Key.

Examples:

1. Rank these logarithms in order from **least to greatest**: $\log_4 62$, $\log_6 36$, $\log_3 10$, $\log_5 20$

2.97 2 2.09 1.86

$\log_5 20, \log_6 36, \log_3 10, \log_4 62.$

2. Evaluate the following logarithms using your calculator:

a) $\log 10 =$ 1
b) $\log 1000 =$ 3
c) $\log_2 8 =$ 3
d) $\log_5 \sqrt{5} =$ $\frac{1}{2}$
e) $\log_6 6 =$ 1
f) $\log_3 81 =$ 4
g) $\log_2 0.25 =$ -2
h) $\log 0.001 =$ -3

3. Evaluate without a calculator

a) $\log_4 64 =$ 3
b) $\log_2\left(\frac{1}{32}\right) =$ -5
c) $5^{\log_5 25} =$ 25
5^2
d) $a^{\log_a a} =$ a
a^1

4. Convert to exponential form and solve where applicable:

a) $\log_2 x = y$
$\boxed{2^y = x}$

b) $3 = \log_5 x$
$5^3 = x$
$\boxed{x = 125}$

c) $\log_5 x = -3$
$5^{-3} = x$
$\boxed{x = \frac{1}{125}}$

d) $\log_5 \sqrt{125} = y$
$5^y = \sqrt{125}$
$5^y = 5^{3/2}$
$\boxed{y = 3/2}$

e) $y = 2\log_8 512$
$y = \log_8 512^2$
$8^y = 262144$
$8^y = 8^6$
$\boxed{y = 6}$

f) $5 = 4\log_b 6$
$b^5 = 6^4$
$b = 6^{4/5} = \sqrt[5]{6^4}$
$= (6^4)^{1/5}$

g) $\log_x 36 = 2$
$\sqrt{x^2} = \sqrt{36}$
$\boxed{x = 6}$

h) $\log_x 9 = \frac{2}{3}$
$x^{2/3 \cdot \frac{3}{2}} = 9^{3/2}$
$x = \sqrt{9}^3$
$\boxed{x = 27}$

i) $\log_{16} x = -\frac{1}{4}$
$16^{-1/4} = x$
$\frac{1}{16^{1/4}} = x$
$\boxed{\frac{1}{2} = x}$

j) $\log_7 \frac{y}{3} = x$
$7^x = \frac{y}{3}$ $\boxed{y = 3(7^x)}$

k) $\log_5(7y) = x$
$5^x = 7y$
$\boxed{y = \frac{5^x}{7}}$

l) $3 = \log_2\left(\frac{y}{4}\right)$
$2^3 = \frac{y}{4}$
$y = 4 \cdot 8 = \boxed{32}$

m) $2 = \log_4 32y$
$4^2 = 32y$
$y = \frac{16}{32} = \boxed{\frac{1}{2}}$

n) $\log_2\left(\frac{y}{5}\right) = -3$
$2^{-3} = \frac{y}{5}$
$5 \cdot \frac{1}{8} = \frac{y}{5} \cdot 5$
$\boxed{y = 5/8}$

5. Convert the following into logarithm form:

a) $y = x^2$

$\log_x y = 2$

d) $5y = 30^x$

$\log_{30}(5y) = x$

g) $\frac{y}{3} = \frac{3(5)^x}{3}$

$\log_5\left(\frac{y}{3}\right) = x$

b) $x^{2y} = 5$

$\log_x 5 = 2y$

e) $y = \frac{3}{2}(10)^x$

$\frac{2y}{3} = 10^x$

$\log\left(\frac{2y}{3}\right) = x$

h) $y = 4^x - 9$

$+9 \quad +9$

$y+9 = 4^x$

$\log_4(y+9) = x$

c) $\frac{y}{2} = \frac{2(3^x)}{2}$

$\log_3\left(\frac{y}{2}\right) = x$

f) $2^{2x-5} = y-1$

$\log_2(y-1) = 2x-5$

6. If $\log_3 x = 15$, then find the value of $\log_3\left(\frac{1}{3}x\right)$.

$3^{15} = x$

or more solutions.

$\log_3\left(\frac{1}{3} \cdot 3^{15}\right)$

$= \log_3\left(\frac{3^{15}}{3^1}\right) = \log_3(3^{14}) = \boxed{14}$

$= 14\log_3 3$

7. If $\log 3 = a$, $\log 5 = b$, then find $\log\left(\frac{3}{25}\right)$ in terms of a and b.

$= \log 3 - \log 25 = \log 3 - \log 5^2$

$= \log 3 - 2\log 5$

$= \boxed{a - 2b}$

8. Solve $\log_x\left(\frac{1}{64}\right) = -\frac{3}{2}$.

$\left(x^{-3/2}\right)^{-\frac{2}{3}} = \left(\frac{1}{64}\right)^{-2/3}$

$\boxed{x = 16}$

9. If $\log_2 x = 3$ and $\log_2 t = x$, then determine the value of t.

$2^3 = x$

$x = 8$

$\log_2 t = 8$

$2^8 = t$

$\boxed{t = 256}$

10. If $\log_a b = 4.5$ and $\log_a c = 3.7$, then the value of $\log_a\left(\frac{b}{c}\right)$ to the nearest tenth is ________.

$= \log_a b - \log_a c$

$= 4.5 - 3.7 = \boxed{0.8}$

11. The point $\left(\frac{1}{8}, -3\right)$ is on the graph of the log function $f(x) = \log_c x$ and the point (4, k) is on the graph of the inverse, $y = f^{-1}(x)$. Determine the value of k.

$-3 = \log_c \frac{1}{8}$

$(c^{-3})^{-\frac{1}{3}} = \left(\frac{1}{8}\right)^{-1/3}$

$c = 2$

$x = \log_c y$

$f^{-1}(x) = y = c^x$

$y = 2^x$

$k = 2^4$

$\boxed{k = 16}$

12. Convert the following logarithm to the base indicated:

a) $\log_6 216$ to base 3

$= 3$

$= \boxed{\log_3 27}$

b) $\log 300$ to base 5

$\log_{10} 300 = \frac{\log 300}{\log 10} = \boxed{\frac{\log_5 300}{\log_5 10}}$

RF8. Demonstrate an understanding of the product, quotient and power laws of logarithms.
8.1 Develop and generalize the laws for logarithms, using numeric examples and exponent laws.
8.2 Derive each law of logarithms.
8.3 Determine, using the laws of logarithms, an equivalent expression for a logarithmic expression.
8.4 Determine, with technology, the approximate value of a logarithmic expression, such as log9.

Notes: Change of base identity can be taught as a strategy for evaluating logarithms.

Key Concepts:

- To solve those exponential equations when the bases are powers of the same number, write the bases with the same power, use the laws of exponents to simplify until you can ignore a single base on each side.
- This outcome deals with algebraic solutions to exponential and logarithmic equations using the laws of exponents and logarithms.
- The following laws of logarithms are on your formula sheet, and are useful in answering questions in this outcome.

$$\log_a(M\times N)=\log_a M+\log_a N$$
$$\log_a\left(\frac{M}{N}\right)=\log_a M-\log_a N$$
$$\log_a M^n = n\log_a M$$

Examples:

1. Solve:

a) $4^{x+2}=64^x$

$4^{x+2}=4^{3x}$
$x+2=3x$
$2=2x$
$\boxed{x=1}$

b) $9^{4x}=27^{x-1}$

$3^{2(4x)}=3^{3(x-1)}$
$8x=3x-3$
$5x=-3$
$\boxed{x=-3/5}$

c) $8^{x+2}=\left(\frac{1}{4}\right)^{x+3}$

$2^{3(x+2)}=2^{-2(x+3)}$
$3x+6=-2x-6$
$5x=-12$
$\boxed{x=-\frac{12}{5}}$

d) $5(3)^x=135$

$\frac{5(3)^x}{5}=\frac{135}{5}$
$3^x=27$
$3^x=3^3$
$\boxed{x=3}$

e) $3^{4x}(3)^1=27^{2x}$

$3^{4x+1}=3^{3(2x)}$
$4x+1=6x$
$1=2x$
$\boxed{x=1/2}$

f) $\left(\frac{1}{9}\right)^x=\frac{27^x}{9^{2x-1}}$

$3^{-2x}=\frac{3^{3x}}{3^{2(2x-1)}}$

$-2x=3x-(4x-2)$
$-2x=3x-4x+2$
$-1x=2$
$\boxed{x=-2}$

g) $5x^{\frac{2}{3}}=45$

$\frac{5x^{2/3}}{5}=\frac{45}{5}$
$x^{2/3\cdot 3/2}=9^{3/2}$
$x=\sqrt{9}^3$
$\boxed{x=27}$

h) $\left(\frac{1}{8}\right)^{x-3}=2(16)^{2x+1}$

$2^{-3(x-3)}=2^1\cdot 2^{4(2x+1)}$
$-3x+9=1+8x+4$
$4=11x$
$\boxed{x=4/11}$

2. Evaluate:

a) $\log_2 12 - \log_2 3 =$ $\log_2\left(\frac{12}{3}\right) = \log_2 4$

$= \boxed{2}$

b) $\log_6 8 + \log_6 8 - \log_6 2 =$

$= \log_6\left(\frac{8\times 8}{2}\right) = \boxed{\log_6(32)}$

c) $\log_5 10 + \log_5 75 - (\log_5 2 + \log_5 3) =$

$= \log_5\left(\frac{10\cdot 75}{2\cdot 3}\right) = \log_5\left(\frac{750}{6}\right)$

$= \log_5 125 = \boxed{3}$

d) $\log_2 2 + \log_2 3 - \log_2 6 - \log_2 8 =$

$= \log_2\left(\frac{2\cdot 3}{6\cdot 8}\right) = \log_2\left(\frac{1}{8}\right) = \boxed{-3}$

e) $\frac{1}{2}\log_2 16 - \frac{1}{3}\log_2 8 =$ $\log_2\sqrt{16} - \log_2\sqrt[3]{8}$

$= \log_2 4 - \log_2 2$

$= \log_2\left(\frac{4}{2}\right) = \boxed{1}$

f) $2\log 5 + 2\log 2 =$

$= \log 5^2 + \log 2^2$

$= \log(25\cdot 4) = \log 100 = \boxed{2}$

g) $3\log x - \log x^3 =$

$= \log x^3 - \log x^3$

$= \log\left(\frac{x^3}{x^3}\right) = \log 1 = \boxed{0}$

3. Write the following as a single logarithm.

a) $\log B + \log D - 5\log E - \log A^2 + \frac{1}{2}\log A =$ $\log\left(\frac{BDA^{1/2}}{E^5A^2}\right) = \boxed{\log\left(\frac{BD}{E^5A^{3/2}}\right)}$

b) $\log_2 x^3 - \cancel{4}\log_2 x^4 - \log_x \sqrt{x}^{1/2} =$

$\log_2\left(\frac{x^3}{x^4}\right) - \frac{1}{2}\log_x x = \boxed{\log_2\left(\frac{1}{x}\right) - \frac{1}{2}}$

c) $4\log_6 y^2 + \log_6 y - \frac{2}{3}\log_6 y =$

$\log_6(y^2)^4 + \log_6 y - \log_6 y^{2/3} = \log_6\left(\frac{y^8\cdot y^1}{y^{2/3}}\right) = \boxed{\log_6\left(y^{25/3}\right)}$

4. Expand each expression using the laws of logarithms.

a) $\log_4 \frac{x^3y}{4z}$ $= \log_4 x^3 + \log_4 y - \log_4 4 - \log_4 z$

$= \boxed{3\log_4 x + \log_4 y - 1 - \log_4 z}$

b) $\log_5 \sqrt{xy^3}$

$= \log_5\left(x^{1/2}y^{3/2}\right)$

$= \log_5 x^{1/2} + \log_5 y^{3/2} = \boxed{\frac{1}{2}\log_5 x + \frac{3}{2}\log_5 y}$

c) $\log \frac{100\sqrt[3]{x^4}}{y^2}$ $= \log 100 + \log x^{4/3} - \log y^2$

$= \boxed{2 + \frac{4\log x}{3} - 2\log y}$

5. If $\log_3 A = t$ then $\log_3(27A^3) =$

$\log_3 27 + \log_3 A^3$

$3 + 3\log_3 A = \boxed{3+3t}$

6. If $\log_3 y = c - \log_3 x$, then y is equal to

$\log_3 y = c\log_3 3 - \log_3 x$

$\log_3 y = \log_3\left(\frac{3^c}{x}\right)$

$\boxed{y = \frac{3^c}{x}}$

7. Solve for x.

a) $4^x = 12$

$\boxed{\log_4 12 = x}$

$\boxed{x \approx 1.79}$

b) $3^{2x+1} = 7$

$\log_3 7 = 2x+1$

$\log_3 7 - 1 = 2x$

$\boxed{x = \frac{(\log_3 7 - 1)}{2}}$

$\boxed{x \approx 0.39}$

c) ~~$6^{3x} = 3^{2x-1}$~~

omit

8. Solve for x.

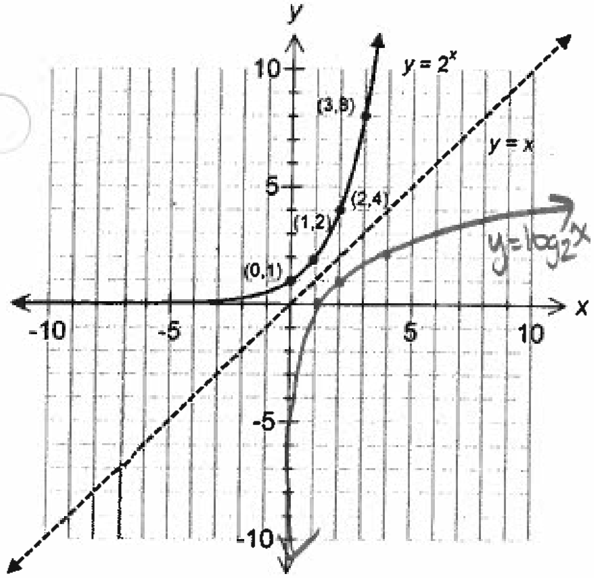
a) $\log_4(x+2) + \log_4(x-4) = 2$

c) $\log_b(4x-3) - \log_b x = \log_b 10$

omit

b) $\log_4(x+3) - \log_4 x = \log_4 7$

d) $\log_2(x+7) = 3 - \log_2(x+5)$



$x = 2^y$	
$y = 2^x$	$y = \log_2 x$
Exponential Form	**Log Form**
Domain: $x \in \mathbb{R}$	Domain: $x > 0$
Range: $y > 0$	Range: $y \in \mathbb{R}$
x-intercept: none	x-intercept: $(1,0)$
y-intercept: $(0,1)$	y-intercept: none
Asymptote: $y = 0$	Asymptote: $x = 0$

Examples:

1. State which of the following are exponential. (variable in exponent)

a) $y = x^7$ ✗

b) $y = 0.5^x$ ✓

c) $y = 4^x$ ✓

d) $y = x^{\frac{1}{3}}$ ✗

e) $y = (-3)^x$ ✓

f) $y = \dfrac{3^{x-2}}{2}$ ✓

g) $y = \dfrac{1}{3x}$ ✗

2. Determine the inverse of each: Switch $x + y$

a) $y = 3^x$

$x = 3^y$

$\boxed{y = \log_3 x}$

b) $y = \log_4 x$

$x = \log_4 y$

$\boxed{y = 4^x}$

c) $x = 20^y$

$\boxed{y = 20^x}$

3. State the transformations, asymptotes and mapping notation for each, compared to $y = \log_3 x$ and $y = 3^x$

Asymptote

a) $y = -2\log_3(x-3)+5$

- VR
- VS of 2
- HT 3 right
- VT 5 up

$x - 3 = 0$

$\boxed{x = 3}$

$(x, y) \to (x+3, -2y+5)$

Asymptote

b) $y = \log_3\left(\frac{1}{3}x + 6\right)$

$\boxed{x = -18}$

$(x, y) \rightarrow (3x - 18, y)$

$y = \log_3\left[\frac{1}{3}(x+18)\right]$

· HS of 3

· HT 18 left

c) $y = 2(3)^{x-4}$

$y = 0$

$(x, y) \rightarrow (x + 4, 2y)$

· VS of 2

· HT 4 right

d) $y = -\frac{1}{2}(3)^{\frac{1}{5}x} - 5$

$y = -5$

$(x, y) \rightarrow \left(5x, -\frac{y}{2} - 5\right)$

· VR

· VS of 1/2

· HS of 5

· VT 5 down

~~4.~~ Describe the transformation which maps the graph of $y = \log_9 x$ to the graph of the following:

increasing function

omit

a) $\log_{\frac{1}{9}} x$

decreasing function

b) $\log_{81} x$

increasing function (slower)

5. Write the equation of the transformed logarithmic function for $y = \log_5 x$ for each of the following:

a) the vertical stretch about the x-axis by a factor of $\frac{1}{2}$ and a horizontal stretch about the y-axis by a factor of $\frac{2}{3}$ and reflected in the x-axis and a horizontal translation of 3 units to the left.

$y = -\frac{1}{2} f\left[\frac{3}{2}(x+3)\right]$

b) A vertical stretch by a factor of 3 about the x-axis, a horizontal stretch by a factor of $\frac{1}{4}$ about the y-axis, reflected on the y-axis, a horizontal translation 1 to the right and a vertical translation of 5 units down.

$y = 3f[-4(x-1)] - 5$

6. The graph of $y = f(x) = b^x$, where b > 1, is translated such that the equation of the new graph is expressed as $y - 2 = f(x-1)$. Identify the range and the y-intercept of the new function.

- HT 1 right
- VT 2 up

$y > 2$

$(0,1) \rightarrow (0+1, 1+2) \rightarrow \boxed{(1,3)}$

7. The equation of the asymptote of $y = 2^{x+1} \underline{+3}$.

$y = 3$

8. State the domain, range, x and y intercepts and asymptotes of $y = \log_2(x+3) - 2$

D: $x > -3$

R: $y \in \mathbb{R}$

9. Identify the equation of the asymptote of $y = \log_2(3x+1) + 1$

$3x + 1 = 0$

$3x = -1$

$\boxed{x = -1/3}$

- For all other word problems, dealing with Growth or Decay, the following formula will be used

$$y = ab^{\frac{t}{p}}$$

y – the final amount

a – is the initial amount

b – is the multiplication factor: 2 for doubles, ½ for half-life, 3 for triples

t – time, must be in the same units as p

p – the period of time for doubling, half-life, or tripling.

Examples:

1. The population of a city was 173 500 on January 1, 1988 and it was 294 000 on January 1, 2002. If the growth of the city can be modeled as an exponential function, then find the average annual growth rate of the city, expressed to the nearest tenth of a percent. $P=1$

$$\frac{294\,000}{173500} = \frac{173\,500(b)^{14/1}}{173500}$$

$$\left(\frac{588}{347}\right)^{\frac{1}{14}} = (b^{14})^{\frac{1}{14}} \qquad b = 1.03839 \Rightarrow 3.8\% \text{ growth}$$

2. A sports car depreciates at a rate of 14% per year ($p=1$) and was bought for \$60 000. How long will it take to depreciate to \$18 000? 86% value remains

$$18000 = 60000(0.86)^x$$

$$0.3 = 0.86^x$$

$$x = \log_{0.86} 0.3 = 7.98 \Rightarrow \boxed{8 \text{ years}}$$

3. Write an exponential expression that will determine the value, V, of the investment at any given time, t, in years.

a) \$3000 is invested at 5.2% per year compounded semi-annually

$$V = 3000\left(1+\frac{0.052}{2}\right)^{2x} \qquad V = 3000(1.026)^{2x}$$

b) \$2500 is invested at 4% per year compounded quarterly

$$V = 2500\left(1+\frac{0.04}{4}\right)^{4x} \qquad V = 2500(1.01)^{4x}$$

c) \$8000 is invested at 6% per annum compounded monthly

$$V = 8000\left(1+\frac{0.06}{12}\right)^{12x} \qquad V = 8000(1.005)^{12x}$$

4. A student borrowed \$6000. Interest is charged at 5% per year compounded semi-annually. The loan is paid off in one final payment of \$6958. What is the length of the loan?

$$6958 = 6000 \cdot \left(1+\frac{0.05}{2}\right)^{2x}$$

$$\frac{3479}{3000} = 1.025^{2x}$$

$$\frac{2x}{2} = \frac{\log_{1.025}\left(\frac{3479}{3000}\right)}{2} \qquad x = 2.99 \Rightarrow 3 \text{ years}$$

5. Jamie borrows \$6000 from the bank at a rate of 8% per annum compounded monthly. How much would he owe at the end of the one month, if he does not make his first payment?

$y = 6000\left(1+\frac{0.08}{12}\right)^{12x}$

$y = 6000(1.00\overline{6})^{12(1)} = \boxed{\$6498.00}$

6. The population of rabbits in a park is increasing by 70% every 6 months. Presently there are 200 rabbits in the park. $p=6$

a) Write an exponential function that represents this scenario. Use P to represent the rabbit population and t to represent the time in months.

$P = 200(1.7)^{t/6}$

b) How many rabbits will there be in 15 months?

$P = 200(1.7)^{15/6} = \boxed{\approx 753 \text{ rabbits}}$

7. A sample of water contains 200 grams of pollutants. Each time the sample is passed through a filter, 20% of its pollutants are removed. 80% remains

a) Write a function that relates the amount of pollutant, P, that remains in the same to the number of times, t, the sample is filtered.

$P = 200(0.80)^t$

b) Determine an expression that gives the amount of pollutants still in the water after it passes through 5 filters. How many grams are there after 5 filters, rounded to the tenth of a gram?

$200(0.80)^5 = \boxed{65.5g}$

8. A colony of 100 insects triples its population every 5 days. How long will it take for the population to grow to 5000? $P=5$

$5000 = 100(3)^{x/5}$

$50 = 3^{x/5} \rightarrow \log_3 50 = \frac{x}{5}$

$x = 5 \cdot \log_3 50 \approx 17.8 \Rightarrow \boxed{18 \text{ days}}$

9. A type of bacterium doubles each hour.

a) If there are 4 bacteria in a sample, write an exponential function that models the sample's growth over time.

$y = 4(2)^{x/1}$

b) Use your equation to determine the time it takes for the sample to become 4096 bacteria.

$4096 = 4(2)^x$

$1024 = 2^x$

$x = \log_2 1024 = \boxed{10 \text{ hours}}$

10. The half-life of radioactive 134 is 8.1 days. Determine the number of days it took for there to be only 2% remaining?

P

$2 = 100\left(\tfrac{1}{2}\right)^{x/8.1}$

$0.02 = \left(\tfrac{1}{2}\right)^{x/8.1}$

$\to \frac{x}{8.1} = \log_{1/2} 0.02 \quad \times 8.1$

$x = 45.7 \Rightarrow \boxed{46\text{ days}}$

11. The sound intensity, B, in decibels, is defined as $B = 10\log\frac{I}{I_0}$, where I is the intensity of the sound. A fine can occur when a motorcycle is idling at 20 times as intense as the sound of an automobile. If the level of an automobile is 80 dB, at what decibel level can a fine occur to a motorcycle operator?

$80 = 10\log(I)$

$8 = \log I$

$I = 10^8$

$\to B = 10\log(20I)$

$\boxed{B = 93\text{dB}}$

12. A sound is 1000 times more intense than a sound you can just hear. What is the measure of its loudness in decibels?

$B = 10\log(1000)$

$B = \boxed{30\text{dB}}$

13. How many more times intense is the sound of normal conversation (60 dB) than the sound of a whisper (30 dB)?

$60 = 10\log(I_1)$

$I = 10^6$

$30 = 10\log(I_2)$

$I = 10^3$

$\frac{I_1}{I_2} = \frac{10^6}{10^3} = 10^3$

= 1000 times louder

14. The pH scale is used to measure acidity or alkalinity of a substance. The formula for pH is $pH = -\log\left[H^+\right]$, where $[H^+]$ is the hydrogen ion concentration.

a) If a solution has a hydrogen ion concentration of 1.21×10^{-2} ml/L, determine the pH value of the solution to the nearest tenth.

$pH = -\log(1.21 \times 10^{-2}) = \boxed{1.9}$

b) If vinegar has a pH of 3.2. Determine the $[H^+]$ in scientific notation to one decimal place.

$\frac{3.2}{-1} = \frac{-1\log[H^+]}{-1}$

$-3.2 = \log_{10}[H^+]$

$\to 10^{-3.2} = [H^+]$

$\boxed{[H^+] = 6.3 \times 10^{-4}\text{ ml/L}}$

15. Earthquake intensity is given by $I = I_0(10)^m$, where I_0 is the reference intensity and m is the magnitude. An earthquake with magnitude 6.8 is followed by an aftershock with magnitude 5.2. How many time more intense was the earthquake than its aftershock?

$\frac{I}{I_0} = \frac{10^{6.8}}{10^{5.2}} =$ 40 times more intense

same equations

16. The formula for the Ritcher magnitude Scale of an earthquake is $M = \log\frac{I}{I_0}$, where I is the magnitude of the largest earthquake and I_0 is the magnitude of the smallest earthquake. How much more intense is an earthquake with a magnitude of 6.9 vs a 3.9?

$\frac{10^{6.9}}{10^{3.9}} = 10^3 =$ 1000 times

17. Earthquake intensity is given by $I = I_0(10)^m$, where I_0 is the reference intensity and m is the magnitude. A particular major earthquake of magnitude is 120 times as intense as a particular minor earthquake. The magnitude, to the nearest tenth, of the minor earthquake is 2.1.

$\frac{I}{I_0} = 10^m$ $\quad 120 = 10^m$

$m = \log 120 = 2.1$

Practice Test

1. The solution for the equation $9^{2x} = \left(\frac{1}{3}\right)^{x+6}$ is

A. 6
B. 2
C. $-\frac{4}{3}$
D. $-\frac{6}{5}$

$3^{2(2x)} = 3^{-1(x+6)}$

$4x = -x-6$

$5x = -6$

$x = -6/5$

2. In logarithmic form, the solution of $5^{x-3} = 50$ is

A. $\log_{10}(50) - \log_{10}(5) + 3$
B. $\log_{10}(10) + 3$
C. $\frac{\log_{10}(50)}{\log_{10}(5)} + 3$
D. $\frac{\log_{10}(53)}{\log_{10}(5)}$

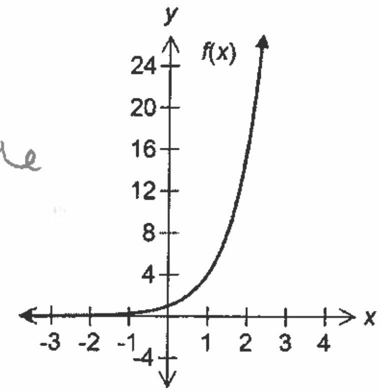
$\log 5^{x-3} = \log 50$

$(x-3)\log 5 = \log 50$

$x\log 5 - 3\log 5 = \log 50$

$\frac{x\log 5}{\log 5} = \frac{\log 50 + 3\log 5}{\log 5}$ $\quad x = \frac{\log 50}{\log 5} + 3$

3. The partial graph of the exponential function $f(x)=4^x$ is shown to the right.



The domain of the inverse function, $f^{-1}(x)$ is the range of the original $y>0$ $\Downarrow$ $x>0$

A. $x>0, x\in\Re$

B. $x<0, x\in\Re$

C. $x\geq 0, x\in\Re$

D. $x\in\Re$

4. The value of x in the equation $3^{x-1}=26$ is

A. 1.97

B. 2.97

C. 3.97

D. 0.96

$\log_3 26 = x-1$

$x=\log_3 26+1 = \boxed{3.97}$

5. If $\log_8(x+5)-\log_8(x-2)=1$, then the value of x is

A. -3

B. 1

C. 3

D. 8

$\log_8\left(\frac{x+5}{x-2}\right)=1$

$8^1=\frac{x+5}{x-2}$

$8(x-2)=x+5$

$8x-16=x+5$

$7x=21$

$\boxed{x=3}$

6. $\frac{\log(r)}{t}+\frac{\log(d)}{t}$ is equal to

$\frac{1}{t}\log r + \frac{1}{t}\log d$

$\log r^{1/t} + \log d^{1/t}$

$\log(r^{1/t}\cdot d^{1/t})$

$\log((rd)^{1/t}) = \log(\sqrt[t]{rd})$

A. $\log(rd)^t$

B. $\log(\sqrt[t]{rd})$

C. $\log(\sqrt[t]{r+d})$

D. $\log(r+d)^t$

7. The expression $\log_a(a^4b)-\log_a(ab)$ is equivalent to

$\log_a a^4 + \log_a b - (\log_a a + \log_a b)$

$4\log_a a + \cancel{\log_a b} - \log_a a - \cancel{\log_a b}$

$4(1) - 1 = 3$

A. 3

B. 4

C. 3a

D. a^3

8. A student graphed the following equations.

Equation I $y_1 = \log_{10} x$

Equation II $y_2 = 5^{x-3}$

Equation III $y_3 = x-3$

Equation IV $y_4 = x$

The student could estimate the solution to the equation $\log_5 x = x-3$ by using the graphs of equations

$\underbrace{5^{x-3}}_{y_1} = \underbrace{x}_{y_2}$

A. I and II

B. I and III

C. II and III

D. II and IV

9. The growth of bacteria can be written $N(t)=N_0\left(2^{\frac{t}{40}}\right)$, where $N(t)$ = the final number of bacteria, N_0 = the initial number of bacteria, and t = time in minutes. The logarithmic expression for time (t) it takes for the number of bacteria to increase from 50 000 to 700 000 is **(SE)**

$700000 = 50000(2)^{t/40}$

$14 = 2^{t/40}$

$\frac{t}{40} = \log_2 14$

$t = 40\log_2 14$

A. $\frac{40}{\log_2(14)}$

B. $\frac{\log_2(14)}{40}$

C. $40\log_2(14)$

D. $\log_2\left(2^{\frac{t}{40}}\right)$

10. The value of i in the compound interest equation $5 = (1+i)^6$ is

A. $i = \sqrt[6]{4}$

B. $i = \sqrt[6]{5} - 1$

C. $i = \dfrac{\log 5}{\log 6} - 1$

D. $i = \dfrac{\log 6}{\log 5} - 1$

$\sqrt[6]{5} = \sqrt[6]{(1+i)^6}$

$\sqrt[6]{5} = 1+i$

$i = \sqrt[6]{5} - 1$

11. The gain, G, measured in decibels, of an amplifier is defined by the equation $G = 10\log\left(\dfrac{P}{P_1}\right)$ where P is the output power of the amplifier, in watts, and P1 is the input power of the amplifier, in watts. If the gain of the amplifier is 23 decibels, then the ratio $\dfrac{P}{P_1}$ is

A. $10\log(23)$

B. $\log(2.3)$

C. 10^{23}

D. $10^{2.3}$

$23 = 10\log\left(\frac{P}{P_1}\right)$

$2.3 = \log_{10}\left(\frac{P}{P_1}\right)$

$\frac{P}{P_1} = 10^{2.3}$

12. The equation that defines the decibel level for any sound is $L = 10\log_{10}\left(\dfrac{I}{I_0}\right)$, where L is the loudness in decibels, I is the intensity of sound being measured, and I_0 is the intensity of sound at the threshold of hearing. Given that normal conversation is 1 000 000 times intense as I_0, then the loudness of normal conversation is

A. 5 decibels

B. 6 decibels

C. 16 decibels

D. 60 decibels

$L = 10\log_{10}(1\,000\,000)$

$= \boxed{60\text{dB}}$

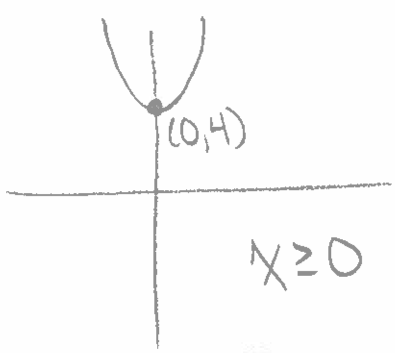
13. A restriction on the domain of $f(x) = x^2 + 4$ such that its inverse is also a function, could be **(SE)**

A. $x \geq -4$

B. $x \geq 0$

C. $x \leq 2$

D. $x \leq 4$



$x \geq 0$ or $x \leq 0$

14. The graph of $y = 3^x$ is reflected in the line y = x. The equation of the transformed graph is

A. $y = \log_3 x$
B. $y = 3\log x$
C. $y = 3^{-x}$
D. $y = -3^x$

$x = 3^y$

$y = \log_3 x$

15. The equation $m\log_p n = q$ can be written in exponential form as

A. $p^m = nq$
B. $p^{\frac{q}{m}} = n$
C. $p^q = mn$
D. $p^{qm} = n$

$\log_p n = \frac{q}{m}$

$p^{q/m} = n$

16. The expression $\left(3^{\log x}\right)\left(3^{\log x}\right)$ is equivalent to

A. $3^{\log x^2}$
B. $9^{\log x^2}$
C. $3^{(\log x)^2}$
D. $9^{(\log x)^2}$

$3^{(\log x + \log x)} = 3^{\log(x \cdot x)} = 3^{\log x^2}$

17. Written as a single logarithm, $2\log x - \frac{\log z}{2} + 3\log y$ is

A. $\log\left(\frac{x^2y^3}{\sqrt{z}}\right)$
B. $3\log\left(\frac{xy}{z}\right)$
C. $\log\left(\frac{x^2}{y^3\sqrt{z}}\right)$
D. $\log\left(x^2 - \sqrt{z} + y^3\right)$

$= \log x^2 - \log z^{1/2} + \log y^3$

$= \log\left(\frac{x^2y^3}{\sqrt{z}}\right)$

Use the following information to answer the next question.

A student's work to simplify a logarithmic expression is shown below, where a > 1.

STEP 1 $2\log_a x^4 - 3\log_a x^2 + 4\log_a x^3$

STEP 2 $\log_a x^8 - \log_a x^6 + \log_a x^{12}$ ✓

STEP 3 $\log_a \left(\dfrac{x^8}{x^6 \times x^{12}} \right)$ ✗

STEP 4 $\log_a \left(\dfrac{x^8}{x^{18}} \right)$

STEP 5 $\log_a x^{10}$

18. The student made his **first** error when going from:

   A. Step 1 to Step 2
   B. Step 2 to Step 3
   C. Step 3 to Step 4
   D. Step 4 to Step 5

19. The equation of the asymptote for the graph of $y = \log_b (x-3) + 2$, where $b > 0$ and $b \neq 1$, is

   A. $x = 2$
   B. $x = -2$
   C. $x = 3$
   D. $x = -3$

$x-3=0$
$x=3$

20. *The graph of $y = \log x$ is transformed into the graph of $y - 4 = \log(x+5)$ by a translation of 5 units ____i____ and 4 units ____ii____.*

+4 +4

left up

$y = \log(x+5) + 4$

The statement above is completed by the information in row

Row	*i*	*ii*
A.	right	up
B.	left	up
C.	right	down
D.	left	down

21. For the graph of $y = \log_b(3x+12)$, where 0 < b < 1, the domain is

A. $x > -4$
B. $x > 4$
C. $x > -12$
D. $x > 12$

$3x+12>0$
$3x>-12$
$x>-4$

22. The y-intercepts on the graph of $f(x) = a^{(x+1)} + b$ is

A. a
B. b
C. $1+b$
D. $a+b$

$y = a^{(0+1)} + b$
$y = a+b$

Numerical Response

1. If $\log_n(a) = 3.6$ and $\log_n(b) = 2.7$ then $\log_n(ab)$ correct to the nearest tenth is ____.

$= \log_n a + \log_n b$
$= 3.6 + 2.7 = \boxed{6.3}$

2. If ab = 24, then to the nearest hundredth, the value of $2\log_{10} a + 2\log_{10} b$, where $a, b > 0$, is ________.

$= 2(\log a + \log b)$
$= 2\log(ab) = 2\log 24 = \boxed{2.76}$

3. The inverse of $f(x) = 2x-3$ is written in the form $f^{-1}(x) = \frac{a}{b}x + \frac{c}{d}$, the values of a, b, c, d are 1, 2, 3, 2.

$x = 2y-3$
$x+3 = 2y$
$y = \frac{1x}{2} + \frac{3}{2}$

4. The value of $\log_5 625 + 3\log_7 49 + \log_2 \frac{1}{16} + \log_b b + \log_a 1$ is ________.

$\cancel{4} + 3(2) + \cancel{-4} + 1 + 0 = 6+1 = \boxed{7}$

5. Given that $\log_3 a = 6$ and $\log_3 b = 5$, determine the value of $\log_3(9ab^2)$.

$= \log_3 9 + \log_3 a + \log_3 b^2$
$= 2 + 6 + 2(5) = \boxed{18}$

6. Algebraically solve the equation $8^{(3x+4)} = 4^{(x-9)}$.

$2^{3(3x+4)} = 2^{2(x-9)}$
$9x+12 = 2x-18$
$\frac{7x}{7} = \frac{-30}{7}$
$\boxed{x = -4.3}$

$(2x+1)\log 3 = (x-3)\log(1/5)$

$2x\log 3 + \log 3 = x\log(1/5) - 3\log 1/5$

$2x\log 3 - x\log(1/5) = -\log 3 - 3\log 1/5$

7. Solve the equation $3^{(2x+1)} = \left(\frac{1}{5}\right)^{(x-3)}$ algebraically. Round to the nearest hundredth, if necessary.

omit for now

$x = \dfrac{-\log 3 - 3\log(1/5)}{2\log 3 - \log(1/5)} = \boxed{0.98}$

8. Solve algebraically $\log_7(x+1)+\log_7(x-5)=1$. What is the extraneous root? **(SE)**

omit for now

$\log_7[(x+1)(x-5)] = 1$

$7^1 = x^2 - 4x - 5$

$0 = x^2 - 4x - 12 \rightarrow 0 = (x-6)(x+2)$

$\boxed{x=6}$ $\cancel{x=-2}$ extraneous

Use the following information to answer the next question.

Earthquake intensity is given by $I = I_0 \times 10^M$, where I_0 is the reference intensity and M is the magnitude. An earthquake measuring 5.3 on the Richter scale is 125 times more intense than a second earthquake.

9. Determine, to the nearest tenth, the Richter scale measure of the second earthquake.

$125 = \dfrac{10^{5.3}}{10^R}$ $\quad 10^R = \dfrac{10^{5.3}}{125}$ $\quad R = 3.2$

10. The population of a particular town on July 1, 2011 was 20 000. If the population decreases at an average annual rate of 1.4%, how long will it take for the population to reach 15 300? **(SE)**

98.6% remain

P

$15300 = 20\,000(0.986)^x$

$0.765 = 0.986^x$

$x = \log_{0.986} 0.765 = 18.9 \Rightarrow \boxed{19 \text{ years}}$

Exponents and Logarithm Practice Test Answers:

MULTIPLE CHOICE:

1. D
2. C
3. A
4. C
5. C
6. B
7. A
8. D
9. C
10. B
11. D
12. D
13. B
14. A
15. B
16. A
17. A
18. B
19. C
20. B
21. A
22. D

NUMERICAL RESPONSE:

1. 6.3
2. 2.76
3. 1, 2, 3, 2
4. 7
5. 18
6. 4.3 negative *
7. ~~1.0~~ 0.98
8. 2
9. 3.2
10. 19