Mathematics 30-1

Student Workbook

Unit 6

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

Lesson 1: Permutations
Approximate Completion Time: 4 Days

$$ {}_nC_r = \frac{n!}{(n-r)!r!} $$

Lesson 2: Combinations
Approximate Completion Time: 4 Days

$$ t_{k+1} = {}_nC_k(x)^{n-k}(y)^k $$

Lesson 3: The Binomial Theorem
Approximate Completion Time: 2 Days

UNIT SIX

Permutations and Combinations

Mathematics 30-1

Student Workbook

Unit 6

Complete this workbook by watching the videos on **www.math30.ca**.
Work neatly and use proper mathematical form in your notes.

UNIT SIX

Permutations and Combinations

$$_nP_r = \frac{n!}{(n - r)!}$$

Permutations and Combinations

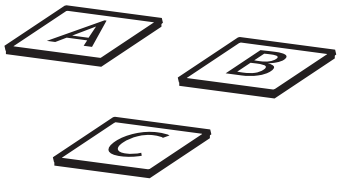
LESSON ONE - *Permutations*

Lesson Notes

Example 1 Introduction to Permutations.

Permutations

Three letters (A, B, and C) are taken from a set of letter tiles and arranged to form "words". In this question, *ACB* counts as a word - even though it's not an actual English word.



a) Use a tree diagram to find the number of unique words.

b) Use the Fundamental Counting Principle to find the number of unique words.

c) Use permutation notation to find the number of unique words. Evaluate using a calculator.

d) What is meant by the terms ***single-case permutation*** and ***multi-case permutation***?

e) Use permutations to find the number of ways a one-, two-, or three-letter word can be formed.

Permutations and Combinations
LESSON ONE - *Permutations*
Lesson Notes

$${}_nP_r = \frac{n!}{(n - r)!}$$

Example 2 Evaluate each of the following factorial expressions.

Factorial Notation

a) 4!

b) 1!

c) 0!

d) (-2)!

e) $\frac{5!}{3!}$

f) $\frac{8!}{7! \bullet 2!}$

g) $\frac{n!}{(n - 2)!}$

h) $\frac{(n + 1)!}{(n - 1)!}$

$$_nP_r = \frac{n!}{(n-r)!}$$

Permutations and Combinations

LESSON ONE - *Permutations*

Lesson Notes

Example 3

Permutations with Repetitions NOT Allowed. *(Finite Sample Sets)*

Simple Permutations

Single-Case Permutations

a) A Grade 12 student is taking Biology, English, Math, and Physics in her first term. If a student timetable has room for five courses *(meaning the student has a spare)*, how many ways can she schedule her courses?

i) Fundamental Counting Principle **ii) Permutation Notation**

One Possible Timetable

Block	*Course*
Block 1	Math 30-1
Block 2	**Spare**
Block 3	Physics 30
Block 4	English 30-1
Block 5	Biology 30

b) A singing competition has three rounds. In each round, the singer has to perform one song from a particular genre. How many different ways can the performer select the genres?

Round 1	Round 2	Round 3
Rock Metal Punk Alternative	Pop Dance	Country Blues Folk

i) Fundamental Counting Principle **ii) Permutation Notation**

c) A web development team of three members is to be formed from a selection pool of 10 people. The team members will be assigned roles of programmer, graphic designer, and database analyst. How many unique teams are possible? You can assume that each person in the selection pool is capable of performing each task.

i) Fundamental Counting Principle **ii) Permutation Notation**

d) There are 13 letter tiles in a bag, and no letter is repeated. Using all of the letters from the bag, a six-letter word, a five-letter word, and a two-letter word are made. How many ways can this be done?

i) Fundamental Counting Principle **ii) Permutation Notation**

Permutations and Combinations
LESSON ONE - *Permutations*
Lesson Notes

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

Example 4 | Permutations with Repetitions NOT Allowed. *(Finite Sample Sets)* | Repetitions NOT Allowed | Single-Case Permutations

a) How many ways can the letters in the word SEE be arranged?

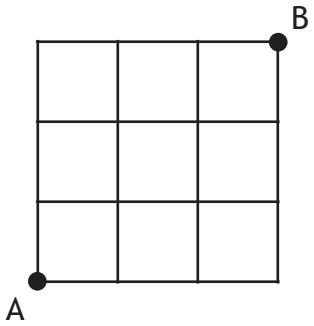
i) Tree Diagram

ii) Fundamental Counting Principle

b) How many ways can the letters in the word MISSISSAUGA be arranged?

c) A multiple-choice test has 10 questions. Three questions have an answer of A, four questions have an answer of B, one question has an answer of C, and two questions have an answer of D. How many unique answer keys are possible?

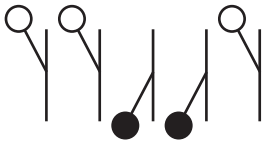
d) How many pathways exist from point A to point B if the only directions allowed are north and east?



e) How many ways can three cars *(red, green, blue)* be parked in five parking stalls?

f) An electrical panel has five switches. How many ways can the switches be positioned up or down if three switches must be up and two must be down?

One possible switch arrangement.



$$_nP_r = \frac{n!}{(n-r)!}$$

Permutations and Combinations

LESSON ONE - *Permutations*

Lesson Notes

Example 5

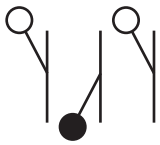
Permutations where Repetitions ARE Allowed. *(Infinite Sample Sets)*

Repetitions ARE Allowed

Single-Case Permutations

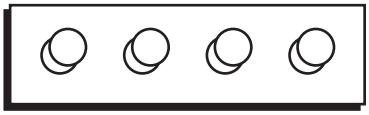
a) There are three switches on an electrical panel. How many unique up/down sequences are there?

One possible switch arrangement.



b) How many two-letter "words" can be created using the letters A, B, C, and D?

c) A coat hanger has four knobs, and each knob can be painted any color. If six different colors of paint are available, how many ways can the knobs be painted?



d) A phone number in British Columbia consists of one of four area codes (236, 250, 604, and 778), followed by a 7-digit number that cannot begin with a 0 or 1.
How many unique phone numbers are there?

e) An identification code consists of any two letters followed by any three digits.
How many identification codes can be created?

Permutations and Combinations
LESSON ONE - *Permutations*
Lesson Notes

$$_nP_r = \frac{n!}{(n - r)!}$$

Example 6

Permutations with Repetitions NOT Allowed. *(Finite Sample Sets)*

Constraints and Line Formations

Single-Case Permutations

Six people (Andrew, Brenda, Cory, Danielle, Eliza, Frank) are going to be seated in a line. How many unique lines can be formed if:

a) Frank must be seated in the third chair?

___ ___ ___ ___ ___ ___

b) Brenda or Cory must be in the second chair, and Eliza must be in the third chair?

___ ___ ___ ___ ___ ___

c) Danielle can't be at either end of the line?

___ ___ ___ ___ ___ ___

d) men and women alternate positions, with a woman sitting in the first chair?

___ ___ ___ ___ ___ ___

e) the line starts with the pattern man-man-woman?

___ ___ ___ ___ ___ ___

$$_nP_r = \frac{n!}{(n - r)!}$$

Permutations and Combinations

LESSON ONE - *Permutations*

Lesson Notes

Example 7

Permutations with Repetitions NOT Allowed. *(Finite Sample Sets)*

Constraints and Words

Single-Case Permutations

How many ways can you order the letters from the word TREES if:

a) a vowel must be at the beginning?

___ ___ ___ ___ ___

b) it must start with a consonant and end with a vowel?

___ ___ ___ ___ ___

c) the R must be in the middle?

___ ___ ___ ___ ___

d) it begins with exactly one E?

___ ___ ___ ___ ___

e) it ends with TR?

___ ___ ___ ___ ___

f) consonants and vowels alternate?

___ ___ ___ ___ ___

Permutations and Combinations
LESSON ONE - *Permutations*
Lesson Notes

$$_nP_r = \frac{n!}{(n - r)!}$$

Example 8

Permutations with Repetitions NOT Allowed. *(Finite Sample Sets)*

Objects ALWAYS Together

Single-Case Permutations

a) How many ways can 3 chemistry books, 4 math books, and 5 physics books be arranged if books on each subject must be kept together?

b) How many arrangements of the word ACTIVE are there if C&E must always be together?

c) How many arrangements of the word ACTIVE are there if C&E must always be together, and in the order CE?

d) Six people (Andrew, Brenda, Cory, Danielle, Eliza, Frank) are going to be seated in a line. How many unique lines can be formed if Cory, Danielle, and Frank must be seated together?

$$_nP_r = \frac{n!}{(n - r)!}$$

Permutations and Combinations

LESSON ONE - *Permutations*

Lesson Notes

Example 9

Permutations with Repetitions NOT Allowed. *(Finite Sample Sets)*

Objects NEVER Together

Single-Case Permutations

a) How many ways can the letters in QUEST be arranged if the vowels must never be together?

i) Use a shortcut that works for separating two items.

ii) Use a general method.

b) Eight cars (3 red, 3 blue, and 2 yellow) are to be parked in a line. How many unique lines can be formed if the yellow cars must not be together? Assume that cars of each color are identical.

i) Use a shortcut that works for separating two items.

ii) Use a general method.

c) How many ways can the letters in READING be arranged if the vowels must never be together?

Permutations and Combinations
LESSON ONE - *Permutations*
Lesson Notes

$$_nP_r = \frac{n!}{(n - r)!}$$

Example 10 More Than One Case. (At Least/At Most)

Multi-Case Permutations

a) How many words (with at most three letters) can be formed from the letter tiles SUNDAY?

b) How many words (with at least five letters) can be formed from the letter tiles SUNDAY?

c) How many 3-digit odd numbers greater than 600 can be formed using the digits 2, 3, 4, 5, 6, and 7, if a number contains no repeating digits?

d) Six vehicles (3 different brands of cars and 3 different brands of trucks) are going to be parked in a line. How many unique lines can be formed if the row starts with at least two trucks?

e) Six vehicles (3 different brands of cars and 3 different brands of trucks) are going to be parked in a line. How many unique lines can be formed if trucks and cars alternate positions?

$${}_nP_r = \frac{n!}{(n - r)!}$$

Permutations and Combinations

LESSON ONE - *Permutations*

Lesson Notes

Example 11

Permutation Formula:
Solve each of the following without using a calculator.

$${}_nP_r = \frac{n!}{(n - r)!}$$

a) Evaluate ${}_4P_3$

b) Evaluate ${}_{12}P_3$

c) Write $\frac{5!}{3!}$ as a permutation.

d) Write 3! as a permutation.

Permutations and Combinations
LESSON ONE - *Permutations*
Lesson Notes

$${}_nP_r = \frac{n!}{(n - r)!}$$

Example 12 Equations with Factorials and Permutations.
Solve each of the following without using a calculator.

$${}_nP_r = \frac{n!}{(n - r)!}$$

a) $\frac{n!}{(n - 2)!} = 5n$

b) $(n + 2)! = 12n!$

c) $\frac{n!}{10} = {}_{n-1}P_{n-3}$

d) $\frac{(2n + 1)!}{(2n - 1)!} = 4n + 2$

$${}_nP_r = \frac{n!}{(n - r)!}$$

Permutations and Combinations

LESSON ONE - *Permutations*

Lesson Notes

Example 13 Equations with Factorials and Permutations.
Solve each of the following without using a calculator.

$${}_nP_r = \frac{n!}{(n - r)!}$$

a) ${}_nP_2 = 56$

b) ${}_6P_r = 120$

c) ${}_{n+3}P_2 = 20$

d) ${}_{n-3}P_1 = 2 \bullet {}_{n-4}P_1$

$$_nP_r = \frac{n!}{(n - r)!}$$

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$$_nC_r = \frac{n!}{(n - r)!r!}$$

Permutations and Combinations

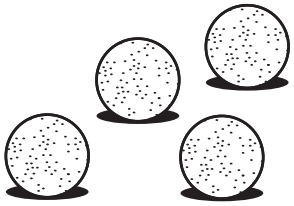
LESSON TWO - *Combinations*

Lesson Notes

Example 1 Introduction to Combinations.

Combinations

There are four marbles on a table, and each marble is a different color (red, green, blue, and yellow). Two marbles are selected from the table at random and put in a bag.



a) Is the order of the marbles, or the order of their colors, important?

b) Use a tree diagram to find the number of unique color combinations for the two marbles.

c) Use combination notation to find the number of unique color combinations.

d) What is meant by the terms ***single-case combination*** and ***multi-case combination?***

e) How many ways can three or four marbles be chosen?

Permutations and Combinations
LESSON TWO - *Combinations*
Lesson Notes

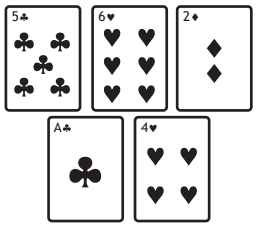
$$ {}_nC_r = \frac{n!}{(n-r)!r!} $$

Example 2 | Combinations with Repetitions NOT Allowed. *(Finite Sample Sets)* | Sample Sets with NO Subdivisions | Single-Case Combinations

a) There are five toppings available for a pizza *(mushrooms, onions, pineapple, spinach, and tomatoes)*. If a pizza is ordered with three toppings, and no topping may be repeated, how many different pizzas can be created?

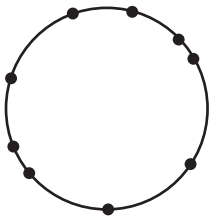
b) A committee of 4 people is to be formed from a selection pool of 9 people. How many possible committees can be formed?

c) How many 5-card hands can be made from a standard deck of 52 cards?



d) There are 9 dots randomly placed on a circle.
i) How many lines can be formed within the circle by connecting two dots?

ii) How many triangles can be formed within the circle?



$$_nC_r = \frac{n!}{(n-r)!r!}$$

Permutations and Combinations

LESSON TWO - *Combinations*

Lesson Notes

Example 3 | Combinations with Repetitions NOT Allowed. *(Finite Sample Sets)* | Sample Sets with Subdivisions | Single-Case Combinations

a) How many 6-person committees can be formed from 11 men and 9 women if 3 men and 3 women must be on the committee?

b) A crate of toy cars contains 10 working cars and 4 defective cars. How many ways can 5 cars be selected if only 3 work?

c) From a deck of 52 cards, a 6-card hand is dealt. How many distinct hands are there if the hand must contain 2 spades and 3 diamonds?

d) A bouquet contains four types of flowers:

Flower Type	Examples
Focal Flowers: Large and eye-catching flowers that draw attention to one area of the bouquet.	Roses, Peonies, Hydrangeas, Chrysanthemums, Tulips, and Lilies
Fragrant Flowers: Flowers that add a pleasant fragrance to the bouquet.	Petunia, Daffodils, Daphnes, Gardenia, Lilacs, Violets, Magnolias
Line Flowers: Tall and narrow flowers used to establish the height of a floral bouquet.	Delphiniums, Snapdragons, Bells of Ireland, Gladioli, and Liatris
Filler Flowers: Unobtrusive flowers that give depth to the bouquet.	Daisies, Baby's Breath, Wax Flowers, Solidago, and Caspia

A florist is making a bouquet that uses one type of focal flower, no fragrant flowers, three types of line flowers and all of the filler flowers. How many different bouquets can be made?

Permutations and Combinations
LESSON TWO - *Combinations*
Lesson Notes

$$_nC_r = \frac{n!}{(n - r)!r!}$$

Example 4 | Combinations with Repetitions NOT Allowed. *(Finite Sample Sets)* | More Sample Sets with Subdivisions | Single-Case Combinations

a) A committee of 5 people is to be formed from a selection pool of 12 people.
If Carmen must be on the committee, how many unique committees can be formed?

b) A committee of 6 people is to be formed from a selection pool of 11 people.
If Grant and Helen must be on the committee, but Aaron must not be on the committee, how many unique committees can be formed?

c) Nine students are split into three equal-sized groups to work on a collaborative assignment.
How many ways can this be done? Does the sample set need to be subdivided in this question?

d) From a deck of 52 cards, a 5-card hand is dealt. How many distinct 5-card hands are there if the ace of spades and two of diamonds must be in the hand?

e) A lottery ticket has 6 numbers from 1-49. Duplicate numbers are not allowed, and the order of the numbers does not matter. How many different lottery tickets contain the numbers 12, 24 and 48, but exclude the numbers 30 and 40?

$$_nC_r = \frac{n!}{(n - r)!r!}$$

Permutations and Combinations

LESSON TWO - *Combinations*

Lesson Notes

Example 5

Combinations with Repetitions NOT Allowed. *(Finite Sample Sets)*

Permutations and Combinations Together

Single-Case Combinations

a) How many five-letter words using letters from TRIANGLE can be made if the five-letter word must have two vowels and three consonants?

b) There are 4 men and 5 women on a committee selection pool. A three-person committee consisting of President, Vice-President, and Treasurer is being formed. How many ways can exactly two men be on the committee?

c) A music teacher is organizing a concert for her students. If there are six piano students and seven violin students, how many different concert programs are possible if four piano students and three violin students perform in an alternating arrangement?

Permutations and Combinations
LESSON TWO - *Combinations*
Lesson Notes

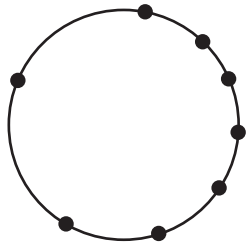
$$_nC_r = \frac{n!}{(n-r)!r!}$$

Example 6 | Combinations with Repetitions NOT Allowed. *(Finite Sample Sets)* | Handshakes, Teams, and Shapes. | Single-Case Combinations

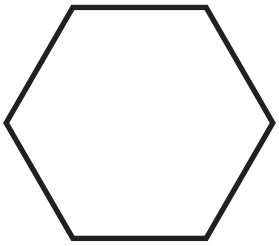
a) Twelve people at a party shake hands once with everyone else in the room. How many handshakes took place?

b) If each of the 8 teams in a league must play each other three times, how many games will be played? *(Note: This is a multi-case combination)*

c) If there are 8 dots on a circle, how many quadrilaterals can be formed?



d) A polygon has 6 sides. How many diagonals can be formed?



$$ {}_nC_r = \frac{n!}{(n - r)!r!} $$

Permutations and Combinations

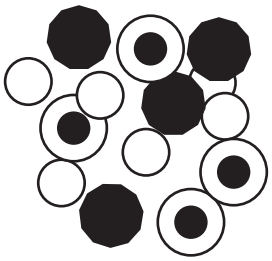
LESSON TWO - *Combinations*

Lesson Notes

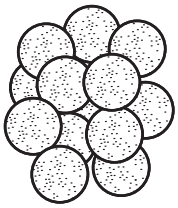
Example 7 Combinations where Repetitions ARE Allowed. *(Infinite Sample Sets)*

Single-Case Combinations

a) A jar contains quarters, loonies, and toonies. If four coins are selected from the jar, how many unique coin combinations are there?



b) A bag contains marbles with four different colors (red, green, blue, and yellow). If three marbles are selected from the bag, how many unique color combinations are there?



Permutations and Combinations
LESSON TWO - *Combinations*
Lesson Notes

$$_nC_r = \frac{n!}{(n - r)!r!}$$

Example 8 More Than One Case (At Least/At Most).

Multi-Case Combinations

a) A committee of 5 people is to be formed from a group of 4 men and 5 women. How many committees can be formed if at least 3 women are on the committee?

b) From a deck of 52 cards, a 5-card hand is dealt. How many distinct hands can be formed if there are at most 2 queens?

c) From a deck of 52 cards, a 5-card hand is dealt. How many distinct hands can be formed if there is at least 1 red card?

d) A research team of 5 people is to be formed from 3 biologists, 5 chemists, 4 engineers, and 2 programmers. How many teams have exactly one chemist and at least 2 engineers?

e) In how many ways can you choose one or more of 5 different candies?

Permutations and Combinations
LESSON TWO - *Combinations*
Lesson Notes

$$ {}_nC_r = \frac{n!}{(n-r)!r!} $$

Example 9 Combination Formula.

$$ {}_nC_r = \frac{n!}{(n-r)!r!} $$

a) Evaluate ${}_7C_5$

b) Evaluate ${}_3C_3$

c) Evaluate $\binom{4}{2}$

d) Write $\frac{6!}{4!2!}$ as a combination.

e) Write $\frac{5!}{4!}$ as a combination.

$${}_nC_r = \frac{n!}{(n - r)!r!}$$

Permutations and Combinations

LESSON TWO - *Combinations*

Lesson Notes

Example 10

Combination Formula.
Solve for the unknown algebraically.

$${}_nC_r = \frac{n!}{(n - r)!r!}$$

a) ${}_nC_2 = 21$

b) ${}_4C_r = 6$

c) $\binom{n}{3} = 10$

d) $\binom{n}{n-2} = 15$

Permutations and Combinations
LESSON TWO - *Combinations*
Lesson Notes

$$ {}_nC_r = \frac{n!}{(n - r)!r!} $$

Example 11

Combination Formula.
Solve for the unknown algebraically.

$$ {}_nC_r = \frac{n!}{(n - r)!r!} $$

a) $\dfrac{{}_nC_4}{{}_{n-2}C_2} = 1$

b) $\dfrac{{}_nC_r}{{}_nC_{n-r}} = 1$

c) ${}_{n-1}P_3 = 2 \times {}_{n-1}C_2$

d) ${}_{n+1}C_2 = \dfrac{1}{2} \times {}_{n+2}C_3$

$$_nC_r = \frac{n!}{(n-r)!r!}$$

Permutations and Combinations

LESSON TWO - *Combinations*

Lesson Notes

Example 12 Assorted Mix I

Assorted Mix I

a) A six-character code has the pattern shown below, and the same letter or digit may be used more than once. How many unique codes can be created?

Letter	Digit	Digit	Digit	Digit	Letter

STRATEGY: Organize your thoughts with these guiding questions:

1) Permutation or Combination?
2) Single-Case or Multi-Case?
3) Are repetitions allowed?
4) What is the sample set?
Are there subdivisions?
5) Are there any tricks or shortcuts?

b) If there are 2 different parkas, 5 different scarves, and 4 different tuques, how many winter outfits can be made if an outfit consists of one type of each garment?

c) If a 5-card hand is dealt from a deck of 52 cards, how many hands have at most one diamond?

d) If there are three cars and four motorcycles, how many ways can the vehicles park in a line such that cars and motorcycles alternate positions?

$$ {}_nC_r = \frac{n!}{(n - r)!r!} $$

e) Show that ${}_nC_r = {}_nC_{n-r}$.

f) There are nine people participating in a raffle. Three $50 gift cards from the same store are to be given out as prizes. How many ways can the gift cards be awarded?

g) There are nine competitors in an Olympic event. How many ways can the bronze, silver, and gold medals be awarded?

h) A stir-fry dish comes with a base of rice and the choice of five toppings: broccoli, carrots, eggplant, mushrooms, and tofu. How many different stir-fry dishes can be prepared if the customer can choose zero or more toppings?

$$_nC_r = \frac{n!}{(n - r)!r!}$$

Permutations and Combinations

LESSON TWO - *Combinations*

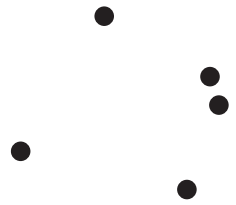
Lesson Notes

Example 13 Assorted Mix II

Assorted Mix II

a) A set of tiles contains eight letters, A - H. If two of these sets are combined, how many ways can all the tiles be arranged? Leave your answer as an exact value.

b) A pattern has five dots such that no three points are collinear.
How many lines can be drawn if each dot is connected to every other dot?



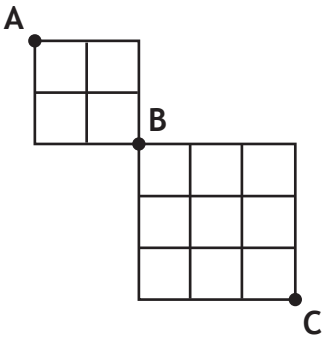
c) How many ways can the letters in CALGARY be arranged if L and G must be separated?

d) A five-person committee is to be formed from 11 people. If Ron and Sara must be included, but Tracy must be excluded due to a conflict of interest, how many committees can be formed?

Permutations and Combinations
LESSON TWO - *Combinations*
Lesson Notes

$$_nC_r = \frac{n!}{(n - r)!r!}$$

e) Moving only south and east, how many unique pathways connect points A and C?



f) How many ways can the letters in SASKATOON be arranged if the letters K and T must be kept together, and in that order?

g) A 5-card hand is dealt from a deck of 52 cards. How many hands are possible containing at least three hearts?

h) A healthy snack contains an assortment of four vegetables. How many ways can one or more of the vegetables be selected for eating?

$$_nC_r = \frac{n!}{(n - r)!r!}$$

Permutations and Combinations

LESSON TWO - *Combinations*

Lesson Notes

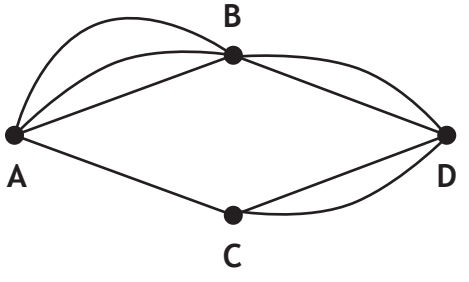
Example 14 Assorted Mix III

Assorted Mix III

a) How many ways can the letters in EDMONTON be arranged if repetitions are not allowed?

b) A bookshelf has *n* fiction books and six non-fiction books. If there are 150 ways to choose two books of each type, how many fiction books are on the bookshelf?

c) How many different pathways exist between points A and D?



d) How many numbers less than 60 can be made using only the digits 1, 5, and 8, if the numbers formed may contain repeated digits?

Permutations and Combinations
LESSON TWO - *Combinations*
Lesson Notes

$$_nC_r = \frac{n!}{(n - r)!r!}$$

e) A particular college in Alberta has a list of approved pre-requisite courses:

Math	Science	English	Other
Math 30-1 or Math 30-2	Biology 30 Chemistry 30 Physics 30	English 30-1	Option A Option B Option C Option D Option E

Five courses are required for admission to the college. Math 30-1 (or Math 30-2) and English 30-1 are mandatory requirements, and at least one science course must be selected as well.
How many different ways could a student select five courses on their college application form?

f) How many ways can four bottles of different spices be arranged on a spice rack with holes for six spice bottles?

g) If there are 8 rock songs and 9 pop songs available, how many unique playlists containing 3 rock songs and 2 pop songs are possible?

h) A hockey team roster contains 12 forwards, 6 defencemen, and 2 goalies. During play, only six players are allowed on the ice - 3 forwards, 2 defencemen, and 1 goalie. How many different ways can the active players be selected?

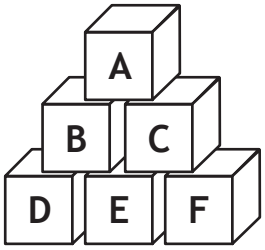
Example 15

Assorted Mix IV

Assorted Mix IV

a) A fruit mix contains blueberries, grapes, mango slices, pineapple slices, and strawberries. If six pieces of fruit are selected from the fruit mix and put on a plate, how many ways can this be done?

b) How many ways can six letter blocks be arranged in a pyramid, if all of the blocks are used?



c) If a 5-card hand is dealt from a deck of 52 cards, how many hands have cards that are all the same color?

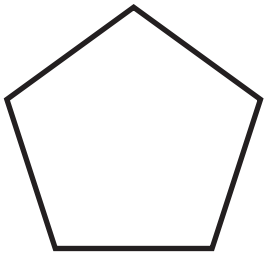
d) If a 5-card hand is dealt from a deck of 52 cards, how many hands have cards that are all the same suit?

Permutations and Combinations
LESSON TWO - *Combinations*
Lesson Notes

$$_nC_r = \frac{n!}{(n - r)!r!}$$

e) A multiple choice test contains 5 questions, and each question has four possible responses. How many different answer keys are possible?

f) How many diagonals are there in a pentagon?



g) How many ways can eight books, each covering a different subject, be arranged on a shelf such that books on biology, history, or programming are never together?

h) If a 5-card hand is dealt from a deck of 52 cards, how many hands have two pairs?

$$_nC_r = \frac{n!}{(n - r)!r!}$$

Permutations and Combinations

LESSON TWO - *Combinations*

Lesson Notes

Example 16 Assorted Mix V

Assorted Mix V

a) How many ways can six people be split into two equal-sized groups?

b) Show that 25! + 26! = 27 × 25!

c) Five different types of fruit and six different types of vegetables are available for a healthy snack tray. The snack tray is to contain two fruits and three vegetables. How many different snack trays can be made if blueberries or carrots must be served, but not both together?

d) In genetics, a codon is a sequence of three letters that specifies a particular amino acid. A fragment of a particular protein yields the amino acid sequence:

Met - Gly - Ser - Arg - Cys - Gly.

How many unique codon arrangements could yield this amino acid sequence?

Amino Acid	Codon(s)
Arginine (Arg)	CGU, CGC, CGA, CGG, AGA, AGG
Cysteine (Cys)	UGU, UGC
Glycine (Gly)	GGU, GGC, GGA, GGG
Methionine (Met)	AUG
Serine (Ser)	UCU, UCC, UCA, UCG, AGU, AGC

Permutations and Combinations
LESSON TWO - *Combinations*
Lesson Notes

$$ {}_nC_r = \frac{n!}{(n - r)!r!} $$

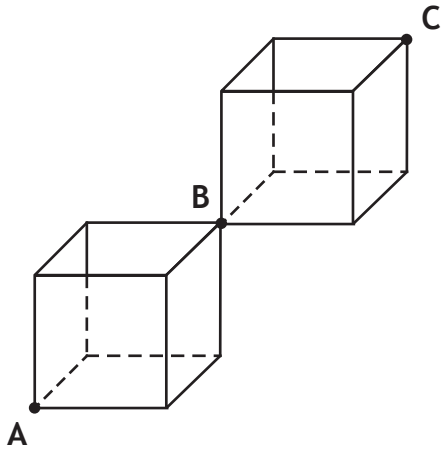
e) In a tournament, each player plays every other player twice. If there are 56 games, how many people are in the tournament?

f) The discount shelf in a bookstore has a variety of books on computers, history, music, and travel. The bookstore is running a promotion where any five books from the discount shelf can be purchased for $20. How many ways can five books be purchased?

g) Show that ${}_nC_r + {}_nC_{r+1} = {}_{n+1}C_{r+1}$.

***Note**: This question will require more paper than is provided on this page.*

h) How many pathways are there from point A to point C, passing through point B? Each step of the pathway must be getting closer to point C.



$$t_{k+1} = {}_nC_k(x)^{n-k}(y)^k$$

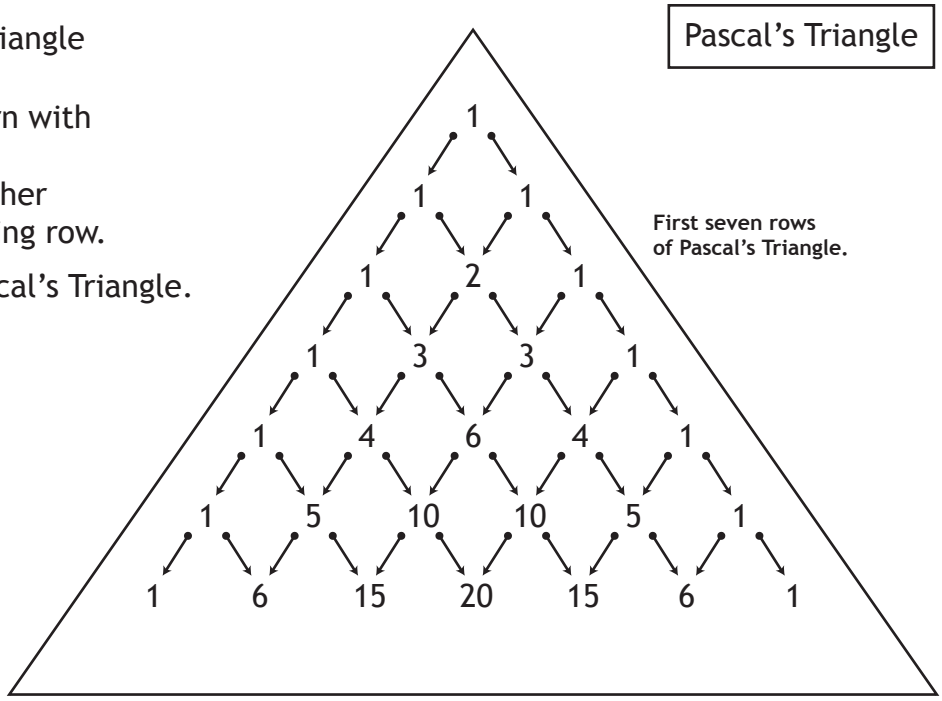
Permutations and Combinations
LESSON THREE - *The Binomial Theorem*
Lesson Notes

Example 1 Pascal's Triangle

Pascal's Triangle is a number pattern with useful applications in mathematics. Each row is formed by adding together adjacent numbers from the preceding row.

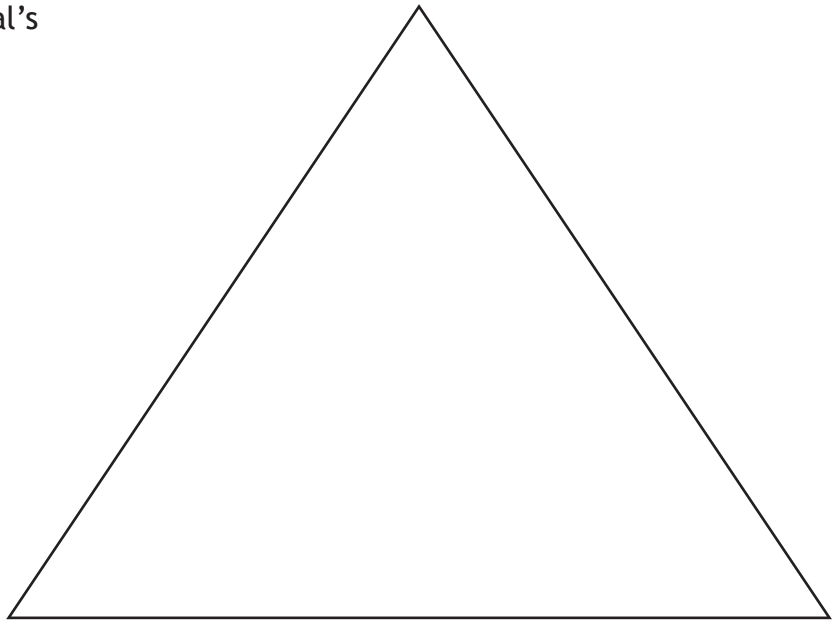
a) Determine the eighth row of Pascal's Triangle.

Pascal's Triangle



First seven rows of Pascal's Triangle.

b) Rewrite the first seven rows of Pascal's Triangle, but use combination notation instead of numbers.



c) Using the triangles from parts (a & b) as a reference, explain what is meant by ${}_nC_k = {}_nC_{n-k}$.

Permutations and Combinations
LESSON THREE - *The Binomial Theorem*
Lesson Notes

$$t_{k+1} = {}_nC_k(x)^{n-k}(y)^k$$

Example 2 Rows and Terms of Pascal's Triangle.

Pascal's Triangle

a) Given the following rows from Pascal's Triangle, write the circled number as a combination.

i) 1 8 28 56 70 56 28 8 1

ii) 1 12 66 220 495 792 924 792 495 220 66 12 1

b) Use a combination to find the third term in row 22 of Pascal's Triangle.

c) Which positions in the 12th row of Pascal's Triangle have a value of 165?

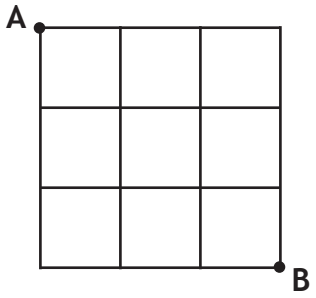
d) Find the sum of the numbers in each of the first four rows of Pascal's Triangle. Use your result to derive a function, S(n), for the sum of all numbers in the nth row of Pascal's Triangle. What is the sum of all numbers in the eleventh row?

Example 3

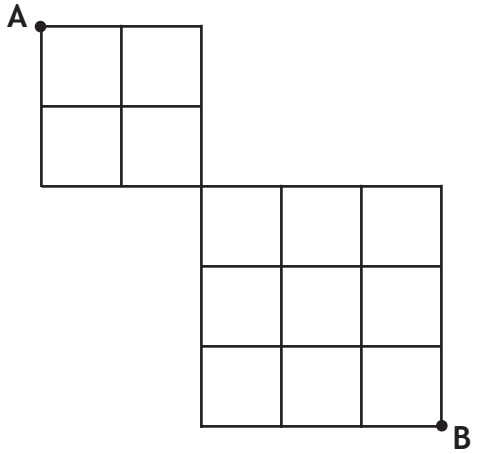
Use Pascal's Triangle to determine the number of paths from point A to point B if east and south are the only possible directions.

Pascal's Triangle and Pathways

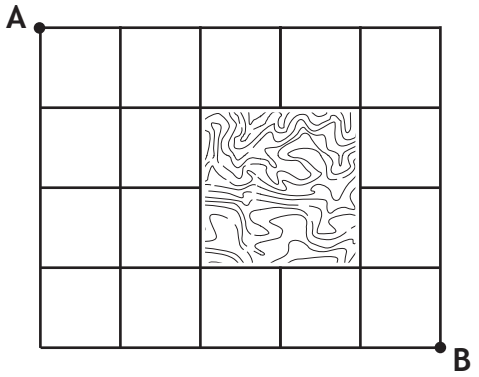
a)



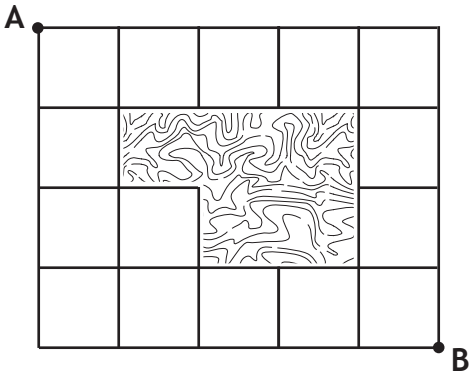
b)



c)



d)



Permutations and Combinations
LESSON THREE - *The Binomial Theorem*
Lesson Notes

$$t_{k+1} = {}_nC_k(x)^{n-k}(y)^k$$

Example 4 The Binomial Theorem.

The Binomial Theorem

a) Define the binomial theorem and explain how it is used to expand $(x + 1)^3$.

Expand the expressions in parts (b) and (c) using the binomial theorem.

b) $(x + 2)^6$

c) $(2x - 3)^4$

Example 5 Expand each expression.

The Binomial Theorem

a) $(x^2 - 2y)^4$

b) $\left(3x^2 - \frac{1}{2}\right)^4$

c) $\left(2x^3 - \frac{3}{x}\right)^5$

Permutations and Combinations
LESSON THREE - *The Binomial Theorem*
Lesson Notes

$$t_{k+1} = {}_nC_k(x)^{n-k}(y)^k$$

Example 6 Write each expression as a binomial power.

Condense to a Binomial

a) $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

b) $32a^5 - 240a^4b + 720a^3b^2 - 1080a^2b^3 + 810ab^4 - 243b^5$

c) $27a^3 - \frac{27a^2}{4} + \frac{9a}{16} - \frac{1}{64}$

$$t_{k+1} = {}_nC_k(x)^{n-k}(y)^k$$

Permutations and Combinations
LESSON THREE - *The Binomial Theorem*
Lesson Notes

General Term

$$t_{k+1} = {}_nC_k(x)^{n-k}(y)^k$$

Example 7 Use the general term formula to find the requested term in a binomial expansion.

a) Find the third term in the expansion of $(x - 3)^4$.

b) Find the fifth term in the expansion of $(3a^3 - 2b^2)^8$.

c) Find the fourth term in the expansion of $\left(x^2 - \frac{1}{x}\right)^6$.

$$t_{k+1} = {}_nC_k(x)^{n-k}(y)^k$$

Example 8 Answer each of the following questions.

Finding Specific Values

a) In the expansion of $(5a - 2b)^9$, what is the coefficient of the term containing a^5?

b) In the expansion of $(4a^3 + 3b^3)^5$, what is the coefficient of the term containing b^{12}?

c) In the expansion of $(3a - 4)^8$, what is the middle term?

d) If there are 23 terms are in the expansion of $(a - 2)^{3k-5}$, what is the value of *k*?

Example 9 Answer each of the following questions.

Finding Specific Values

a) A term in the expansion of $(ma - 4)^5$ is $-5760a^2$. What is the value of m?

b) The term $-1080a^2b^3$ occurs in the expansion of $(2a - 3b)^n$. What is the value of n?

c) A term in the expansion of $(a + m)^7$ is $21504\frac{a^5}{b^4}$. What is the value of m.

$$t_{k+1} = {}_nC_k(x)^{n-k}(y)^k$$

Example 10 Answer each of the following questions.

Finding Specific Values

a) In the expansion of $\left(2x^2 - \frac{1}{x}\right)^6$, what is the constant term?

b) In the expansion of $\left(\frac{3}{x^2} - x^8\right)^{10}$, what is the constant term?

c) In the expansion of $\left(\sqrt{x} + b\right)^6$, one of the terms is $240x^2$. What is the value of b?

Permutations and Combinations Lesson One: Permutations

Example 1:
a) Six words can be formed. **b)** 3 × 2 × 1 = 6 **c)** ${}_3P_3$
d) See Video **e)** ${}_3P_1 + {}_3P_2 + {}_3P_3$
Example 2: a) 24 **b)** 1 **c)** 1 **d)** (-2)! Does not exist.
e) 20 **f)** 4 **g)** $n^2 - n$ **h)** $n^2 + n$
Example 3: a) 120 **b)** 24 **c)** 720 **d)** 13!
Example 4: a) 3 **b)** 415800 **c)** 12600 **d)** 20 **e)** 60 **f)** 10
Example 5: a) 8 **b)** 16 **c)** 1296 **d)** 32×10^6 **e)** 676 000
Example 6: a) 120 **b)** 48 **c)** 480 **d)** 108
Example 7: a) 24 **b)** 18 **c)** 12 **d)** 18 **e)** 3 **f)** 6
Example 8: a) 103 680 **b)** 240 **c)** 120 **d)** 144
Example 9: a) 72 **b)** 420 **c)** 1440
Example 10: a) 156 **b)** 1440 **c)** 20 **d)** 144 **e)** 72
Example 11: a) 24 **b)** 1320 **c)** ${}_5P_2$ **d)** ${}_3P_2$ or ${}_3P_3$
Example 12: a) n = 6 **b)** n = 2 **c)** n = 5 **d)** n = 1
Example 13: a) n = 8 **b)** r = 3 **c)** n = 2 **d)** n = 5

Permutations and Combinations Lesson Two: Combinations

Example 1:a) The order of the colors is not important.
b) 6 **c)** ${}_4C_2$ **d)** See Video **e)** ${}_4C_3 + {}_4C_4$
Example 2: a) 10 **b)** 126 **c)** 2598960 **d)** 36; 84
Example 3: a) 13860 **b)** 720 **c)** 580008 **d)** 60
Example 4: a) 330 **b)** 70 **c)** 1680 **d)** 19600 **e)** 13244
Example 5: a) 3600 **b)** 180 **c)** 75600
Example 6: a) 66 **b)** 84 **c)** 70 **d)** 9
Example 7: a) 15 **b)** 20
Example 8: a) 81 **b)** 2594400 **c)** 2533180 **d)** 405 **e)** 31
Example 9: a) 21 **b)** 1 **c)** 6 **d)** ${}_6C_2$ **e)** ${}_5C_1$
Example 10: a) n = 7 **b)** ${}_4C_2$ **c)** n = 5 **d)** n = 6
Example 11: a) n = 4 **b)** All n-values **c)** n = 4 **d)** n = 4
Example 12: a) 6760000 **b)** 40 **c)** 1645020 **d)** 144
e) See Video **f)** 84 **g)** 504 **h)** 32
Example 13: a) $16!/(2!)^8$ **b)** 10 **c)** 1800 **d)** 56
e) 120 **f)** 5040 **g)** 241098 **h)** 15
Example 14: a) 10080 **b)** 5 **c)** 8 **d)** 9
e) 92 **f)** 360 **g)** 241920 **h)** 6600
Example 15: a) 210 **b)** 720 **c)** 5148 **d)** 131560
e) 1024 **f)** 5 **g)** 14400 **h)** 123552
Example 16: a) 20 **b)** See Video **c)** 100 **d)** 1152
e) n = 8 **f)** 56 **g)** See Video **h)** 36

Permutations and Combinations Lesson Three: The Binomial Theorem

Example 1:
a) The eighth row of Pascal's Triangle is: 1, 7, 21, 35, 35, 21, 7, 1.
b) See Video. Note that rows and term positions use a zero-based index.
c) There is symmetry in each row. For example, the second position of the sixth row is equal to the second-last position of the same row.

Example 2:
a) ${}_8C_0$; ${}_{12}C_{10}$
b) ${}_{21}C_2 = 210$
c) k = 3 and 8, so the fourth and ninth positions have a value of 165.
d) 1024

Example 3:
a) 20
b) 120
c) 66
d) 54

Example 4:
a) The binomial theorem states that a binomial power of the form $(x + y)^n$ can be expanded into a series of terms with the form ${}_nC_k x^{n-k}y^k$, where *n* is the exponent of the binomial *(and also the zero-based row of Pascal's Triangle)*, and *k* is the zero-based term position.

$(x+1)^3 = x^3 + 3x^2 + 3x + 1$

b) $(x+2)^6 = x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$

c) $(2x-3)^4 = 16x^4 - 96x^3 + 216x^2 - 216x + 81$

Example 5:

a) $(x^2 - 2y)^4 = x^8 - 8x^6y + 24x^4y^2 - 32x^2y^3 + 16y^4$

b) $\left(3x^2 - \frac{1}{2}\right)^4 = 81x^8 - 54x^6 + \frac{27}{2}x^4 - \frac{3}{2}x^2 + \frac{1}{16}$

c) $\left(2x^3 - \frac{3}{x}\right)^5 = 32x^{15} - 240x^{11} + 720x^7 - 1080x^3 + 810\frac{1}{x} - 243\frac{1}{x^5}$

Example 6:

a) $(x+y)^4$

b) $(2a-3b)^5$

c) $\left(3a - \frac{1}{4}\right)^3$

Example 7:

a) $t_3 = 54x^2$

b) $t_5 = 90720a^{12}b^8$

c) $t_4 = -20x^3$

Example 8:

a) $t_5 = 6300000a^5b^4$

b) $t_5 = 1620a^3b^{12}$

c) $t_5 = 1451520a^4$

d) $k = 9$

Example 9:

a) $m = 3$

b) $t_4 = -1080a^2b^3 \quad (n = 5)$

c) $m = \frac{32}{b^2}$

Example 10:

a) $k = 4$ (fifth term); $t_5 = 60$

b) $k = 2$ (third term); $t_3 = 295245$

c) $b = 4$