
(Revisit, Review and Revive)

## Exponential and Logarithmic Functions

## Key Concepts

- The Exponent Laws
$a^{m} \times a^{n}=a^{m+n}$
$\left(a^{m}\right)^{n}=a^{m n}$
$a^{-m}=\frac{1}{a^{m}}$
$a^{\frac{m}{n}}=\sqrt[n]{a^{m}}$
$\frac{a^{m}}{a^{n}}=a^{m-n}$
$(a b)^{m}=a^{m} b^{m}$
$\left(\frac{a}{b}\right)^{-m}=\left(\frac{b}{a}\right)^{m}$

Simplify each expression.
a. $\frac{14 a^{3} b c^{2}}{7 a b^{4} c}$
b. $\left(\frac{-2 x^{3} y^{3}}{25 x y^{4}}\right)^{2}\left(\frac{-4 x^{2} y^{3}}{5 x^{2} y}\right)^{-3}$

RF7. Demonstrate an understanding of logarithms.
7.1 Explain the relationship between logarithms and exponents.
7.2 Express a logarithmic expression as an exponential expression and vice versa.
7.3 Determine, without technology, the exact value of a logarithm, such as $.2 \log 8$
7.4 Estimate the value of a logarithm, using benchmarks, and explain the reasoning; e.g., since $=3$ and $=4$, is approximately equal to $3.1 .2 \log 82 \log 162 \log 9$
Notes: $\log _{b} b=1$ and $\log _{b} 1=0$

## Key Concepts

- Recall that $a=b^{c}$ implies that $c=\log _{b} a$, and vice versa
- Common Sense Rules:
- $\log _{a} 1=0$ since in exponential form $1=a^{0}$
- $\log _{a} a=1$ since in exponential form $a=a^{1}$
- $\quad a^{\log _{a} x}=x$ since in exponential form $\log _{a} x=\log _{a} x$
- $\log _{a} a^{x}=x$ since $a^{x}=a^{x}$
- Since our number system is based on powers of 10 , logarithms with base 10 are widely used and are called common logarithms. When you write a common logarithm, you do not need to write the base. For example, $\log 3$ means $\log _{10} 3$.
- If you need to work with a base other than 10 , you can convert then to common logarithms as follows:
- 

$$
\log _{b} c=\frac{\log _{a} c}{\log _{a} b}
$$

## Examples:

1. Rank these logarithms in order from least to greatest: $\log _{4} 62, \log _{6} 36, \log _{3} 10, \log _{5} 20$
2. Evaluate the following logarithms using your calculator:
a) $\log 10=$
b) $\log 1000=$
c) $\log _{2} 8=$
d) $\log _{5} \sqrt{5}=$
e) $\log _{6} 6=$
f) $\log _{3} 81=$
g) $\log _{2} 0.25=$
h) $\log 0.001=$
3. Evaluate without a calculator
a) $\log _{4} 64=$
b) $\log _{2}\left(\frac{1}{32}\right)=$
c) $5^{\log _{5} 25}=$
d) $a^{\log _{a} a}=$
4. Convert to exponential form and solve where applicable:
a) $\log _{2} x=y$
f) $5=4 \log _{b} 6$
j) $\quad \log _{7} \frac{y}{3}=x$
b) $3=\log _{5} x$
g) $\log _{x} 36=2$
k) $\log _{5}(7 y)=x$
c) $\log _{5} x=-3$
I) $3=\log _{2}\left(\frac{y}{4}\right)$
h) $\log _{x} 9=\frac{2}{3}$
d) $\log _{5} \sqrt{125}=y$
m) $2=\log _{4} 32 y$
i) $\log _{16} x=-\frac{1}{4}$
e) $y=2 \log _{8} 512$
n) $\log _{2}\left(\frac{y}{5}\right)=-3$
5. Convert the following into logarithm form:
a) $y=x^{2}$
b) $x^{2 y}=5$
d) $5 y=30^{x}$
e) $y=\frac{3}{2}(10)^{x}$
g) $y=3(5)^{x}$
h) $y=4^{x}-9$
c) $y=2\left(3^{x}\right)$
f) $2^{2 x-5}=y-1$
6. If $\log _{3} x=15$, then find the value of $\log _{3}\left(\frac{1}{3} x\right)$.
7. If $\log 3=a, \log 5=b$, then find $\log \left(\frac{3}{25}\right)$ in terms of $a$ and $b$.
8. Solve $\log _{x}\left(\frac{1}{64}\right)=-\frac{3}{2}$.
9. If $\log _{2} x=3$ and $\log _{2} t=x$, then determine the value of $t$.
10. If $\log _{a} b=4.5$ and $\log _{a} c=3.7$, then the value of $\log _{a}\left(\frac{b}{c}\right)$ to the nearest tenth is $\qquad$ .
11. The point $\left(\frac{1}{8},-3\right)$ is on the graph of the log function $f(x)=\log _{c} x$ and the point $(4, k)$ is on the graph of the inverse, $y=f^{-1}(x)$. Determine the value of $k$.
12. Convert the following logarithm to the base indicated:
a) $\log _{6} 216$ to base 3
b) $\log 300$ to base 5

RF8. Demonstrate an understanding of the product, quotient and power laws of logarithms.
8.1 Develop and generalize the laws for logarithms, using numeric examples and exponent laws.
8.2 Derive each law of logarithms.
8.3 Determine, using the laws of logarithms, an equivalent expression for a logarithmic expression.
8.4 Determine, with technology, the approximate value of a logarithmic expression, such as log9.

Notes: Change of base identity can be taught as a strategy for evaluating logarithms.

## Key Concepts:

- To solve those exponential equations when the bases are powers of the same number, write the bases with the same power, use the laws of exponents to simplify until you can ignore a single base on each side.
- This outcome deals with algebraic solutions to exponential and logarithmic equations using the laws of exponents and logarithms.
- The following laws of logarithms are on your formula sheet, and are useful in answering questions in this outcome.

$$
\begin{aligned}
\log _{a}(M \times N) & =\log _{a} M+\log _{a} N \\
\log _{a}\left(\frac{M}{N}\right) & =\log _{a} M-\log _{a} N \\
\log _{a} M^{n} & =n \log _{a} M
\end{aligned}
$$

## Examples:

1. Solve:
a) $4^{x+2}=64^{x}$
d) $5(3)^{x}=135$
g) $5 x^{\frac{2}{3}}=45$
b) $9^{4 x}=27^{x-1}$
e) $3^{4 x}(3)=27^{2 x}$
h) $\left(\frac{1}{8}\right)^{x-3}=2(16)^{2 x+1}$
c) $8^{x+2}=\left(\frac{1}{4}\right)^{x+3}$
f) $\left(\frac{1}{9}\right)^{x}=\frac{27^{x}}{9^{2 x-1}}$
2. Evaluate:
a) $\log _{2} 12-\log _{2} 3=$
e) $\frac{1}{2} \log _{2} 16-\frac{1}{3} \log _{2} 8=$
b) $\log _{6} 8+\log _{6} 8-\log _{6} 2=$

$$
\text { f) } 2 \log 5+2 \log 2=
$$

c) $\log _{5} 10+\log _{5} 75-\left(\log _{5} 2+\log _{5} 3\right)=$
g) $3 \log x-\log x^{3}=$
d) $\log _{2} 2+\log _{2} 3-\log _{2} 6-\log _{2} 8=$
3. Write the following as a single logarithm.
a) $\log B+\log D-5 \log E-\log A^{2}+\frac{1}{2} \log A=$
b) $\log _{2} x^{3}-4 \log _{2} x-\log _{x} \sqrt{x}=$
c) $4 \log _{6} y^{2}+\log _{6} y-\frac{2}{3} \log _{6} y=$
4. Expand each expression using the laws of logarithms.
a) $\log _{4} \frac{x^{3} y}{4 z}$
b) $\log _{5} \sqrt{x y^{3}}$
c) $\log \frac{100 \sqrt[3]{x^{4}}}{y^{2}}$
5. If $\log _{3} A=t$ then $\log _{3} 27 A^{3}=$
6. If $\log _{3} y=c-\log _{3} x$, then $y$ is equal to
7. Solve for $x$.
a) $4^{x}=12$
b) $3^{2 x+1}=7$
c) $6^{3 x}=3^{2 x-1}$
8. Solve for $x$.
a) $\log _{4}(x+2)+\log _{4}(x-4)=2$ c) $\log _{b}(4 x-3)-\log _{b} x=\log _{b} 10$
b) $\log _{4}(x+3)-\log _{4} x=\log _{4} 7$
d) $\log _{2}(x+7)=3-\log _{2}(x+5)$
e) $\log _{4}(5 x+1)=\log _{4}(x+17)$
g) $\log _{2}(x-4)=4-\log _{2}(x+2)$
f) $\log (5 x)-\log (x-1)=1$
h) $\log _{3}(3-3 y)-\log _{3}(y-2)=\log _{3} 3$

RF9. Graph and analyze exponential and logarithmic functions. [C, CN, T, V]
[ICT: C6-4.3, C6-4.4, F1-4.2]
9.1 Sketch, with or without technology, a graph of an exponential function of the form $y=a x, a>0$.
9.2 Identify the characteristics of the graph of an exponential function of the form $y=a x, a>0$, including the domain, range, horizontal asymptote and intercepts, and explain the significance of the horizontal asymptote.
9.3 Sketch the graph of an exponential function by applying a set of transformations to the graph of $y=a x, a>0$, and state the characteristics of the graph.
9.4 Sketch, with or without technology, the graph of a logarithmic function of the form $y=\log b x, b>1$.
9.5 Identify the characteristics of the graph of a logarithmic function of the form $y=\log b x, b>1$, including the domain, range, vertical asymptote and intercepts, and explain the significance of the vertical asymptote.
9.6 Sketch the graph of a logarithmic function by applying a set of transformations to the graph of $y=\log b x, b>$ 1, and state the characteristics of the graph.
9.7 Demonstrate, graphically, that a logarithmic function and an exponential function with the same base are inverses of each other.

## Notes:

- When graphing $y=\log _{b} x$ and $y=b^{x}$, the value of b will be restricted to $b>0, b \neq 1$
- Natural logarithms and base e are beyond the scope of this course.


## Key Concepts

- An exponential function is written in the form $y=a b^{x}$, where b is a constant and x is the exponent. It is increasing when $b>1$, and decreasing when $0<b<1$.

- A logarithmic function is the inverse of an exponential function.
- Remember, the line $y=x$ acts as a mirror, to find the inverse of a function, we take the following steps:

```
Step \#1 switch \(x\) and \(y\)
Step \#2 solve for \(y\)
```

But for the inverse of an exponential function it is difficult to solve for $y$.
In the inverse of $y=2^{x}$ the first step is to switch $x$ and $y$, therefore $x=2^{y}$, but it is difficult to solve for $y$. To solve for $y$ we introduce the logarithmic function as follows:

$$
\log _{a} x=y \quad \Leftrightarrow \quad x=a^{y}
$$

Log form Exponential form


| Exponential Form | Log Form |
| :--- | :--- |
| Domain: | Domain: |
| Range: | Range: |
| x-intercept: | x-intercept: |
| y-intercept: | y-intercept: |
| Asymptote: | Asymptote: |

## Examples:

1. State which of the following are exponential.
a) $y=x^{7}$
b) $y=0.5^{x}$
c) $y=4^{x}$
d) $y=x^{\frac{1}{3}}$
e) $y=(-3)^{x}$
f) $y=\frac{3^{x-2}}{2}$
g) $y=\frac{1}{3 x}$
2. Determine the inverse of each:
a) $y=3^{x}$
b) $y=\log _{4} x$
c) $x=20^{y}$
3. State the transformations, asymptotes and mapping notation for each, compared to $y=\log _{3} x$ and $y=3^{x}$

## Asymptote

a) $y=-2 \log _{3}(x-3)+5$
$(x, y) \rightarrow$

## Asymptote

b) $y=\log _{3}\left(\frac{1}{3} x+6\right) \quad(x, y) \rightarrow$
c) $y=2(3)^{x-4}$
$(x, y) \rightarrow$
d) $y=-\frac{1}{2}(3)^{\frac{1}{5} x}-5$

$$
(x, y) \rightarrow
$$

4. Describe the transformation which maps the graph of $y=\log _{9} x$ to the graph of the following:
a) $\log _{\frac{1}{9}} x$
b) $\log _{81} x$
5. Write the equation of the transformed logarithmic function for $y=\log _{5} x$ for each of the following:
a) the vertical stretch about the $x$-axis by a factor of $\frac{1}{2}$ and a horizontal stretch about the $y$-axis by a factor of $\frac{2}{3}$ and reflected in the $x$-axis and a horizontal translation of 3 units to the left.
b) A vertical stretch by a factor of 3 about the $x$-axis, a horizontal stretch by a factor of $\frac{1}{4}$ about the $y$-axis, reflected on the $y$-axis, a horizontal translation 1 to the right and a vertical translation of 5 units down.
6. The graph of $y=f(x)=b^{x}$, where $\mathrm{b}>1$, is translated such that the equation of the new graph is expressed as $y-2=f(x-1)$. Identify the range and the $y$-intercept of the new function.
7. The equation of the asymptote of $y=2^{x+1}+3$.
8. State the domain, range, x and y intercepts and asymptotes of $y=\log _{2}(x+3)-2$
9. Identify the equation of the asymptote of $y=\log _{2}(3 x+1)+1$

RF10. Solve problems that involve exponential and logarithmic equations. [C, CN, PS, R]
10.1 Determine the solution of an exponential equation in which the bases are powers of one another.
10.2 Determine the solution of an exponential equation in which the bases are not powers of one another, using a variety of strategies.
10.3 Determine the solution of a logarithmic equation, and verify the solution.
10.4 Explain why a value obtained in solving a logarithmic equation may be extraneous.
10.5 Solve a problem that involves exponential growth or decay.
10.6 Solve a problem that involves the application of exponential equations to loans, mortgages and investments.
10.7 Solve a problem that involves logarithmic scales, such as the Richter scale and the pH scale.
10.8 Solve a problem by modelling a situation with an exponential or a logarithmic equation.

## Notes:

- Formulas will be given for all problems involving logarithmic scales such as decibels, earthquake intensity, and pH
- Formulas will be given unless the context fits the form $y=a b^{\frac{t}{p}}$, where y is the final amount, a is the initial amount, b is the growth/decay factor, t is the total time, and p is the period time.
- Logarithmic equations should be restricted to the same bases.


## Key Concept:

- To solve these exponential equations when the bases are powers of the same number, write the bases with the same power, use the laws of exponents to simplify until you can ignore a single base on each side.
- This outcome deals with algebraic solutions to exponential and logarithmic equations using the laws of exponents and logarithms.
- The following laws of logarithms are on your formula sheet, and are useful in answering questions in this outcome.

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\begin{aligned}
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\log _{a} M^{n} & =n \log _{a} M
\end{aligned}
$$

- The compound interest equation, $A=P(1+i)^{n}$, can be used to model the growth or decay of any type of population.
- For applications with interest ...
$\boldsymbol{A}$ is the amount of money at the end of the investment, the final amount
$\boldsymbol{P}$ is the principal amount deposited, the initial amount deposited
$\boldsymbol{i}$ is the interest rate per compounding period, expressed as a decimal
(interest rate / \# compounding periods)
$\boldsymbol{n}$ is the number of compounding periods, the number of times you get interest
(\# years $\times$ \# compounding periods)

|  | $\boldsymbol{i}$ | $\boldsymbol{n}$ |
| :---: | :---: | :---: |
| Annually | $\div 1$ | $\times 1$ |
| Semi-annually | $\div 2$ | $\times 2$ |
| Quarterly | $\div 4$ | $\times 4$ |
| Monthly | $\div 12$ | $\times 12$ |

- For all other word problems, dealing with Growth or Decay, the following formula will be used

$$
y=a b^{\frac{t}{p}}
$$

$y$-the final amount
$a$ - is the initial amount
$b$ - is the multiplication factor: 2 for doubles, $1 / 2$ for half-life, 3 for triples
$t$ - time, must be in the same units as $p$
$p$ - the period of time for doubling, half-life, or tripling.

## Examples:

1. The population of a city was 173500 on January 1,1988 and it was 294000 on January 1, 2002. If the growth of the city can be modeled as an exponential function, then find the average annual growth rate of the city, expressed to the nearest tenth of a percent.
2. A sports car depreciates at a rate of $14 \%$ per year and was bought for $\$ 60000$. How long will it take to depreciate to $\$ 18000$ ?
3. Write an exponential expression that will determine the value, V , of the investment at any given time, t , in years.
a) $\$ 3000$ is invested at $5.2 \%$ per year compounded semi-annually
b) $\$ 2500$ is invested at $4 \%$ per year compounded quarterly
c) $\$ 8000$ is invested at $6 \%$ per annum compounded monthly
4. A student borrowed $\$ 6000$. Interest is charged at $5 \%$ per year compounded semi-annually. The loan is paid off in one final payment of $\$ 6958$. What is the length of the loan?
5. Jamie borrows $\$ 6000$ from the bank at a rate of $8 \%$ per annum compounded monthly. How much would he owe at the end of the one month, if he does not make his first payment?
6. The population of rabbits in a park is increasing by $70 \%$ every 6 months. Presently there are 200 rabbits in the park.
a) Write an exponential function that represents this scenario. Use $P$ to represent the rabbit population and t to represent the time in months.
b) How many rabbits will there be in 15 months?
7. A sample of water contains 200 grams of pollutants. Each time the sample is passed through a filter, $20 \%$ of its pollutants are removed.
a) Write a function that relates the amount of pollutant, $P$, that remains in the same to the number of times, t , the sample is filtered.
b) Determine an expression that gives the amount of pollutants still in the water after it passes through 5 filters. How many grams are there after 5 filters, rounded to the tenth of a gram?
8. A colony of 100 insects triples its population every 5 days. How long will it take for the population to grow to 5000?
9. A type of bacterium doubles each hour.
a) If there are 4 bacteria in a sample, write an exponential function that models the sample's growth over time.
b) Use your equation to determine the time it takes for the sample to become 4096 bacteria.
10. The half-life of radioactive 134 is 8.1 days. Determine the number of days it took for there to be only $2 \%$ remaining?
11. The sound intensity, B , in decibels, is defined as $B=10 \log \frac{I}{I_{0}}$, where I is the intensity of the sound. A fine can occur when a motorcycle is idling at 20 times as intense as the sound of an automobile. If the level of an automobile is 80 dB , at what decibel level can a fine occur to a motorcycle operator?
12. A sound is 1000 times more intense than a sound you can just hear. What is the measure of its loudness in decibels?
13. How many more times intense is the sound of normal conversation ( 60 dB ) than the sound of a whisper $(30 \mathrm{~dB})$ ?
14. The pH scale is used to measure acidity or alkalinity of a substance. The formula for pH is $p H=-\log \left[\mathrm{H}^{+}\right]$, where $\left[\mathrm{H}^{+}\right]$is the hydrogen ion concentration.
a) If a solution has a hydrogen ion concentration of $1.21 \times 10^{-2} \mathrm{ml} / \mathrm{L}$, determine the pH value of the solution to the nearest tenth.
b) If vinegar has a pH of 3.2. Determine the $\left[\mathrm{H}^{+}\right]$in scientific notation to one decimal place.
15. Earthquake intensity is given by $I=I_{0}(10)^{m}$, where $I_{0}$ is the reference intensity and $m$ is the magnitude. An earthquake with magnitude 6.8 is followed by an aftershock with magnitude 5.2. How many time more intense was the earthquake than its aftershock?
16. The formula for the Ritcher magnitude Scale of an earthquake is $M=\log \frac{I}{I_{0}}$, where $I$ is the magnitude of the largest earthquake and $I_{0}$ is the magnitude of the smallest earthquake. How much more intense is an earthquake with a magnitude of 6.9 vs a 3.9 ?
17. Earthquake intensity is given by $I=I_{0}(10)^{m}$, where $I_{0}$ is the reference intensity and $m$ is the magnitude. A particular major earthquake of magnitude is 120 times as intense as a particular minor earthquake. The magnitude, to the nearest tenth, of the minor earthquake is $\qquad$ _.

## Practice Test

1. The solution for the equation $9^{2 x}=\left(\frac{1}{3}\right)^{x+6}$ is
A. 6
B. 2
C. $-\frac{4}{3}$
D. $-\frac{6}{5}$
2. In logarithmic form, the solution of $5^{x-3}=50$ is
A. $\log _{10}(50)-\log _{10}(5)+3$
B. $\log _{10}(10)+3$
C. $\frac{\log _{10}(50)}{\log _{10}(5)}+3$
D. $\frac{\log _{10}(53)}{\log _{10}(5)}$
3. The partial graph of the exponential function $f(x)=4^{x}$ is shown to the right.
The domain of the inverse function, $f^{-1}(x)$ is
A. $x>0, x \in \mathfrak{R}$
B. $x<0, x \in \mathfrak{R}$
C. $x \geq 0, x \in \mathfrak{R}$
D. $x \in \mathfrak{R}$

4. The value of $x$ in the equation $3^{x-1}=26$ is
A. 1.97
B. 2.97
C. 3.97
D. 0.96
5. If $\log _{8}(x+5)-\log _{8}(x-2)=1$, then the value of $x$ is
A. -3
B. 1
C. 3
D. 8
6. $\frac{\log (r)}{t}+\frac{\log (d)}{t}$ is equal to
A. $\log (r d)^{t}$
B. $\log (\sqrt[t]{r d})$
C. $\log (\sqrt[t]{r+d})$
D. $\log (r+d)^{t}$
7. The expression $\log _{a}\left(a^{4} b\right)-\log _{a}(a b)$ is equivalent to
A. 3
B. 4
C. 3 a
D. $a^{3}$
8. A student graphed the following equations.

Equation I $\quad y_{1}=\log _{10} x$
Equation II $\quad y_{2}=5^{x-3}$
Equation III $y_{3}=x-3$
Equation IV $\quad y_{4}=x$
The student could estimate the solution to the equation $\log _{5} x=x-3$ by using the graphs of equations
A. I and II
B. I and III
C. II and III
D. II and IV
9. The growth of bacteria can be written $N(t)=N_{0}\left(2^{\frac{t}{40}}\right)$, where $N(t)=$ the final number of bacteria, $N_{0}$ $=$ the initial number of bacteria, and $t=$ time in minutes. The logarithmic expression for time $(t)$ it takes for the number of bacteria to increase from 50000 to 700000 is (SE)
A. $\frac{40}{\log _{2}(14)}$
B. $\frac{\log _{2}(14)}{40}$
C. $40 \log _{2}(14)$
D. $\log _{2}\left(2^{\frac{t}{40}}\right)$
10. The value of $i$ in the compound interest equation $5=(1+i)^{6}$ is
A. $i=\sqrt[6]{4}$
B. $i=\sqrt[6]{5}-1$
C. $i=\frac{\log 5}{\log 6}-1$
D. $i=\frac{\log 6}{\log 5}-1$
11. The gain, G , measured in decibels, of an amplifier is defined by the equation $G=10 \log \left(\frac{P}{P_{1}}\right)$ where P is the output power of the amplifier, in watts, and P1 is the input power of the amplifier, in watts. If the gain of the amplifier is 23 decibels, then the ratio $\frac{P}{P_{1}}$ is
A. $10 \log (23)$
B. $\log (2.3)$
C. $10^{23}$
D. $10^{2.3}$
12. The equation that defines the decibel level for any sound is $L=10 \log _{10}\left(\frac{I}{I_{0}}\right)$, where L is the loudness in decibels, $I$ is the intensity of sound being measured, and 10 is the intensity of sound at the threshold of hearing. Given that normal conversation is 1000000 times intense as 10 , then the loudness of normal conversation is
A. 5 decibels
B. 6 decibels
C. 16 decibels
D. 60 decibels
13. A restriction on the domain of $f(x)=x^{2}+4$ such that its inverse is also a function, could be (SE)
A. $x \geq-4$
B. $x \geq 0$
C. $x \leq 2$
D. $x \leq 4$
14. The graph of $y=3^{x}$ is reflected in the line $\mathrm{y}=\mathrm{x}$. The equation of the transformed graph is
A. $y=\log _{3} x$
B. $y=3 \log x$
C. $y=3^{-x}$
D. $y=-3^{x}$
15. The equation $m \log _{p} n=q$ can be written in exponential form as
A. $p^{m}=n q$
B. $p^{\frac{q}{m}}=n$
C. $p^{q}=m n$
D. $p^{q m}=n$
16. The expression $\left(3^{\log x}\right)\left(3^{\log x}\right)$ is equivalent to
A. $3^{\log x^{2}}$
B. $9^{\log x^{2}}$
C. $3^{(\log x)^{2}}$
D. $9^{(\log x)^{2}}$
17. Written as a single logarithm, $2 \log x-\frac{\log z}{2}+3 \log y$ is
A. $\log \left(\frac{x^{2} y^{3}}{\sqrt{z}}\right)$
B. $3 \log \left(\frac{x y}{z}\right)$
C. $\log \left(\frac{x^{2}}{y^{3} \sqrt{z}}\right)$
D. $\log \left(x^{2}-\sqrt{z}+y^{3}\right)$

Use the following information to answer the next question.
A student's work to simplify a logarithmic expression is shown below, where a $>1$.
STEP $12 \log _{a} x^{4}-3 \log _{a} x^{2}+4 \log _{a} x^{3}$
STEP $2 \log _{a} x^{8}-\log _{a} x^{6}+\log _{a} x^{12}$
STEP $3 \log _{a}\left(\frac{x^{8}}{x^{6} \times x^{12}}\right)$
STEP $4 \log _{a}\left(\frac{x^{8}}{x^{18}}\right)$
STEP $5 \log _{a} x^{10}$
18. The student made his first error when going from:
A. Step 1 to Step 2
B. Step 2 to Step 3
C. Step 3 to Step 4
D. Step 4 to Step 5
19. The equation of the asymptote for the graph of $y=\log _{b}(x-3)+2$, where $b>0$ and $b \neq 1$, is
A. $x=2$
B. $x=-2$
C. $x=3$
D. $x=-3$
20. The graph of $y=\log x$ is transformed into the graph of $y-4=\log (x+5)$ by a translation of 5 units
$\qquad$ and 4 units $\qquad$ .

The statement above is completed by the information in row

| Row | $i$ | ii |
| ---: | :---: | :---: |
| A. | right | up |
| B. | left | up |
| C. | right | down |
| D. | left | down |

21. For the graph of $y=\log _{b}(3 x+12)$, where $0<\mathrm{b}<1$, the domain is
A. $x>-4$
B. $x>4$
C. $x>-12$
D. $x>12$
22. The $y$-intercepts on the graph of $f(x)=a^{(x+1)}+b$ is
A. $a$
B. $b$
C. $1+b$
D. $a+b$

## Numerical Response

1. If $\log _{n}(a)=3.6$ and $\log _{n}(b)=2.7$ then $\log _{n}(a b)$ correct to the nearest tenth is $\qquad$ .
2. If $a b=24$, then to the nearest hundredth, the value of $2 \log _{10} a+2 \log _{10} b$, where $a, b>0$, is $\qquad$ .
3. The inverse of $f(x)=2 x-3$ is written in the form $f^{-1}(x)=\frac{a}{b} x+\frac{c}{d}$, the values of $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}$ are
$\qquad$ , $\quad$, $\qquad$ .
4. The value of $\log _{5} 625+3 \log _{7} 49+\log _{2} \frac{1}{16}+\log _{b} b+\log _{a} 1$ is $\qquad$ .
5. Given that $\log _{3} a=6$ and $\log _{3} b=5$, determine the value of $\log _{3}\left(9 a b^{2}\right)$.
6. Algebraically solve the equation $8^{(3 x+4)}=4^{(x-9)}$.
7. Solve the equation $3^{(2 x+1)}=\left(\frac{1}{5}\right)^{(x-3)}$ algebraically. Round to the nearest hundredth, if necessary.
8. Solve algebraically $\log _{7}(x+1)+\log _{7}(x-5)=1$. What is the extraneous root?

Use the following information to answer the next question.
Earthquake intensity is given by $I=I_{0} \times 10^{M}$, where $I_{0}$ is the reference intensity and $M$ is the magnitude. An earthquake measuring 5.3 on the Richter scale is 125 times more intense than a second earthquake.
9. Determine, to the nearest tenth, the Richter scale measure of the second earthquake.
10. The population of a particular town on July 1,2011 was 20000 . If the population decreases at an average annual rate of $1.4 \%$, how long will it take for the population to reach 15 300? (SE)

Exponents and Logarithm Practice Test Answers:
MULTIPLE CHOICE:

1. D
2. C
3. A
4. C
5. C
6. B
7. A
8. D
9. C
10. B
11. D
12. D
13. B
14. A
15. B
16. A
17. A
18. B
19. C
20. B
21. A
22. D

NUMERICAL RESPONSE:
1.6.3
2. 2.76
3. 1, 2, 3, 2
4.7
5.18
6.4 .3
7.1.0
8.2
9.3.2
10. 19

