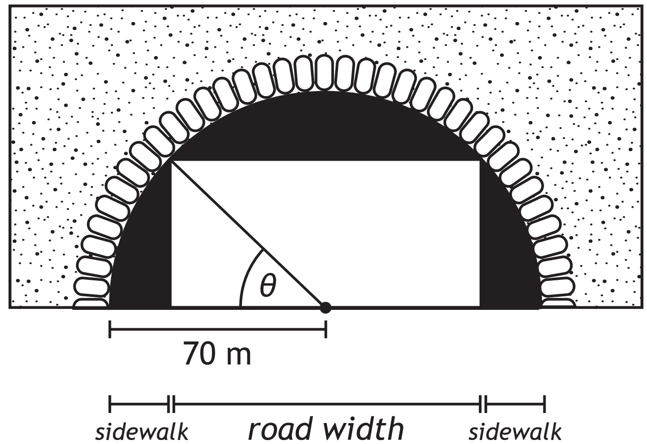
a) Rewrite the function so it involves a single trigonometric identity.

b) Graph the function. Use the graph to describe the trajectory of the cannonball at the following angles: 0°, 45°, and 90°.



c) If the cannonball travels a horizontal distance of 100 m, find the angle of the cannon. Solve graphically, and round your answer to the nearest tenth of a degree.

Example 21

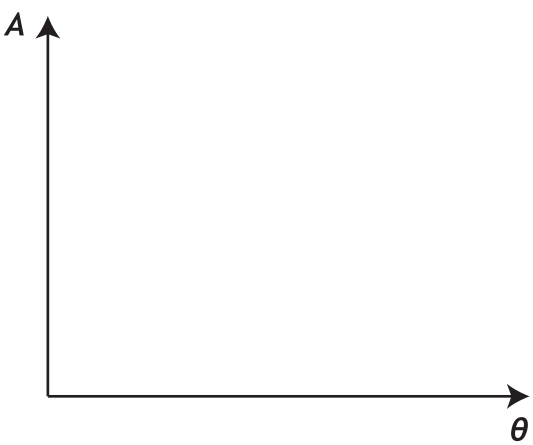


An engineer is planning the construction of a road through a tunnel. In one possible design, the width of the road maximizes the area of a rectangle inscribed within the cross-section of the tunnel.

The angle of elevation from the centre line of the road to the upper corner of the rectangle is θ. Sidewalks on either side of the road are included in the design.

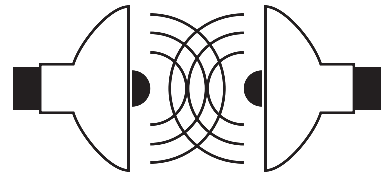
a) If the area of the rectangle can be represented by the function $A(\theta) = m\sin 2\theta$, what is the value of m?

b) What angle maximizes the area of the rectangular cross-section?



c) For the angle that maximizes the area:
i) What is the width of the road?
ii) What is the height of the tallest vehicle that will pass through the tunnel?
iii) What is the width of one of the sidewalks?
Express answers as exact values.

Example 22



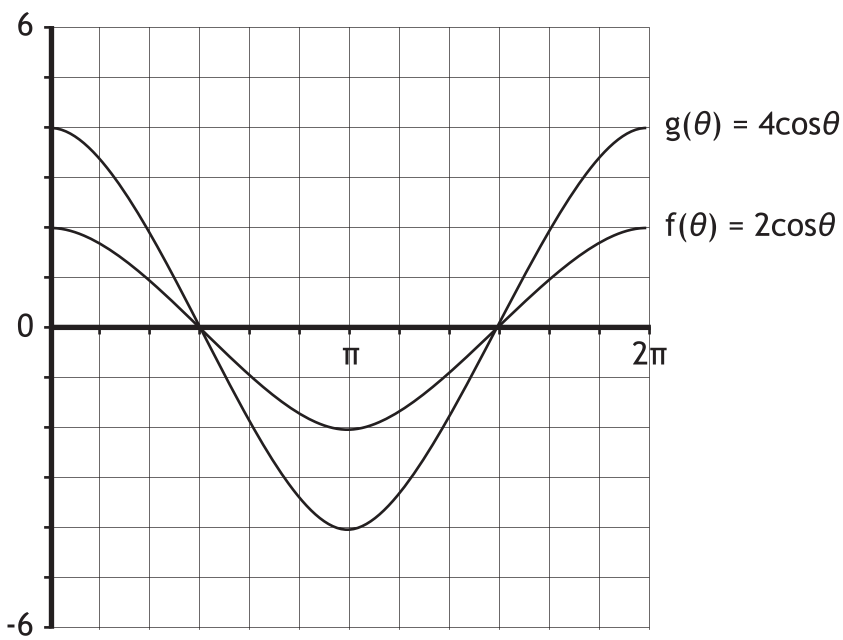
The improper placement of speakers for a home theater system may result in a diminished sound quality at the primary viewing area. This phenomenon occurs because sound waves interact with each other in a process called interference. When two sound waves undergo interference, they combine to form a resultant sound wave that has an amplitude equal to the sum of the component sound wave amplitudes.

If the amplitude of the resultant wave is larger than the component wave amplitudes, we say the component waves experienced constructive interference.

If the amplitude of the resultant wave is smaller than the component wave amplitudes, we say the component waves experienced destructive interference.

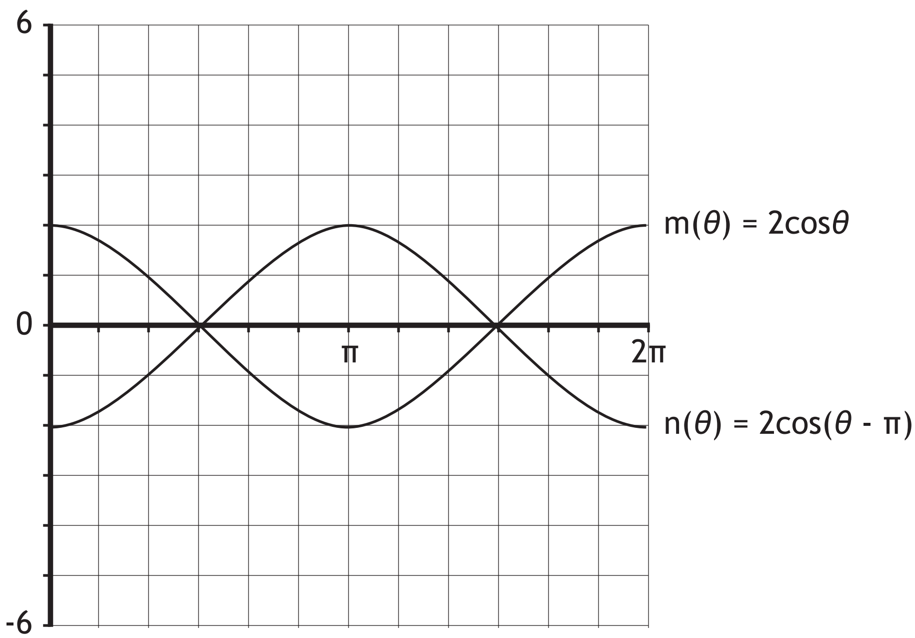
a) Two sound waves are represented with $f(\theta)$ and $g(\theta)$.

i) Draw the graph of $y = f(\theta) + g(\theta)$ and determine the resultant wave function.
ii) Is this constructive or destructive interference?
iii) Will the new sound be louder or quieter than the original sound?



b) A different set of sound waves are represented with m(θ) and n(θ).

i) Draw the graph of y = m(θ) + n(θ) and determine the resultant wave function.
ii) Is this constructive or destructive interference?
iii) Will the new sound be louder or quieter than the original sound?



c) Two sound waves experience total destructive interference if the sum of their wave functions is zero. Given p(θ) = sin(3θ - 3π/4) and q(θ) = sin(3θ - 7π/4), show that these waves experience total destructive interference.

Example 23

Even & Odd Identities

Even & Odd Identities

$\sin(-x) = -\sin x$
$\cos(-x) = \cos x$
$\tan(-x) = -\tan x$

a) Explain what is meant by the terms *even function* and *odd function*.

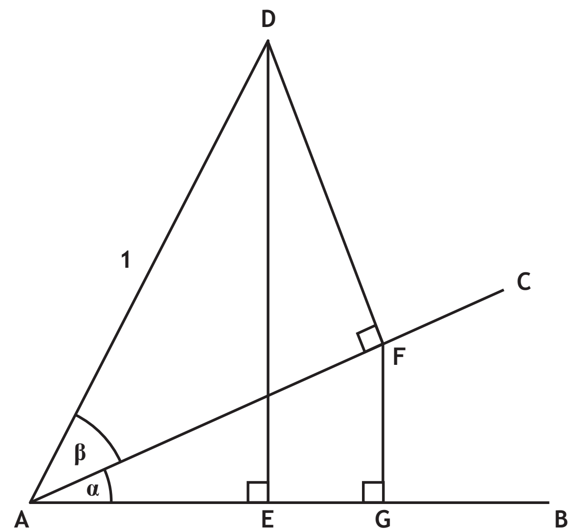
b) Explain how the even & odd identities work.
(Reference the unit circle or trigonometric graphs in your answer.)

c) Prove the three even & odd identities algebraically.

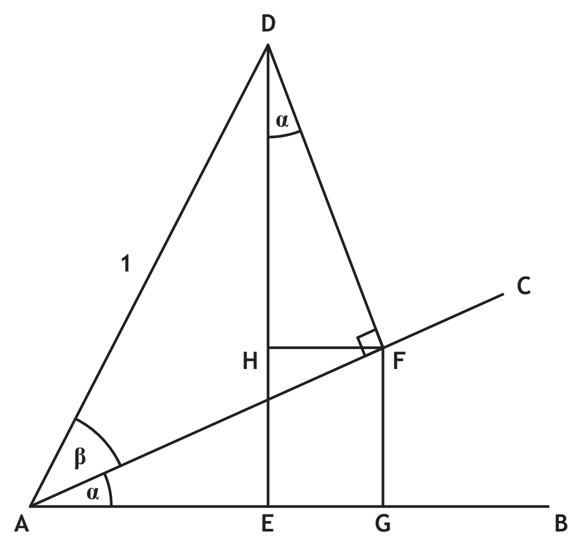
Example 24 Proving the sum and difference identities.

Enrichment Example
Students who plan on taking university calculus should complete this example.

a) Explain how to construct the diagram shown.



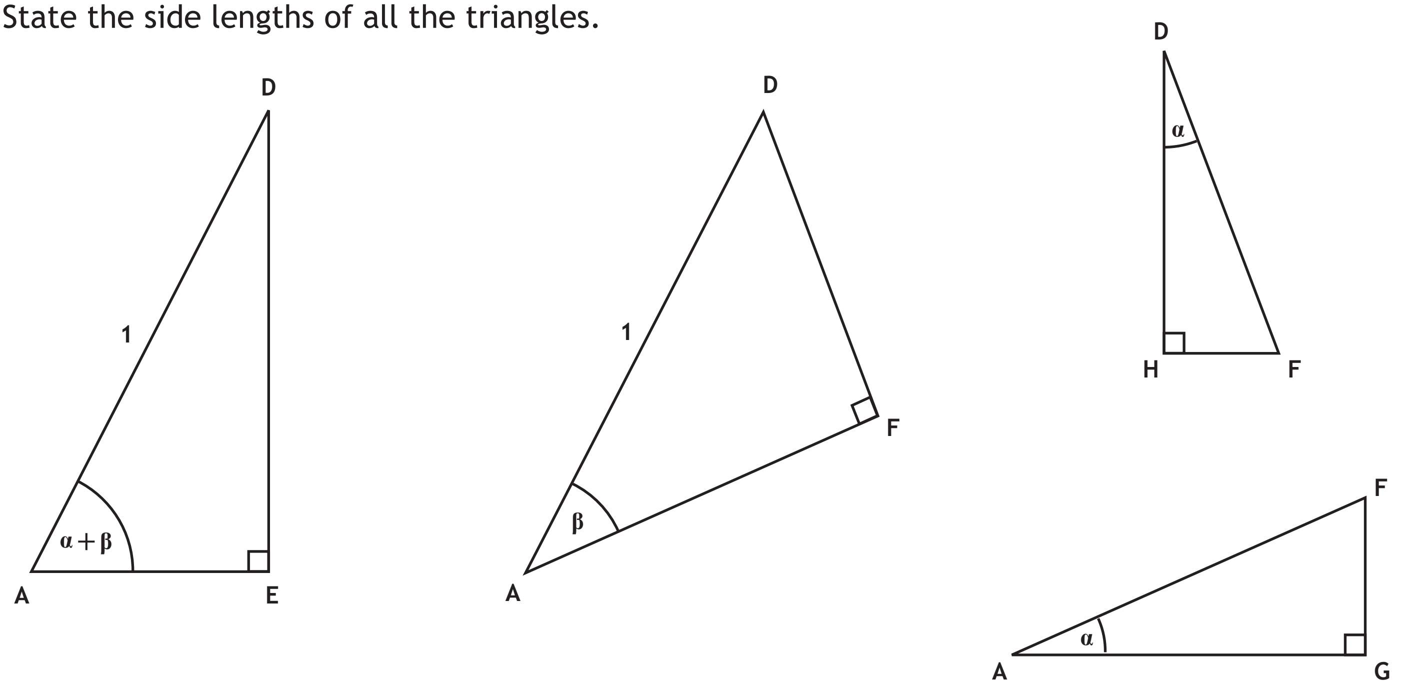
b) Explain the next steps in the construction.



$$\cos 15° = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\sin 15° = \frac{\sqrt{6}-\sqrt{2}}{4}$$

c) State the side lengths of all the triangles.



d) Prove the sum and difference identity for sine.

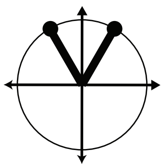
Answer Key

Trigonometry Lesson Five: Trigonometric Equations

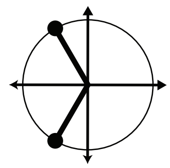
Note: n ε I for all general solutions.

Example 1:

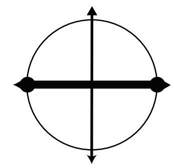
a) $\theta = \frac{\pi}{3} + n(2\pi),\ \ \theta = \frac{2\pi}{3} + n(2\pi)$



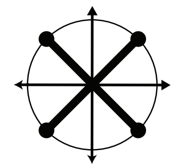
b) $\theta = \frac{2\pi}{3} + n(2\pi),\ \ \theta = \frac{4\pi}{3} + n(2\pi)$



c) $\theta = n\pi$

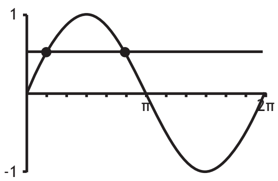


d) $\theta = \frac{\pi}{4} + n\left(\frac{\pi}{2}\right)$

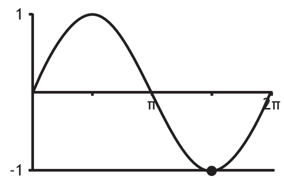


Example 2:

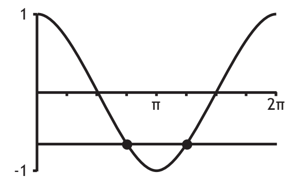
a) $\theta = \frac{\pi}{6} + n(2\pi),\ \theta = \frac{5\pi}{6} + n(2\pi)$



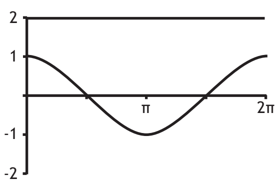
b) $\theta = \frac{3\pi}{2} + n(2\pi)$



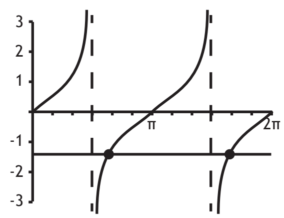
c) $\theta = \frac{3\pi}{4} + n(2\pi),\ \theta = \frac{5\pi}{4} + n(2\pi)$



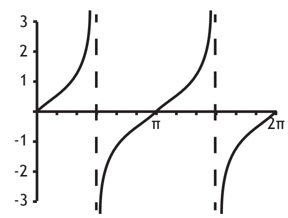
d) no solution



e) $\theta = \frac{2\pi}{3} + n\pi$

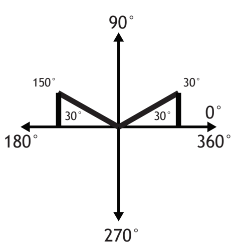


f) $\theta = \frac{\pi}{2} + n\pi$

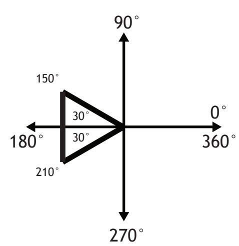


Example 3:

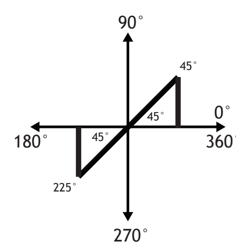
a) $\theta = 30^\circ + n(360^\circ),\ \theta = 150^\circ + n(360^\circ)$



b) $\theta = 150^\circ + n(360^\circ),\ \theta = 210^\circ + n(360^\circ)$

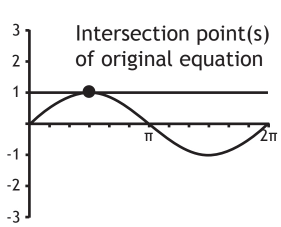


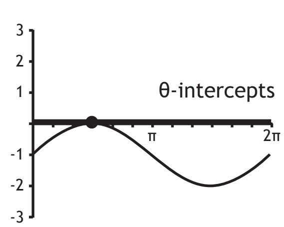
c) $\theta = 45^\circ + n(180^\circ)$



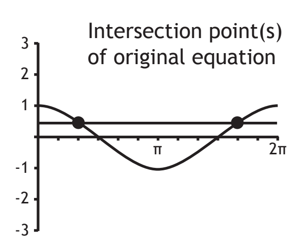
Example 4:

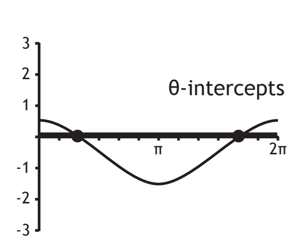
a) $\theta = \frac{\pi}{2}$





b) $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

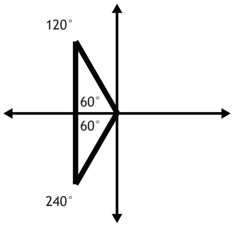




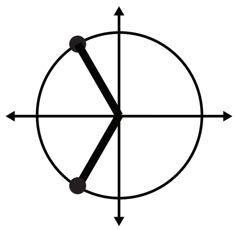
Answer Key

Example 5:

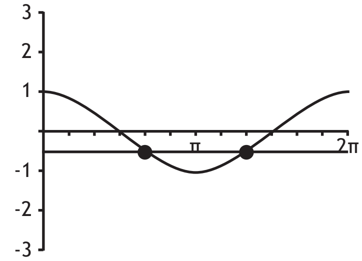
a) $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$



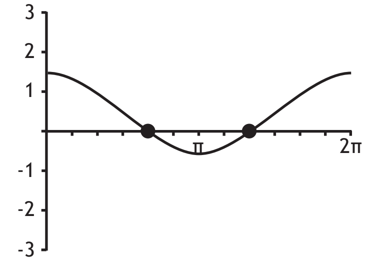
b) $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$



c) $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

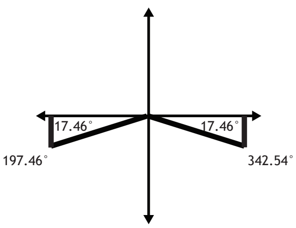


d) $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$



Example 6:

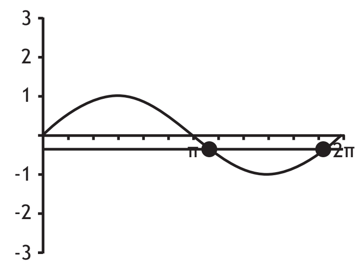
a) 197.46° and 342.54°



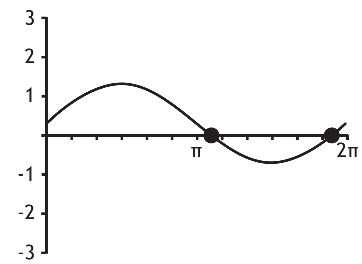
b) 197.46° and 342.54°

The unit circle is not useful for this question.

c) 197.46° and 342.54°

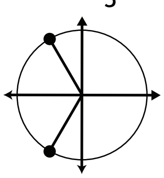


d) 197.46° and 342.54°

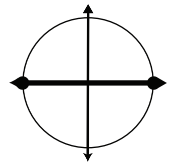


Example 7:

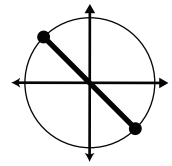
a) $\theta = \frac{2\pi}{3} + n(2\pi),\ \theta = \frac{4\pi}{3} + n(2\pi)$



b) $\theta = n\pi$

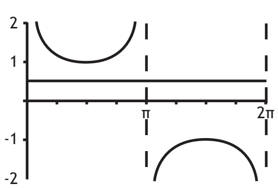


c) $\theta = \frac{3\pi}{4} + n\pi$

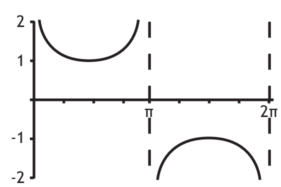


Example 8:

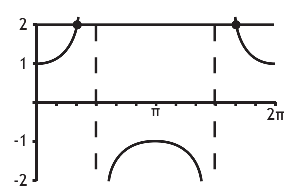
a) No Solution



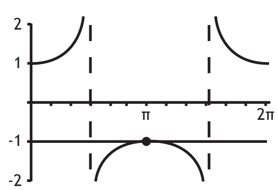
b) $\theta = n\pi$



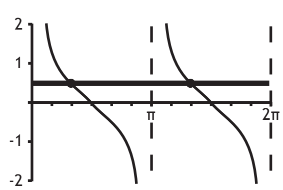
c) $\theta = \frac{\pi}{3} + n(2\pi),\ \theta = \frac{5\pi}{3} + n(2\pi)$



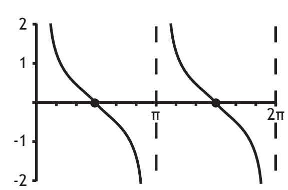
d) $\theta = \pi + n(2\pi)$



e) $\theta = \frac{\pi}{3} + n\pi$



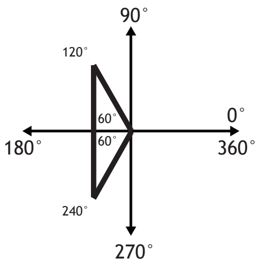
f) $\theta = \frac{\pi}{2} + n\pi$



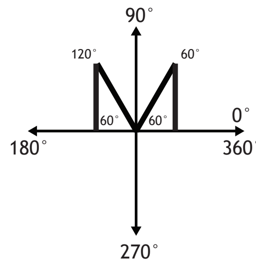
Answer Key

Example 9:

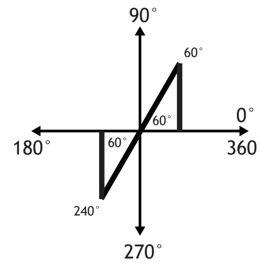
a) $\theta = 120^\circ + n(360^\circ),\ \theta = 240^\circ + n(360^\circ)$



b) $\theta = 60^\circ + n(360^\circ),\ \theta = 120^\circ + n(360^\circ)$



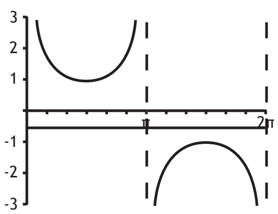
c) $\theta = 60^\circ + n(180^\circ)$



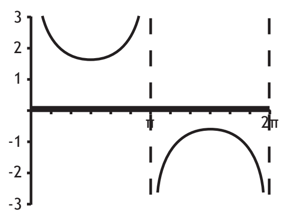
Example 10:

a) No Solution

Intersection point(s) of original equation

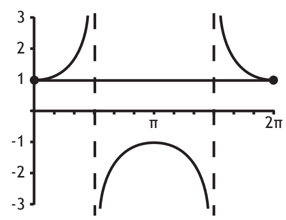


θ-intercepts

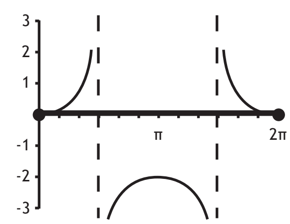


b) $\theta = n(2\pi)$

Intersection point(s) of original equation

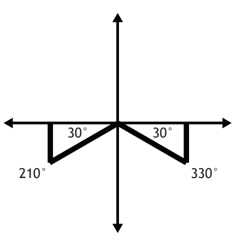


θ-intercepts

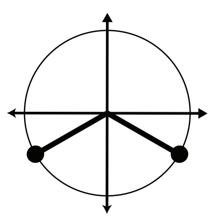


Example 11:

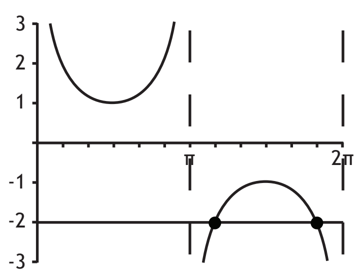
a) $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$



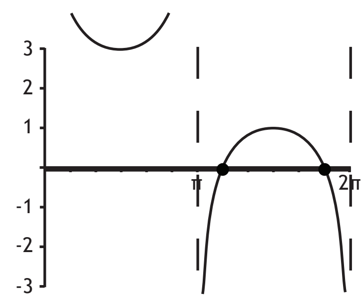
b) $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$



c) $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

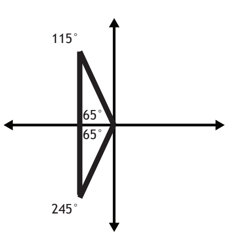


d) $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$



Example 12:

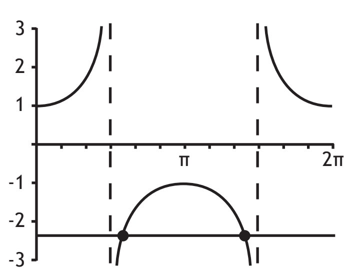
a) $\theta = 115^\circ,\ 245^\circ$



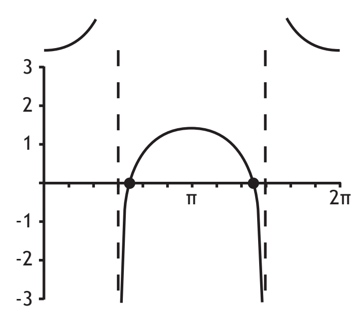
b) $\theta = 115^\circ,\ 245^\circ$

The unit circle is not useful for this question.

c) $\theta = 115^\circ,\ 245^\circ$



d) $\theta = 115^\circ,\ 245^\circ$

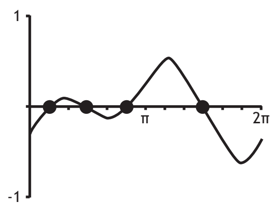
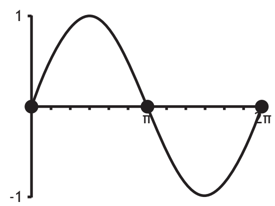
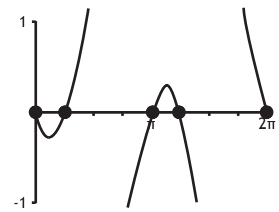
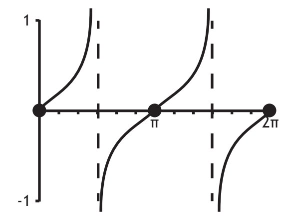


Example 13:

a) $\theta = n(2\pi)$ **b)** $\theta = \frac{\pi}{3} + n(2\pi),\ \theta = \frac{2\pi}{3} + n(2\pi)$ **c)** $\theta = 59^\circ + n(180^\circ)$ or $\theta = 1.03 + n\pi$ **d)** $\theta = \frac{2\pi}{3} + n(2\pi),\ \theta = \frac{4\pi}{3} + n(2\pi)$

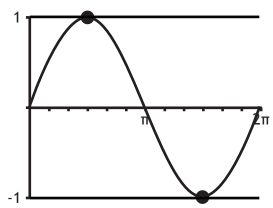
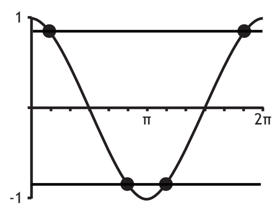
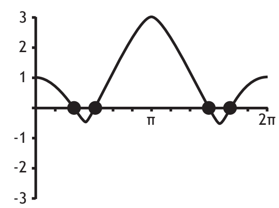
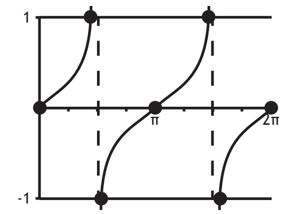
Example 14:

a) $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **b)** $\theta = 0, \pi, 2\pi$ **c)** $\theta = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$ **d)** $\theta = 0, \pi, 2\pi$

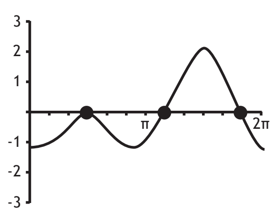
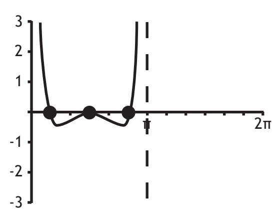
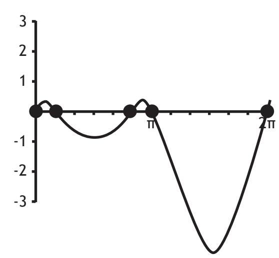
Example 15:

a) $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ **b)** $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **c)** $\theta = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$ **d)** $\theta = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$

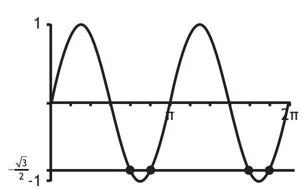
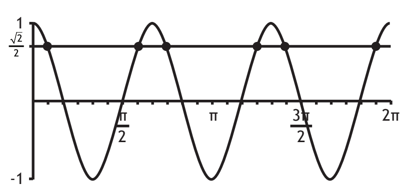
Example 16:

a) $\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b)** $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ **c)** $\theta = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, 2\pi$

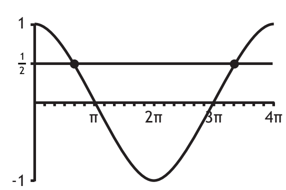
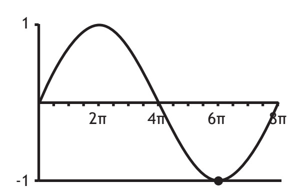
Example 17:

a) $\theta = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$ **b)** $\theta = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{23\pi}{12}$

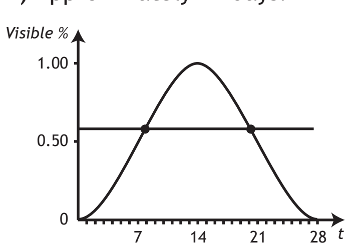
Example 18:

a) $\theta = \frac{2\pi}{3}, \frac{10\pi}{3}$ **b)** $\theta = 6\pi$

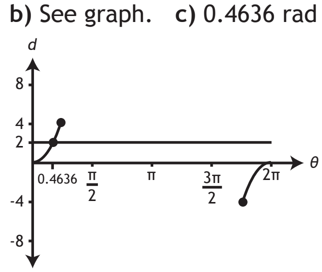
Example 19: a) $P(t) = -0.50\cos\frac{\pi}{14}t + 0.50$

b) Approximately 12 days.



Example 20: a) $d(\theta) = 4\tan\theta$

b) See graph. **c)** 0.4636 rad (or 26.6°)



Example 21: See Video

Answer Key

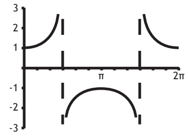
Trigonometry Lesson Six: Trigonometric Identities I

Note: n ε I for all general solutions.

Example 1: a)

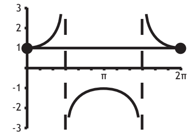
Identity

$$\boxed{\sec x = \frac{1}{\cos x}}$$



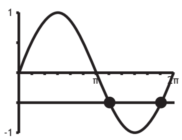
Equation

$$\boxed{\sec x = 1}$$



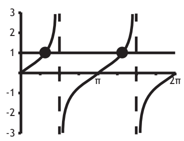
b) i) $\sin x = -\frac{1}{2}$

Not an Identity



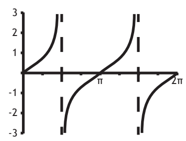
ii) $\tan x = 1$

Not an Identity



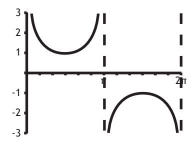
iii) $\tan x = \frac{\sin x}{\cos x}$

Identity



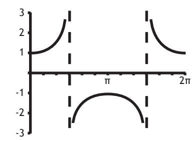
iv) $\csc x = \frac{1}{\sin x}$

Identity



v) $\sec x = \textit{undefined}$

Not an Identity



Example 2:

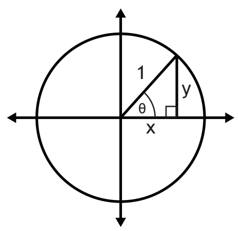
a)

$x^2 + y^2 = 1$

Use basic trigonometry (SOHCAHTOA) to show that x = cosθ and y = sinθ.

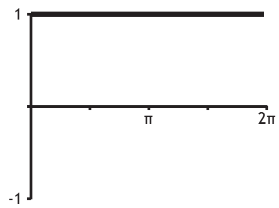
$(\cos\theta)^2 + (\sin\theta)^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$



b) Verify that the L.S. = R.S. for each angle.

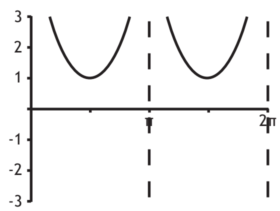
c) The graphs of y = $\sin^2$x + $\cos^2$x and y = 1 are the same.



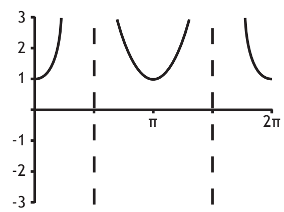
d) Divide both sides of $\sin^2$x + $\cos^2$x = 1 by $\sin^2$x to get 1 + $\cot^2$x = $\csc^2$x.
Divide both sides of $\sin^2$x + $\cos^2$x = 1 by $\cos^2$x to get $\tan^2$x + 1 = $\sec^2$x.

e) Verify that the L.S. = R.S. for each angle.

f) The graphs of y = 1 + $\cot^2$x and y = $\csc^2$x are the same.

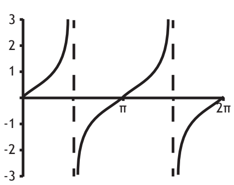


The graphs of y = $\tan^2$x + 1 and y = $\sec^2$x are the same.

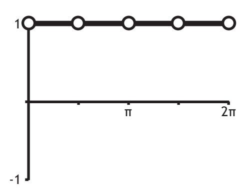


Example 3:

a) $\sin x \sec x = \tan x,\ x \neq \frac{\pi}{2} + n\pi$

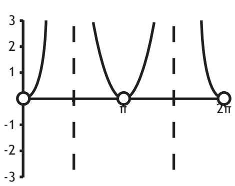


b) $\cot x \sin x \sec x = 1,\ x \neq \frac{n\pi}{2}$

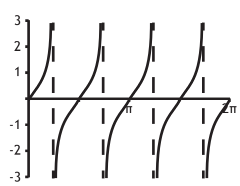


Example 4:

a) $\frac{\sin x \sec x}{\cot x} = \tan^2 x,\ x \neq \frac{n\pi}{2}$

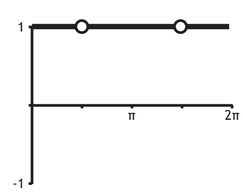


b) $\sin 2x \sec 2x = \tan 2x,\ x \neq \frac{\pi}{4} + n\frac{\pi}{2}$

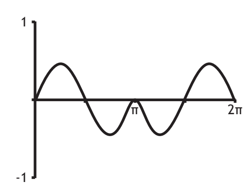


Example 5:

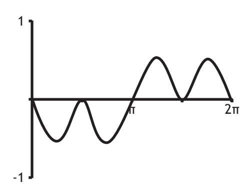
a) $\sin^2 x + \frac{1}{\sec^2 x} = 1$, $x \neq \frac{\pi}{2} + n\pi$



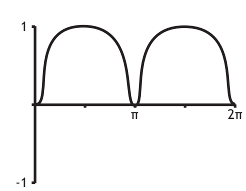
b) $\cos x - \cos^3 x$
$= \cos x \sin^2 x$



c) $\sin^3 x - \sin x$
$= -\sin x \cos^2 x$

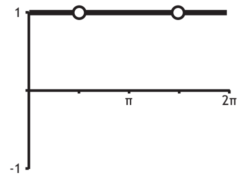


d) $\sin^2 x + \sin^2 x \cos^2 x$
$= \sin^2 x\left(1 + \cos^2 x\right)$

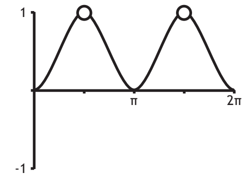


Example 6:

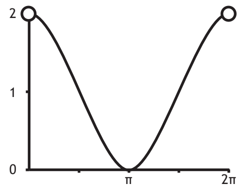
a) $\cos^2 x + \tan^2 x\cos^2 x = 1, \quad x \neq \frac{\pi}{2} + n\pi$



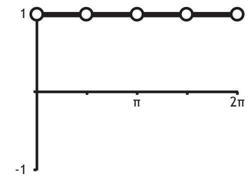
b) $\frac{\sec^2 x - 1}{1 + \tan^2 x} = \sin^2 x, \quad x \neq \frac{\pi}{2} + n\pi$



c) $\frac{\sin^2 x}{1 - \cos x} = 1 + \cos x, \quad x \neq n(2\pi)$

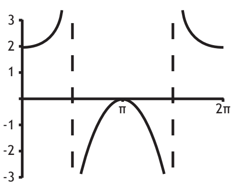


d) $\left(\frac{\sec^2 x}{\csc^2 x}\right)(\csc^2 x - 1) = 1, \quad x \neq \frac{n\pi}{2}$

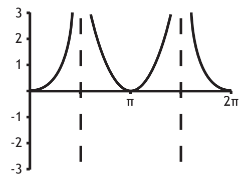


Example 7:

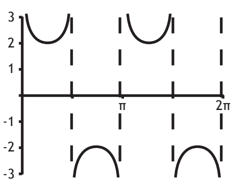
a) $1 + \sec x = \frac{\cos x + 1}{\cos x}, \quad x \neq \frac{\pi}{2} + n\pi$



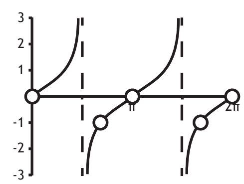
b) $\tan^2 x - \sin^2 x = \sin^2 x\tan^2 x, \quad x \neq \frac{\pi}{2} + n\pi$



c) $\cot x + \tan x = \sec x\csc x, \quad x \neq \frac{n\pi}{2}$

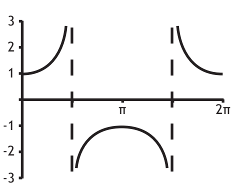


d) $\frac{1 + \tan x}{1 + \cot x} = \tan x, \quad x \neq \frac{n\pi}{2}, \quad x \neq \frac{3\pi}{4} + n\pi$

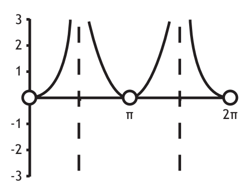


Example 8:

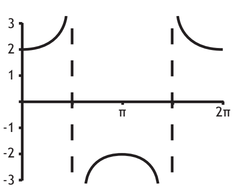
a) $\frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = \sec x, \quad x \neq \frac{\pi}{2} + n\pi$



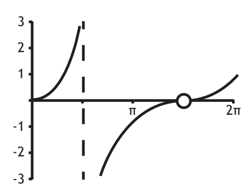
b) $\frac{1 + \tan^2 x}{1 + \cot^2 x} = \tan^2 x, \quad x \neq \frac{n\pi}{2}$



c) $\frac{\cos x}{1 + \sin x} + \frac{\cos x}{1 - \sin x} = 2\sec x, \quad x \neq \frac{\pi}{2} + n\pi$



d) $\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}, \quad x \neq \frac{\pi}{2} + n\pi$



Answer Key

Example 9: See Video **Example 10:** See Video **Example 11:** See Video

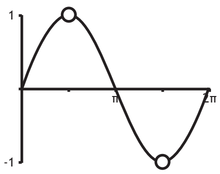
Example 12:

a) See Video

b) $L.S. = R.S. = \dfrac{\sqrt{3}}{2}$

c) $x \neq \dfrac{\pi}{2} + n\pi$

d) $\sin x = \tan x \cos x, \quad x \neq \dfrac{\pi}{2} + n\pi$



The graphs
are NOT identical.
The R.S. has holes.

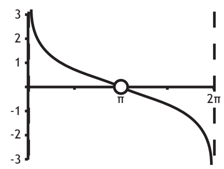
Example 13:

a) See Video

b) $L.S. = R.S. = \sqrt{3}$

c) $x \neq n\pi$

d) $\csc x + \cot x = \dfrac{1 + \cos x}{\sin x}, \quad x \neq n\pi$



The graphs
are identical.

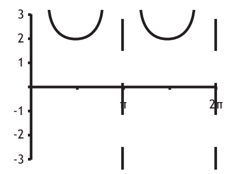
Example 14:

a) See Video

b) $L.S. = R.S. = 2$

c) $x \neq n\pi$

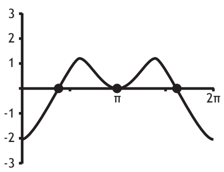
d) $\dfrac{1}{1 - \cos x} + \dfrac{1}{1 + \cos x} = 2\csc^2 x, \quad x \neq n\pi$



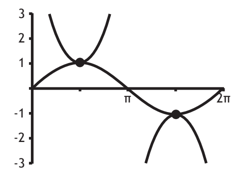
The graphs
are identical.

Example 15:

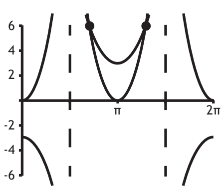
a) $2\sin^2 x - \cos x - 1 = 0, \quad x = \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}$



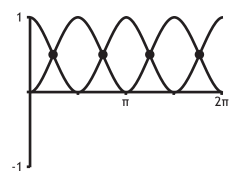
b) $\sin x = \sec x \cot x, \quad x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$



c) $2\tan^2 x = -3\sec x, \quad x = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

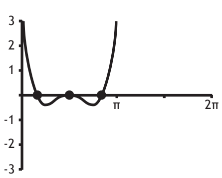


d) $\cos^2 x = \sin^2 x, \quad x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

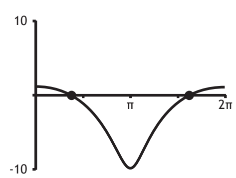


Example 16:

a) $3 - 3\csc x + \cot^2 x = 0, \quad x = \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}$

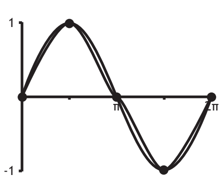


b) $2\sin^2 x + 5\cos x - 4 = 0, \quad x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

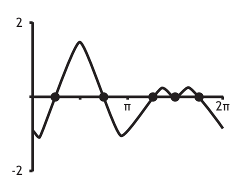


Note: All terms from the original equation were collected on the left side before graphing.

c) $\sin^3 x = \sin x, \quad x = 0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}, 2\pi$

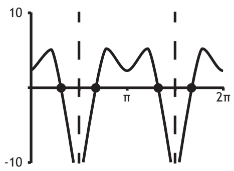


d) $2\sin^3 x - 2\cos^2 x - \sin x + 1 = 0, \quad x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{3\pi}{2}, \dfrac{7\pi}{4}$



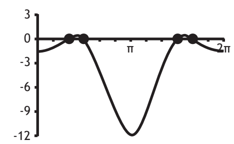
Example 17:

a) $2\sec^2 x - \tan^4 x + 1 = 0$, $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$



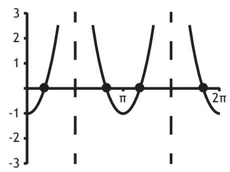
Note: All terms from the original equation were collected on the left side before graphing.

b) $2\cos^3 x - 7\cos^2 x + 3\cos x = 0$, $x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$

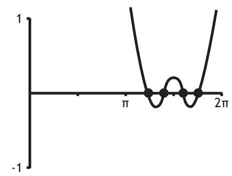


Note: All terms from the original equation were collected on the left side before graphing.

c) $\tan^2 x + 2\sec^2 x - 3 = 0$, $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

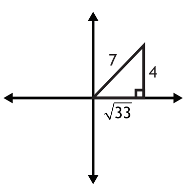


d) $4\sin^2 x + 2\sqrt{2}\sin x + 2\sqrt{3}\sin x + \sqrt{6} = 0$, $x = \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{4}$

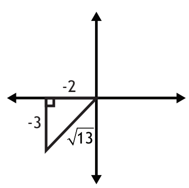


Example 18:

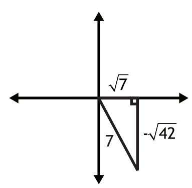
a) $\cos x = \frac{\sqrt{33}}{7}$



b) $\sec A = -\frac{\sqrt{13}}{2}$



c) $\sin\theta = -\frac{\sqrt{42}}{7}$



Example 19: See Video

Answer Key

Trigonometry Lesson Seven: Trigonometric Identities II

Note: n ε I for all general solutions.

Example 1:

a) $\frac{\sqrt{6}+\sqrt{2}}{4}$ **b)** $\frac{\sqrt{3}}{2}$ **c)** $\frac{\sqrt{2}+\sqrt{6}}{4}$ **d)** 0 **e)** $-\sqrt{3}-2$ **f)** $-\frac{\sqrt{3}}{3}$

Example 2:

a) $\sin\left(\frac{2\pi}{3}\right)$ **b)** $\tan\left(\frac{\pi}{12}\right)$ **c)** $\cos\left(\frac{\pi}{6}\right)$

Example 3:

a) $\frac{\sqrt{6}+\sqrt{2}}{4}$ **b)** $\frac{\sqrt{6}+\sqrt{2}}{4}$ **c)** $2-\sqrt{3}$ **d)** See Video

Example 4:

a) $\frac{4}{\sqrt{6}+\sqrt{2}}$ **b)** $\frac{4}{\sqrt{6}+\sqrt{2}}$ **c)** 1

Example 5: See Video

Example 6:

a) i. $\frac{\sqrt{3}}{2}$ ii. 0 iii. undefined

b) *(answers may vary)*

i. $\sin(8x)=2\sin(4x)\cos(4x)$

ii. $\cos(4x)=\cos^2(2x)-\sin^2(2x)$

iii. $\sin x=2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)$

iv. $\cos\left(\frac{1}{2}x\right)=1-2\sin^2\left(\frac{1}{4}x\right)$

c) *(answers may vary)*

i. $\cos(60^\circ)$

ii. $\frac{1}{2}\sin\left(\frac{\pi}{4}\right)$

iii. $\cos(x)$

iv. $\tan\left(\frac{1}{4}x\right)$

Examples 7 - 13: Proofs. See Video.

Example 14:

a) $x=0,\pi,2\pi$

b) $x=\frac{3\pi}{4},\frac{5\pi}{4}$

c) $x=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}$

d) $x=\frac{\pi}{3},\frac{5\pi}{3}$

Example 15:

a) $x=\frac{\pi}{6},\frac{5\pi}{6}$

b) $x=\frac{\pi}{6},\frac{\pi}{2},\frac{5\pi}{6},\frac{3\pi}{2}$

c) $x=\frac{\pi}{2}$

d) $x=\frac{\pi}{12},\frac{5\pi}{12},\frac{13\pi}{12},\frac{17\pi}{12}$

Example 16:

a) $x=0,\frac{2\pi}{3},\frac{4\pi}{3},2\pi$

b) $x=\frac{\pi}{3},\frac{5\pi}{3}$

c) $x=0,\pi,2\pi$

d) $x=\frac{\pi}{6},\frac{5\pi}{6}$

Example 17:

a) $x=\frac{\pi}{12},\frac{5\pi}{12},\frac{13\pi}{12},\frac{17\pi}{12}$

b) $x=0,\frac{\pi}{2},\pi,\frac{3\pi}{2},2\pi$

c) $x=\frac{3\pi}{2}$

d) $x=\frac{\pi}{2},\pi,\frac{3\pi}{2}$

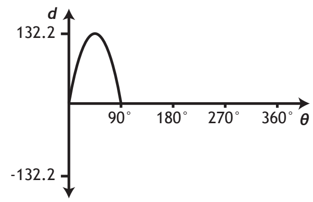
Example 18: 57°

Example 19: 92.9

Example 20:

a) $d(\theta)=\frac{1296}{9.8}\sin 2\theta$

b)



c) θ = 24.6° and θ = 65.4°

At 0°, the cannonball hits the ground as soon as it leaves the cannon, so the horizontal distance is 0 m.

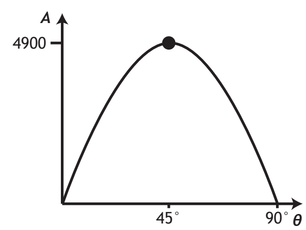
At 45°, the cannonball hits the ground at the maximum horizontal distance, 132.2 m.

At 90°, the cannonball goes straight up and down, landing on the cannon at a horizontal distance of 0 m

Example 21:

a) $A(\theta)=4900\sin(2\theta)$

b)

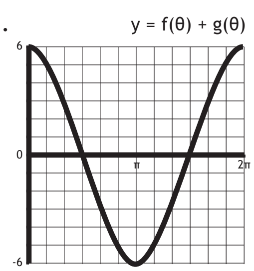


The maximum area occurs when θ = 45°. At this angle, the rectangle is the top half of a square.

c) i. $70\sqrt{2}$ m ii. $35\sqrt{2}$ m iii. $35\left(2-\sqrt{2}\right)$ m

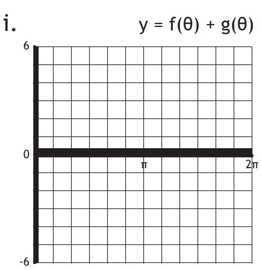
Example 22:

a) i.



ii. The waves experience constructive interference.
iii. The new sound will be louder than either original sound.

b) i.



ii. The waves experience destructive interference.
iii. The new sound will be quieter than either original sound.

c) All of the terms subtract out leaving y = 0,
A flat line indicating no wave activity.

Example 23: See Video.

Example 24: See Video.

Mathematics 30-1

Formula Sheet

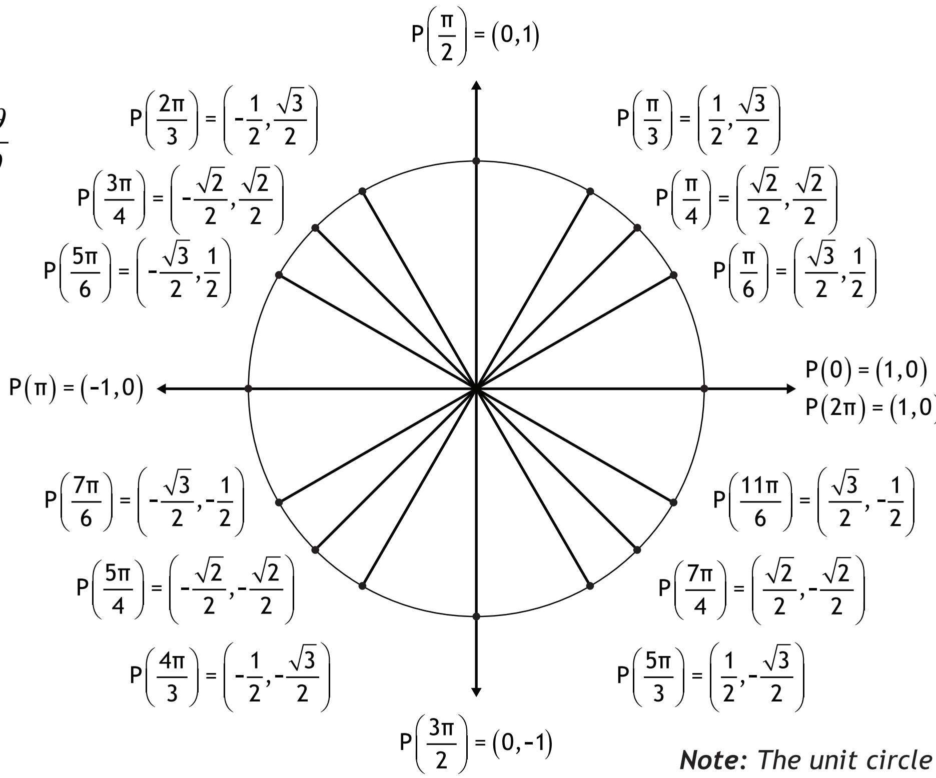
Trigonometry I

$\theta = \frac{a}{r}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$

The Unit Circle



***Note:** The unit circle is **NOT** included on the official formula sheet.*

Trigonometry II

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

$\sin(2A) = 2\sin A\cos A$

$\cos(2A) = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$

Transformations & Operations

$y = af\left[b(x-h)\right] + k$

Polynomial, Radical & Rational Functions

$x : \left[x_{\min}, x_{\max}, x_{\text{scl}}\right]$

$y : \left[y_{\min}, y_{\max}, y_{\text{scl}}\right]$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Exponential and Logarithmic Functions

$\log_b(M \times N) = \log_b M + \log_b N$

$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

$\log_b\left(M^n\right) = n\log_b M$

$\log_b c = \frac{\log_a c}{\log_a b}$

$y = ab^{\frac{t}{P}}$

Permutations & Combinations

$n! = n(n-1)(n-2)\ldots 3 \times 2 \times 1$

${}_nP_r = \frac{n!}{(n-r)!}$

${}_nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$

$t_{k+1} = {}_nC_k x^{n-k} y^k$